

Lecture TWO

Analog Computing Technique

REAL-TIME SYSTEMS

Basic Op-Amp

The basic circuit connection using an op-amp is shown in Fig. 1. The circuit shown provides operation as a constant-gain multiplier. An input signal, V_i , is applied through resistor R_i to the minus input. The output is then connected back to the same minus input through resistor R_f . The plus input is connected to ground. Since the signal V_i is essentially applied to the minus input, the resulting output is opposite in phase to the input signal. Figure 2a shows the op-amp replaced by its ac equivalent circuit. If we use the ideal op-amp equivalent circuit, replacing R_i by an infinite resistance and R_o by zero resistance, the ac equivalent circuit is that shown in Fig. 2b. The circuit is then redrawn, as shown in Fig. 2c, from which circuit analysis carried out.

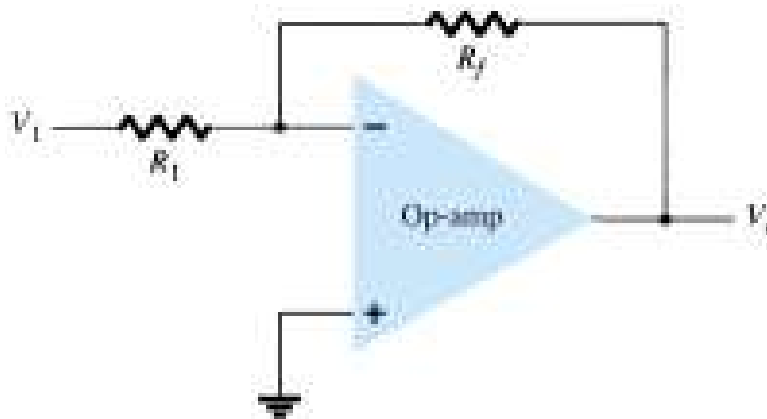


Fig. 1

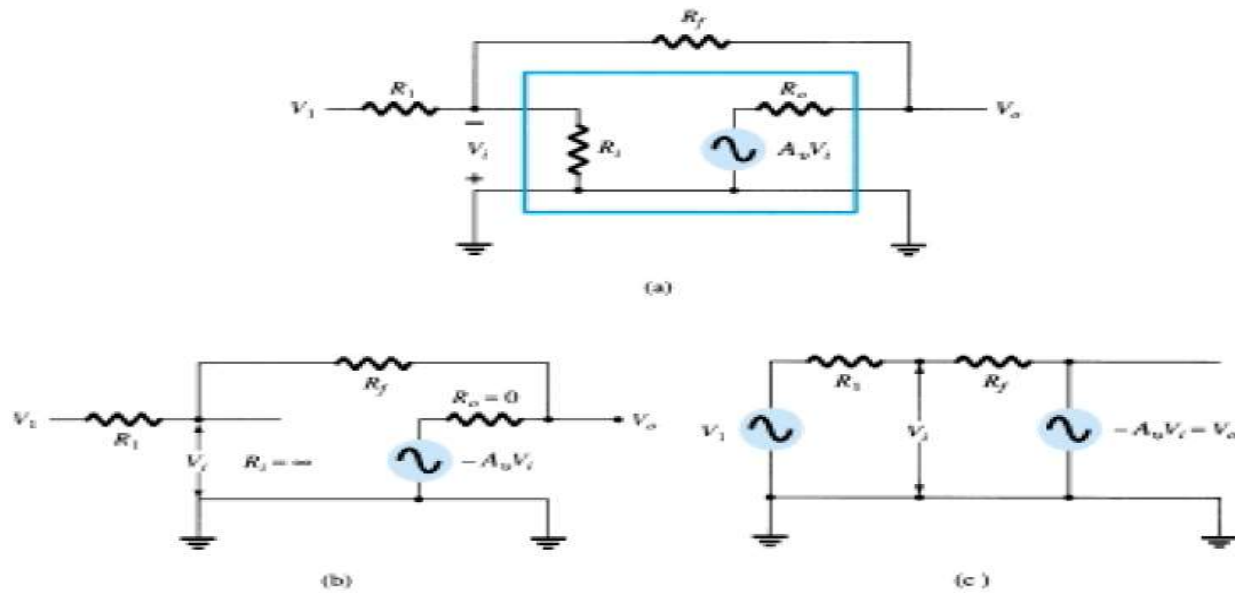


Fig.2

$$V_{i1} = \frac{R_f}{R_1 + R_f} V_1$$

For source $-A_v V_i$ only (V_1 set to zero),

$$V_{i2} = \frac{R_1}{R_1 + R_f} (-A_v V_i)$$

The total voltage V_i is then

$$V_i = V_{i1} + V_{i2} = \frac{R_f}{R_1 + R_f} V_1 + \frac{R_1}{R_1 + R_f} (-A_v V_i)$$

which can be solved for V_i as

$$V_i = \frac{R_f}{R_f + (1 + A_v)R_1} V_1$$

If $A_v \gg 1$ and $A_v R_1 \gg R_f$, as is usually true, then

$$V_i = \frac{R_f}{A_v R_1} V_1$$

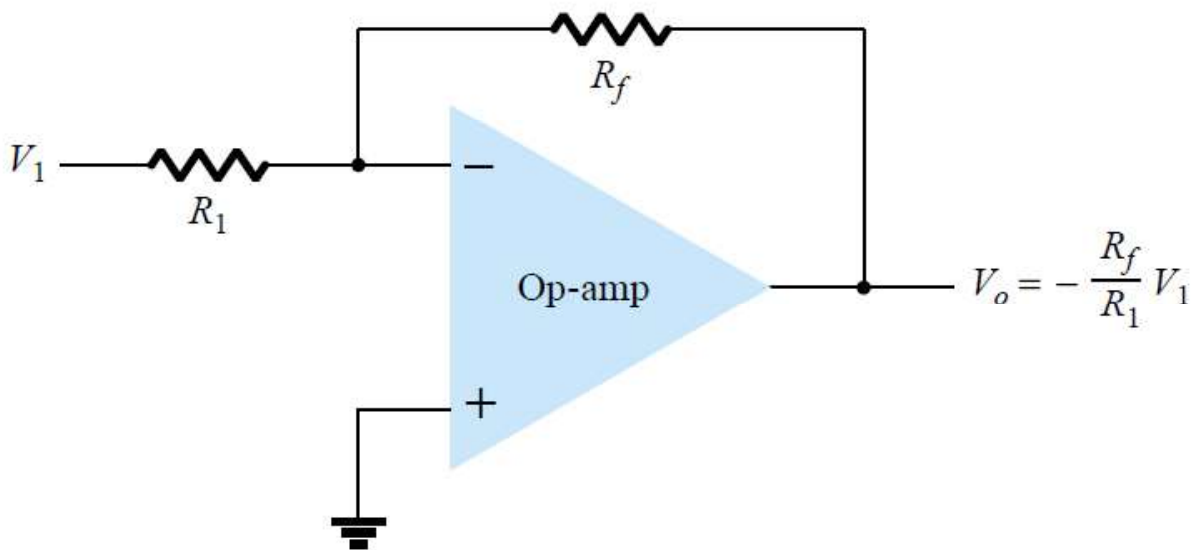
Solving for V_o/V_i , we get

$$\frac{V_o}{V_i} = \frac{-A_v V_i}{V_i} = \frac{-A_v}{V_i} \frac{R_f V_1}{A_v R_1} = -\frac{R_f}{R_1} \frac{V_1}{V_i}$$

