

Summary of Digital Communication

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Chapter 1: introduction

Signals are analog in nature, when the communication needs to be established over a distance, then the analog signals are sent by using different techniques for effective transmission.

Why convert signal analog to digital?

When used analog signals for long distance communications, which suffer from many losses such as distortion, interference, and other losses. Including security breach.

In order to overcome these problems, The signals are digitized using different techniques. The digitized signals allow the communication to be more clear and accurate without losses.



Analog Signal



Digital Signal

Representation of Signals

Advantages of Digital communication

The advantages is 14, We classifications the advantages by four group:

1. Errors and distortions (5):

- The effect of distortion, noise, and interference is much less in digital signals as they are less affected.
- The occurrence of cross-talk is very rare in digital communication.
- The probability of error occurrence is reduced by employing error detecting and error correcting codes.
- Spread spectrum technique is used to avoid signal jamming.
- The signal is un-altered as the pulse needs a high disturbance to alter its properties, which is very difficult.

2. Component build(3):

- Digital circuits are more reliable.
- Digital circuits are easy to design and cheaper than analog circuit.
- The hardware implementation in digital circuits, is more flexible than analog.

3. security(2):

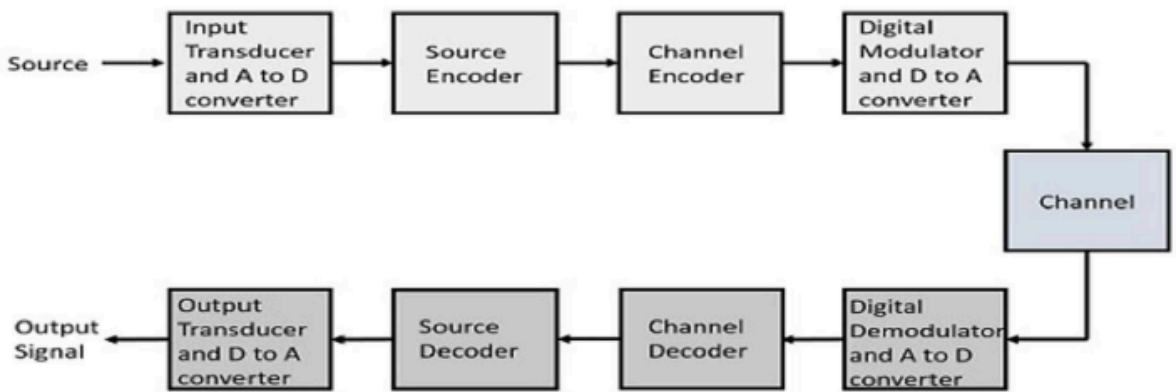
- Signal processing functions such as encryption and compression are employed in digital circuits to maintain the secrecy of the information
- Many of the digital circuits have almost common encoding techniques and hence similar devices can be used for a number of purposes

4. Signal properties and specifications(4):

- Combining digital signals using Time Division Multiplexing (TDM) is easier than combining analog signals using Frequency Division Multiplexing (FDM).
- The configuring process of digital signals is easier than analog signals.
- Digital signals can be saved and retrieved more conveniently than analog signals.
- The capacity of the channel is effectively utilized by digital signals.

Elements of Digital Communication

The elements which form a digital communication system is represented by the following block diagram for the ease of understanding.



Element of digital communication system:

1. Source Encoder

The source compresses the data into minimum number of bits. This process helps in effective utilization of the bandwidth. It removes the redundant bits (unnecessary excess bits, .i.e., zeroes) encoder

2. Channel Encoder

The channel encoder, does the coding for error correction. During the transmission of the signal, due to the noise in the channel, the signal may get altered and hence to avoid this, the channel encoder adds some redundant bits to the transmitted data. These are the error correcting bits.

3. Digital Modulator

The signal to be transmitted is modulated here by a carrier. The signal is also converted to analog from the digital sequence, in order to make it travel through the channel or medium.

4. Channel

The channel or a medium, allows the analog signal to transmit from the transmitter end to the receiver end.

5. Digital Demodulator

This is the first step at the receiver end. The received signal is demodulated as well as converted again from analog to digital. The signal gets reconstructed here.

6. Channel Decoder

The channel decoder, after detecting the sequence, does some error corrections. The distortions which might occur during the transmission, are corrected by adding some redundant bits. This addition of bits helps in the complete recovery of the source output.

7. Source Decoder

The resultant signal is once again digitized by sampling and quantizing so that the pure digital output is obtained without the loss of information. The source decoder recreates the source output.

8. Output Transducer

This is the last block which converts the signal into the original physical form, which was at the input of the transmitter. It converts the electrical signal into physical output (Example: loud speaker).

9. Output Signal

This is the output which is produced after the whole process. Example – The sound signal received.

Chapter 2:sampling

Sampling Theorem

1. The modern communication systems are **digital**.
2. The analog signal(continues- time signal) to be transmitted by digital communication system should be converted to **Digital form** (Discrete-time signal).
3. The first step in this direction, is to take **samples** from original signal (Discretization).

Ho - many samples must be taken?, in order to good signal representation and ability of original signal reconstruction ?

This problem is solved by a mathematical tool known as Sampling theorem, a continuous – time signal may be completely represented and recovered from its samples taken uniformly to reconstruct the original signal completely.

the sampling theorem may be stated as under:

A continuous – time signal may be completely represented in its samples and recovered back , If the sampling frequency is **$F_s \geq 2 F_m$** .

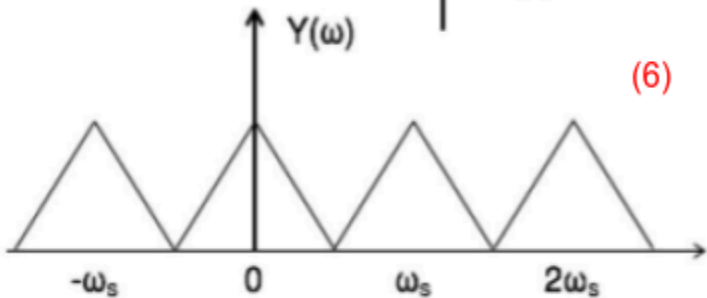
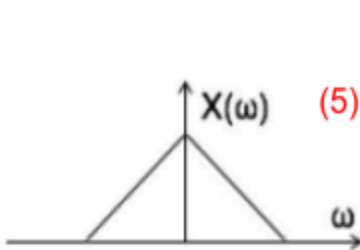
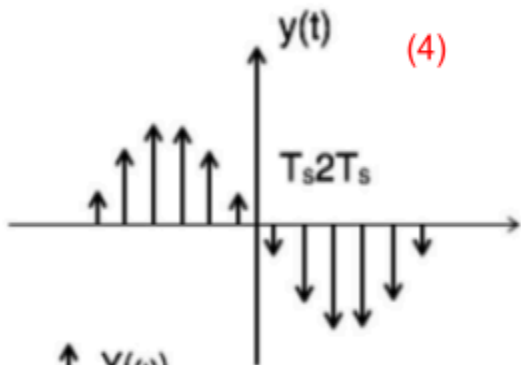