Inverting Amplifier

The most widely used constant-gain amplifier circuit is the inverting amplifier, as shown in Fig. 14.15. The output is obtained by multiplying the input by a fixed or constant gain, set by the input resistor (R_1) and feedback resistor (R_f)—this output also being inverted from the input. Using Eq. (14.8) we can write

$$V_o = -\frac{R_f}{R_1} V_1$$

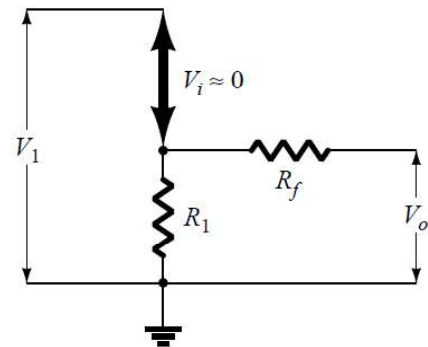
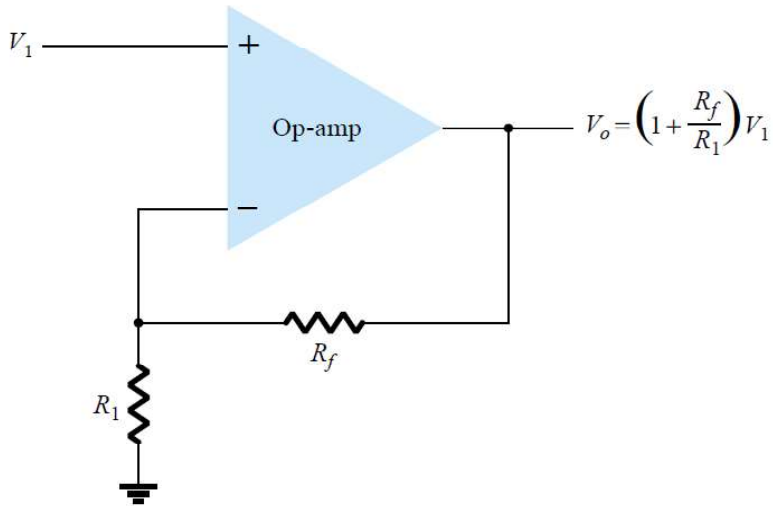


Noninverting Amplifier

The connection of Fig. 14.16a shows an op-amp circuit that works as a noninverting amplifier or constant-gain multiplier. It should be noted that the inverting amplifier connection is more widely used because it has better frequency stability (discussed later). To determine the voltage gain of the circuit, we can use the equivalent representation shown in Fig. 14.16b. Note that the voltage across R_1 is V_1 since $V_i \approx 0$ V. This must be equal to the output voltage, through a voltage divider of R_1 and R_f , so that

$$V_1 = \frac{R_1}{R_1 + R_f} V_o$$

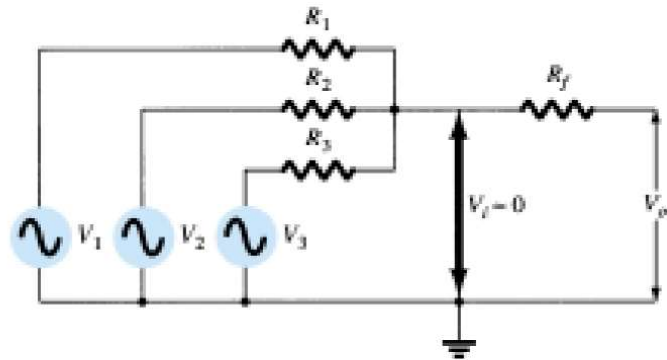
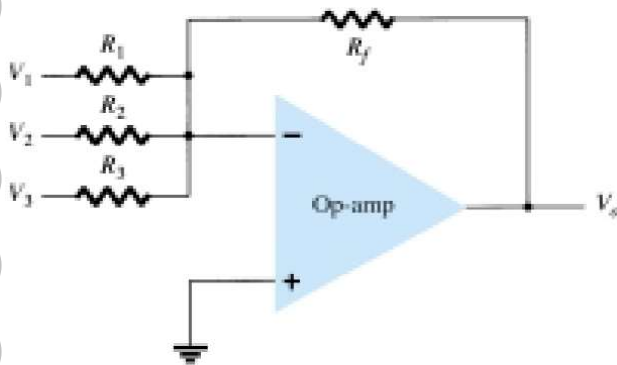
$$\frac{V_o}{V_1} = \frac{R_1 + R_f}{R_1} = 1 + \frac{R_f}{R_1}$$



Summing Amplifier

Probably the most used of the op-amp circuits is the summing amplifier circuit shown in Fig. 14.18a. The circuit shows a three-input summing amplifier circuit, which provides a means of algebraically summing (adding) three voltages, each multiplied by a constant-gain factor. Using the equivalent representation shown in Fig. 14.18b, the output voltage can be expressed in terms of the inputs as

$$V_o = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3\right)$$



Calculate the output voltage of an op-amp summing amplifier for the following sets of voltages and resistors. Use $R_f = 1 \text{ M}\Omega$ in all cases.

EXAMPLE 14.5

- (a) $V_1 = +1 \text{ V}$, $V_2 = +2 \text{ V}$, $V_3 = +3 \text{ V}$, $R_1 = 500 \text{ k}\Omega$, $R_2 = 1 \text{ M}\Omega$, $R_3 = 1 \text{ M}\Omega$.
 (b) $V_1 = -2 \text{ V}$, $V_2 = +3 \text{ V}$, $V_3 = +1 \text{ V}$, $R_1 = 200 \text{ k}\Omega$, $R_2 = 500 \text{ k}\Omega$, $R_3 = 1 \text{ M}\Omega$.

Solution

Using Eq. (14.11):

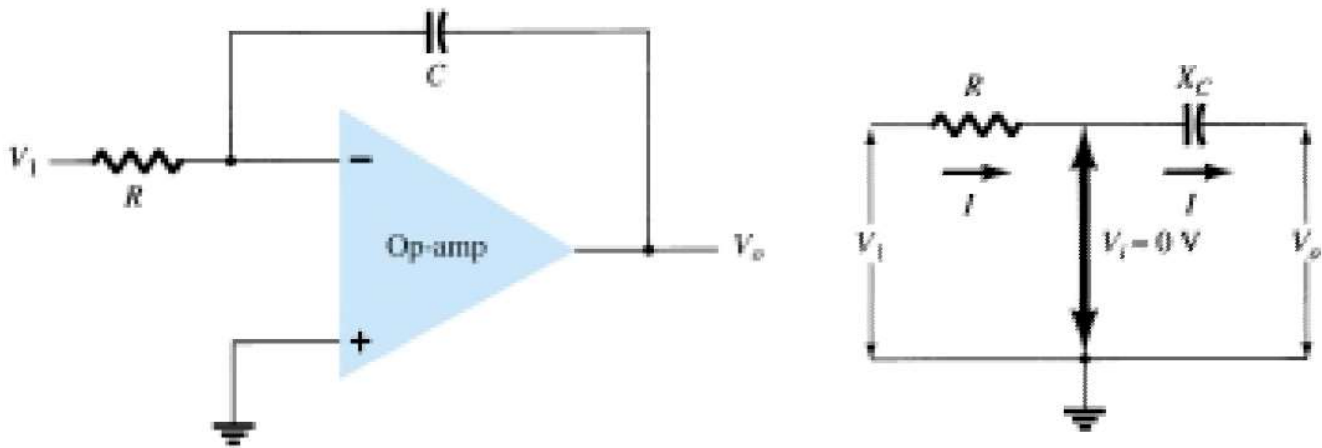
$$\begin{aligned} \text{(a) } V_o &= - \left[\frac{1000 \text{ k}\Omega}{500 \text{ k}\Omega} (+1 \text{ V}) + \frac{1000 \text{ k}\Omega}{1000 \text{ k}\Omega} (+2 \text{ V}) + \frac{1000 \text{ k}\Omega}{1000 \text{ k}\Omega} (+3 \text{ V}) \right] \\ &= - [2(1 \text{ V}) + 1(2 \text{ V}) + 1(3 \text{ V})] = -7 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{(b) } V_o &= - \left[\frac{1000 \text{ k}\Omega}{200 \text{ k}\Omega} (-2 \text{ V}) + \frac{1000 \text{ k}\Omega}{500 \text{ k}\Omega} (+3 \text{ V}) + \frac{1000 \text{ k}\Omega}{1000 \text{ k}\Omega} (+1 \text{ V}) \right] \\ &= - [5(-2 \text{ V}) + 2(3 \text{ V}) + 1(1 \text{ V})] = +3 \text{ V} \end{aligned}$$

Integrator

So far, the input and feedback components have been resistors. If the feedback component used is a capacitor, as shown in Fig. 14.19a, the resulting connection is called an *integrator*. The virtual-ground equivalent circuit (Fig. 14.19b) shows that an expression for the voltage between input and output can be derived in terms of the current I , from input to output. Recall that virtual ground means that we can consider the voltage at the junction of R and X_C to be ground (since $V_i \approx 0$ V) but that no current goes into ground at that point. The capacitive impedance can be expressed as

$$X_C = \frac{1}{j\omega C} = \frac{1}{sC}$$



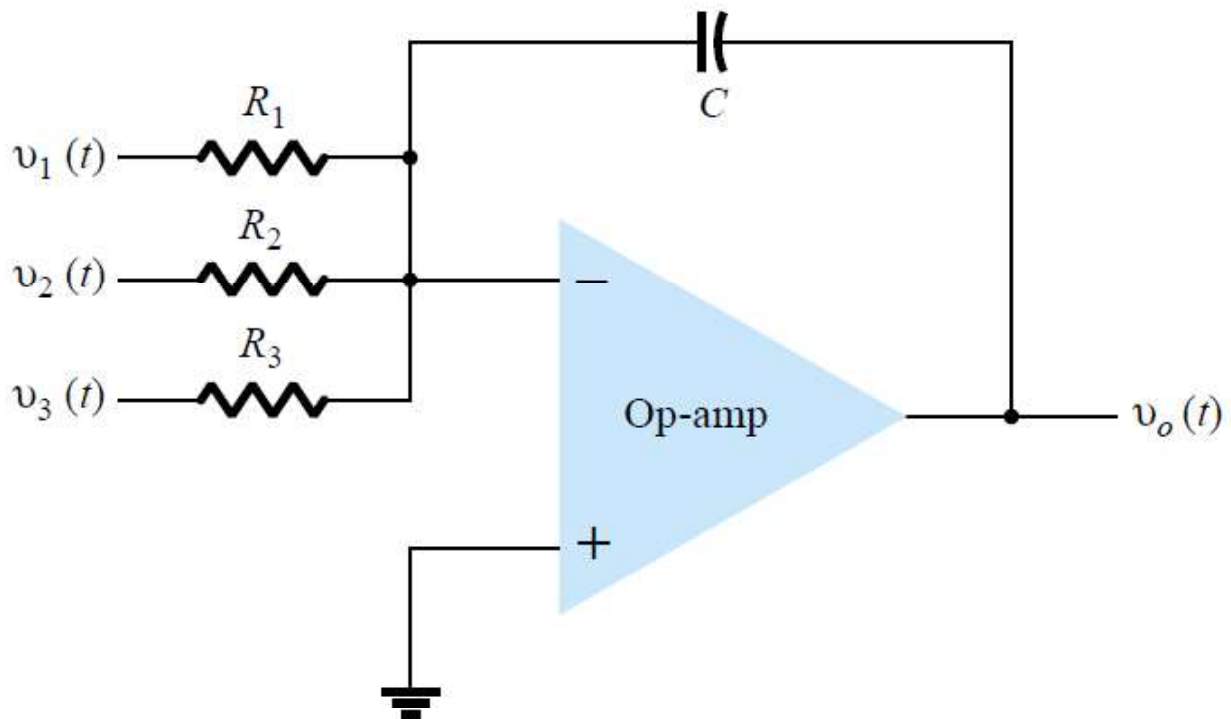
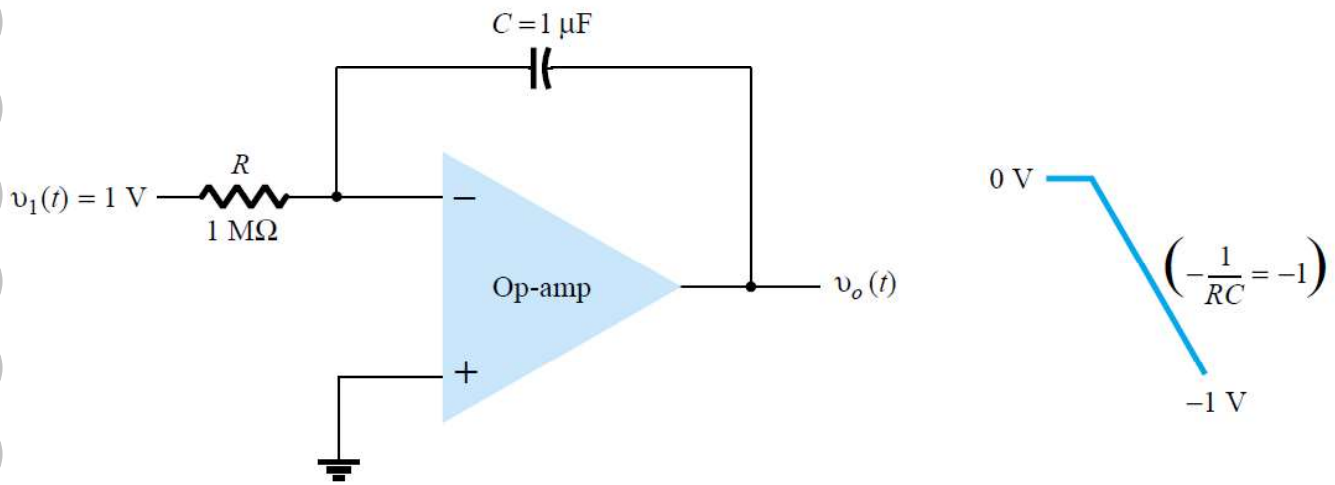
where $s = j\omega$ is in the Laplace notation.* Solving for V_o/V_1 yields

$$I = \frac{V_1}{R} = -\frac{V_o}{X_C} = \frac{-V_o}{1/sC} = -sCV_o$$

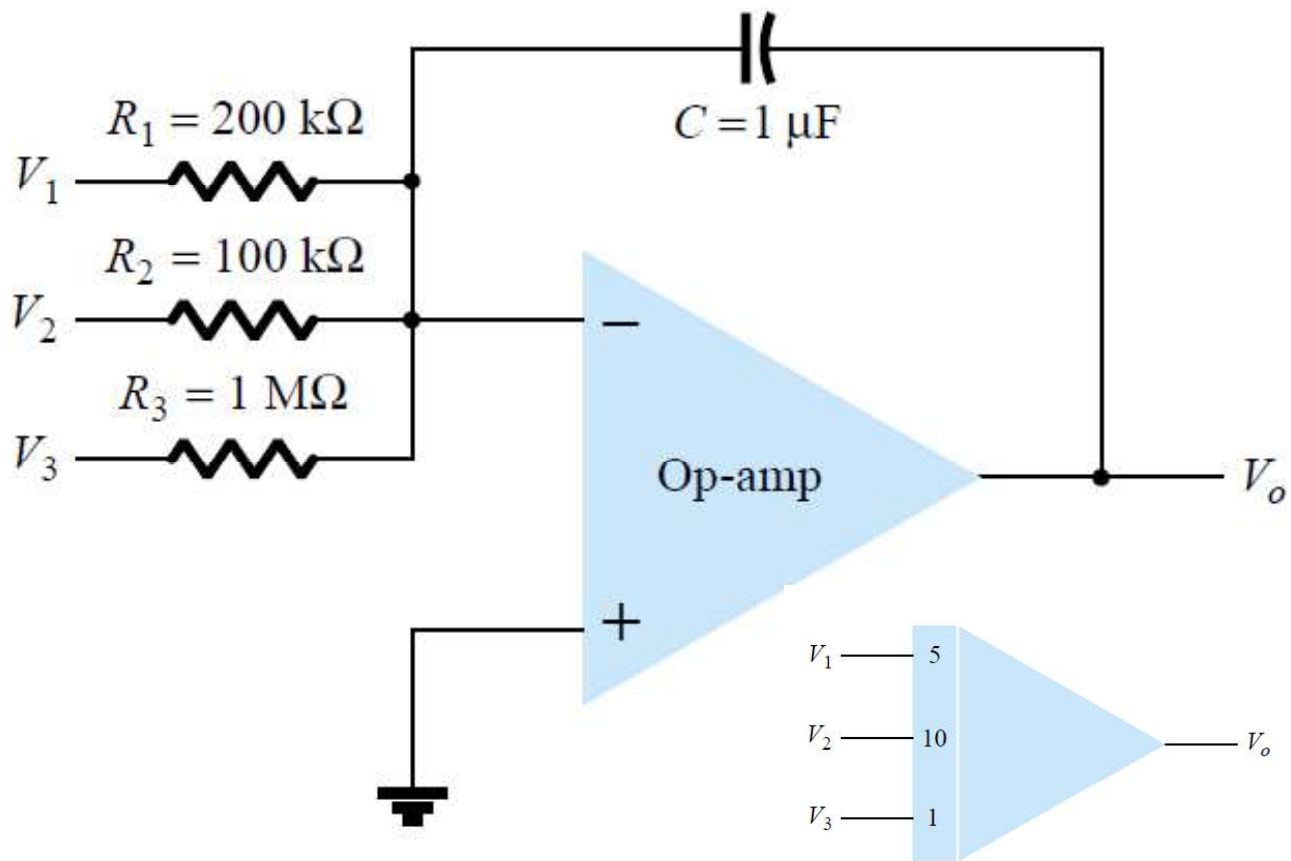
$$\frac{V_o}{V_1} = \frac{-1}{sCR}$$

The expression above can be rewritten in the time domain as

$$v_o(t) = -\frac{1}{RC} \int v_1(t) dt$$

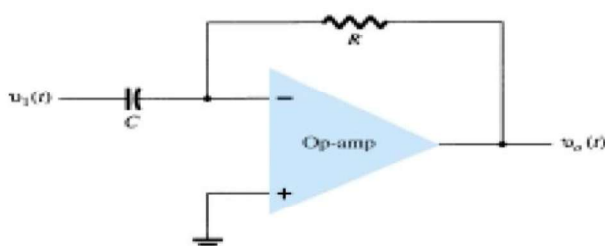


$$v_o(t) = -\left[\frac{1}{R_1 C} \int v_1(t) dt + \frac{1}{R_2 C} \int v_2(t) dt + \frac{1}{R_3 C} \int v_3(t) dt\right]$$



Differentiator

A differentiator circuit is shown in Fig. 14.22. While not as useful as the circuit forms covered above, the differentiator does provide a useful operation, the resulting relation for the circuit being

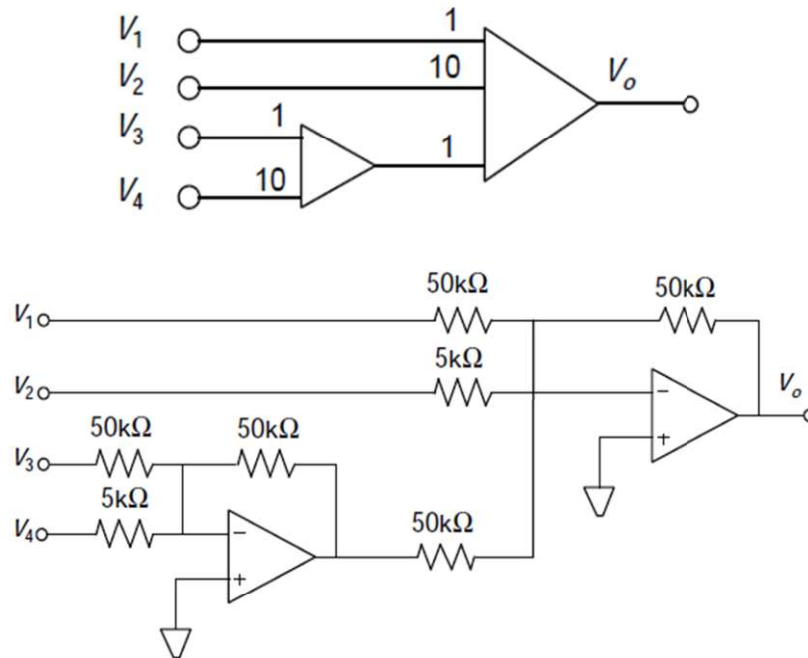


$$v_o(t) = -RC \frac{dv_1(t)}{dt}$$

where the scale factor is $-RC$.

Example : Determine the circuit to produce the output voltage given by

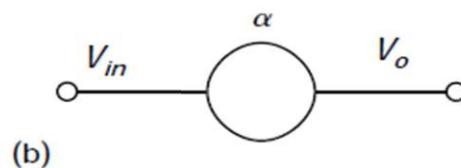
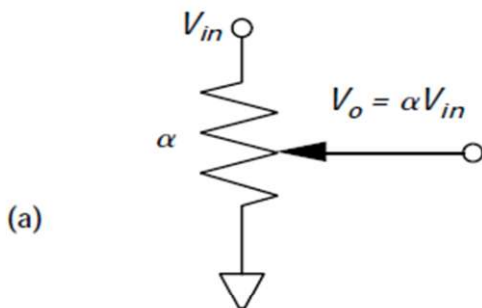
$$V_o = -V_1 - 10V_2 + V_3 + 10V_4.$$



Multiplication by a Constant ; Reference Voltages

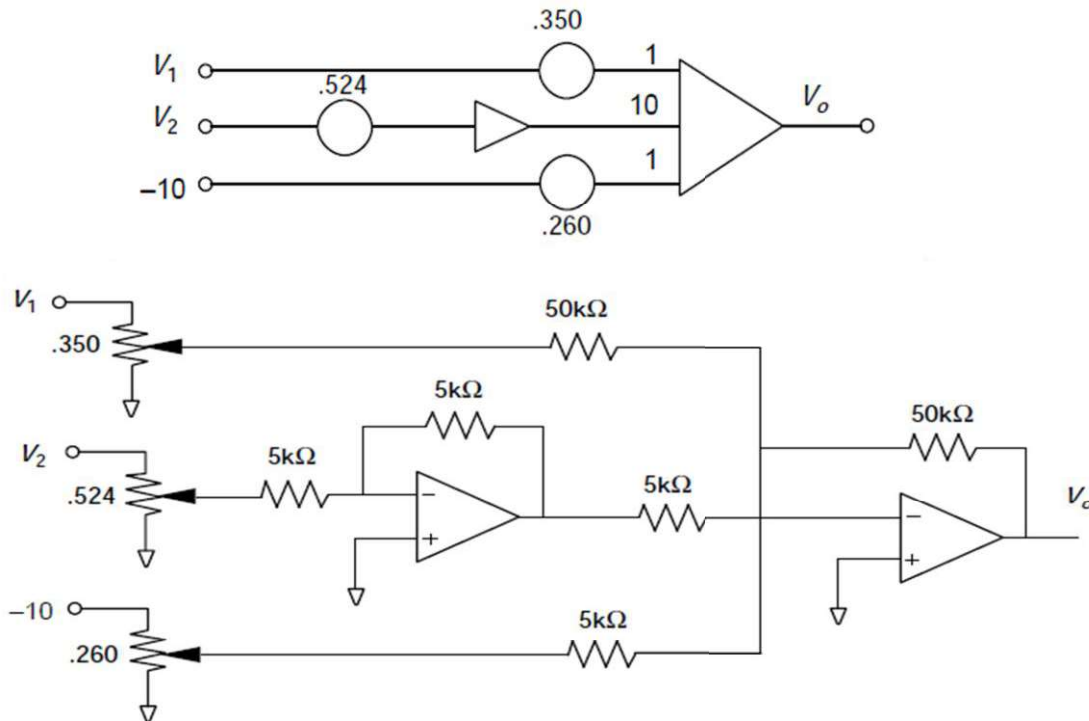
The coefficient potentiometer (pot) is a voltage divider which allows the output voltage to be some fraction of the input voltage. A pot thus has a gain of less than unity. The electrical circuit diagram and analog computer diagram are shown in Figure . The transfer characteristic is given by

$$V_o = \alpha V_{in}, \quad 0 \leq \alpha \leq 1. \quad (1.16)$$



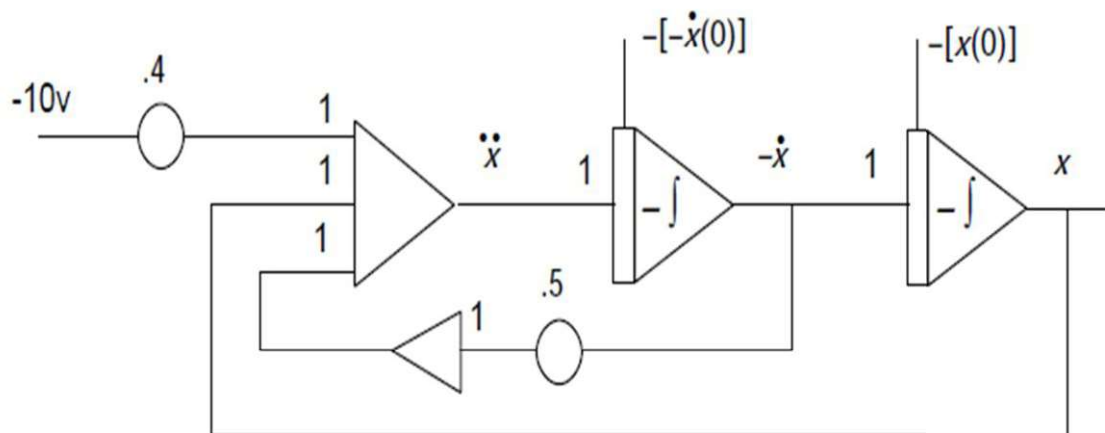
Example : Determine the analog diagram and circuit to implement the

$$V_o = -0.35V_1 + 5.24V_2 + 2.6.$$



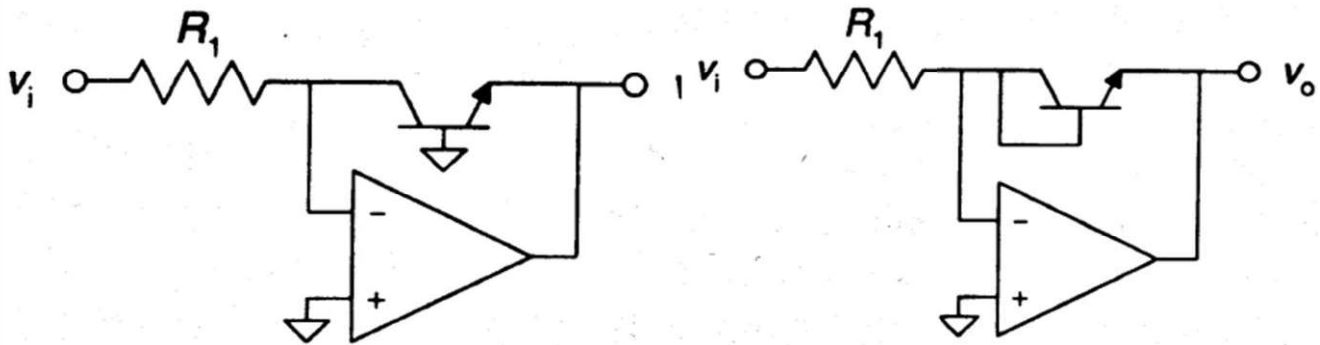
Example : Determine the analog diagram and circuit to implement the equation

$$\frac{d^2x}{dt^2} + 0.5\frac{dx}{dt} + x = 4, \quad x(0) = 0, \dot{x}(0) = 1.$$



Logarithmic circuit

- ❖ There are two basic circuits for logarithmic amplifiers
 - (a) transdiode and
 - (b) diode connected transistor
- ❖ Most logarithmic amplifiers are based on the inherent logarithmic relationship between the collector current, I_c , and the base-emitter voltage, v_{be} , in silicon bipolar transistors.



The input voltage is converted by R_1 into a current, which then flows through the transistor's collector modulating the base-emitter voltage according to the input voltage.

- ❖ The opamp forces the collector voltage to that at the noninverting input, 0 V
- ❖ From Ebers-Moll model the collector current is

$$I_c = I_s (e^{qV_{be}/kT} - 1) = I_s (e^{V_{be}/V_T} - 1) \approx I_s \cdot e^{V_{be}/V_T}$$

- ❖ where I_s is saturation current, q is the charge of the electron 1.6×10^{-19} Coulombs, k is the Boltzman's constant 1.38×10^{-23} Joules, T is absolute temperature, V_T is thermal voltage.

- ❖ For room temperature 300°K

$$I_c = I_s (e^{38.6V_{be}} - 1) \approx I_s \cdot e^{38.6V_{be}}$$

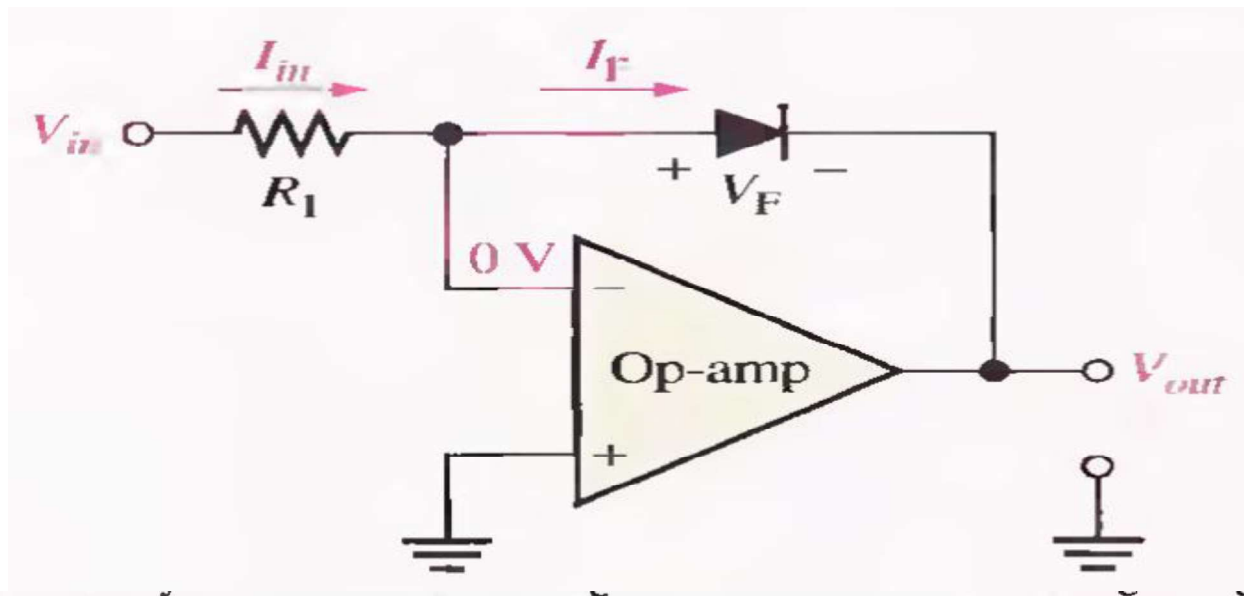
- ❖ The output voltage is therefore

$$V_{out} = -V_{be} = -V_T \ln\left(\frac{i_c}{I_s}\right) = -\frac{V_T}{2.3} \lg\left(\frac{v_i}{R_1 I_s}\right) = \left(-0.0259 \cdot \ln \frac{V_{in}}{R_1 \cdot I_s}\right)$$

- ❖ This equation yields the desired logarithmic relationship over a wide range of currents, but is temperature-sensitive because of V_T and I_S resulting in scale-factor and offset temperature-dependent errors.
- ❖ The system bandwidth is narrower for small signals because emitter resistance increases for small currents.
- ❖ The source impedance of voltage signals applied to the circuit must be small compared to R_1 . Omitting R_1 yields a current-input log amp.
- ❖ Using a $p-n-p$ transistor changes the polarity of input signals acceptable but limits the logarithmic range because of the degraded performance of $p-n-p$ transistors compared to $n-p-n$ transistors

$$v_o = -V_T \ln \left(\frac{v_i}{R_1 I_S} \right)$$

- ❖ The equation for output voltage shows that the scale factor of the basic transdiode log amp depends on temperature because of V_T and
- ❖ that there is also a temperature-dependent offset because of I_S .
- ❖ Temperature compensation must correct both error sources.



$V_{out} = -V_F$ and $I_F = I_{in}$ because there is no current at the inverting input.

$$V_{out} = -V_F$$

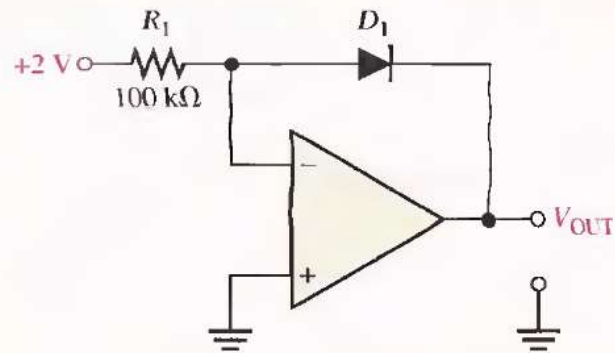
$$I_F = I_{in} = \frac{V_{in}}{R_1}$$

Substituting into the formula for V_F ,

$$V_{out} = -\left(\frac{kT}{q}\right) \ln\left(\frac{V_{in}}{I_R R_1}\right)$$

The term kT/q is a constant equal to approximately 25 mV at 25°C. Therefore, voltage can be expressed as

$$V_{out} \cong -(0.025 \text{ V}) \ln\left(\frac{V_{in}}{I_R R_1}\right)$$



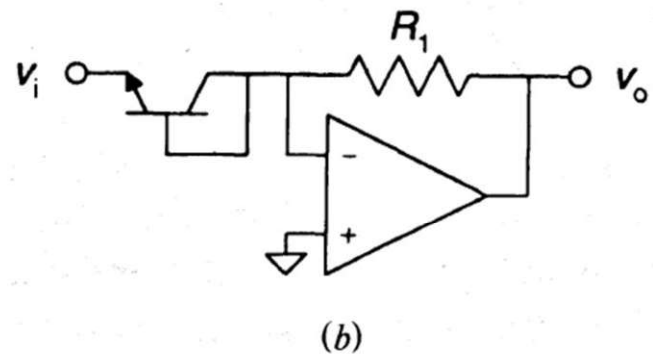
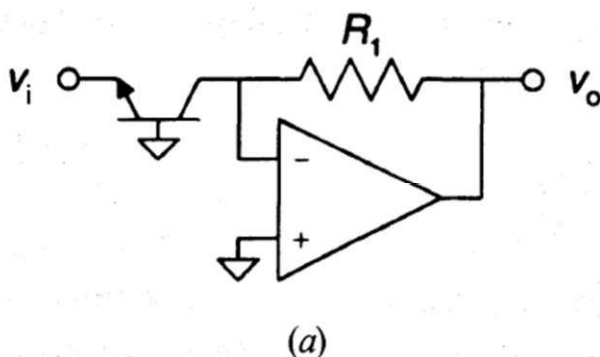
Solution The input voltage and the resistor value are given in Figure 14–28.

$$V_{OUT} = -(0.025 \text{ V}) \ln \left(\frac{V_{in}}{I_R R_1} \right) = -(0.025 \text{ V}) \ln \left(\frac{2 \text{ V}}{50 \text{ nA} \times 100 \text{ k}\Omega} \right)$$

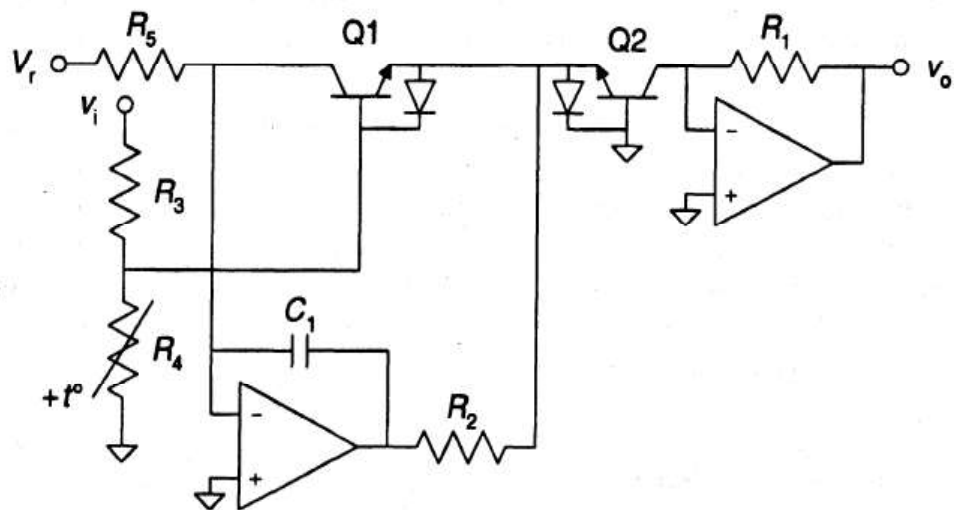
$$= -(0.025 \text{ V}) \ln(400) = -(0.025 \text{ V})(5.99) = -0.150 \text{ V}$$

exponential circuit

- ❖ An exponential or antilogarithmic amplifier (antilog amp), performs the function inverse to that of log amps:
- ❖ its output voltage is proportional to a base (10, e) elevated to the ratio between two voltages.
- ❖ Antilog amps are used together with log amps to perform analog computation.
- ❖ Similar to Log Amps there are two basic circuits for logarithmic amplifiers
 - (a) transdiode and
 - (b) diode connected transistor



- ❖ Interchanging the position of resistor and transistor in a log amp yields a basic antilog amp.
- ❖ The base-collector voltage is kept at 0 V, so that collector current is given by
- ❖
$$i_c \approx I_s \cdot \exp(v_{BE} / V_T)$$
- ❖ and for negative input voltages we have:
$$v_o = i_c R_1 = I_s R_1 \exp(-v_i / V_T)$$
- ❖ There is again a double temperature dependence because of I_s and V_T .
- ❖ Temperature compensation can be achieved by the same technique shown for log amps.



- ❖ The multiplication procedure take three steps:
1. To get the logarithm of a signal voltage use a Log amplifier.

$$V_1^* = \ln(V_1) \quad \text{and} \quad V_2^* = \ln(V_2)$$

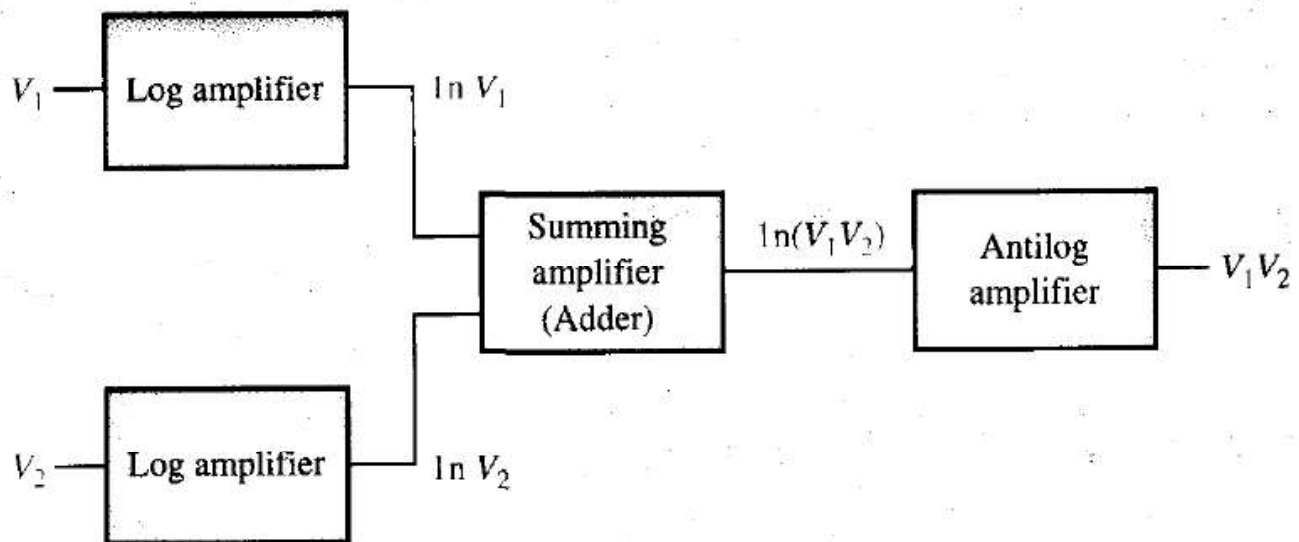
2. By summing the outputs of two log amplifiers, you get the logarithm of the product of the two original input voltages.

$$V_o^* = V_1^* + V_2^* = \ln(V_1) + \ln(V_2) = \ln(V_1 \cdot V_2)$$

3. Then, by taking the antilogarithm, you get the product of the two input voltages as indicated in the following equations:

$$V_o = \exp(V_o^*) = \exp[\ln(V_1 \cdot V_2)] = V_1 \cdot V_2$$

- ❖ The block diagram shows how the functions are connected to multiply two input voltages.
- ❖ Constant terms are omitted for simplicity.



❖ The outputs of the log amplifier are stated as follows:

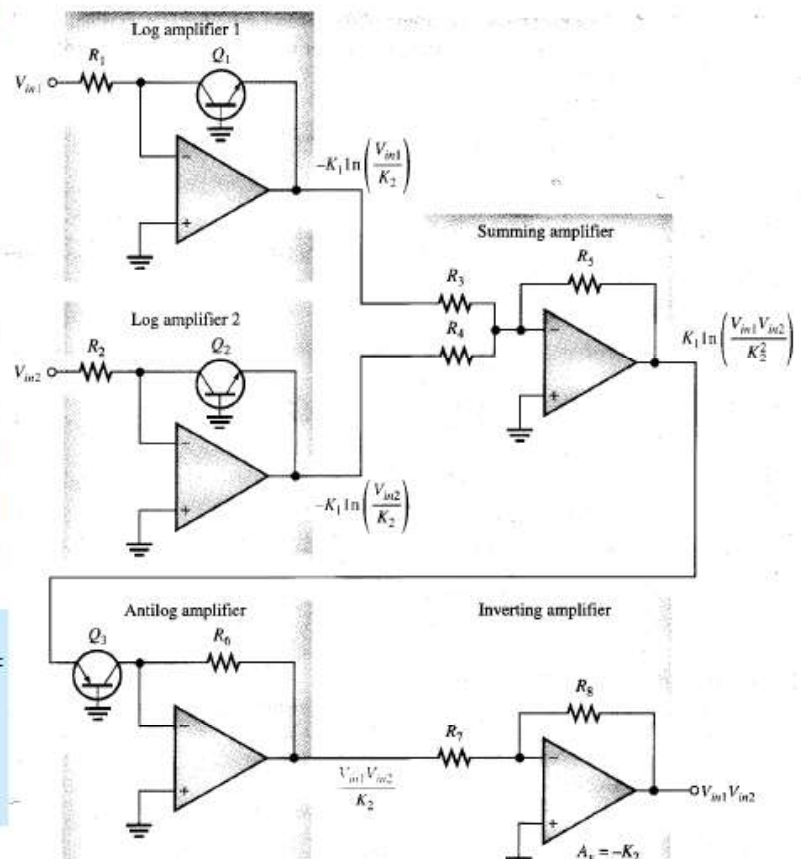
$$V_{out(log1)} = -K_1 \cdot \ln \left(\frac{V_{in1}}{K_2} \right)$$

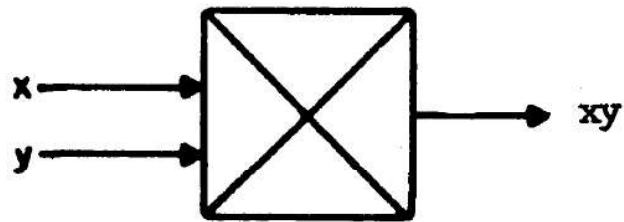
$$V_{out(log2)} = -K_1 \cdot \ln \left(\frac{V_{in2}}{K_2} \right)$$

where $K_1 = 0.025 \text{ V}$, $K_2 = R \cdot I_{ebo}$ and $R = R1 = R2 = R6$.

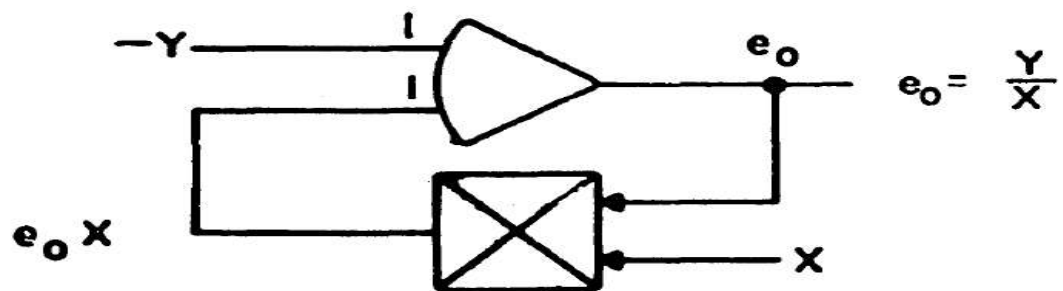
The two output voltages from the log amplifiers are added and inverted by the unity-gain summing amplifier to produce the following result:

$$\begin{aligned} V_{out(sum)} &= K_1 \cdot \ln \left[\ln \left(\frac{V_{in1}}{K_2} \right) + \ln \left(\frac{V_{in2}}{K_2} \right) \right] = \\ &= K_1 \cdot \ln \left(\frac{V_{in1} \cdot V_{in2}}{K_2^2} \right) \end{aligned}$$





DIVISION



SQUARE ROOTING

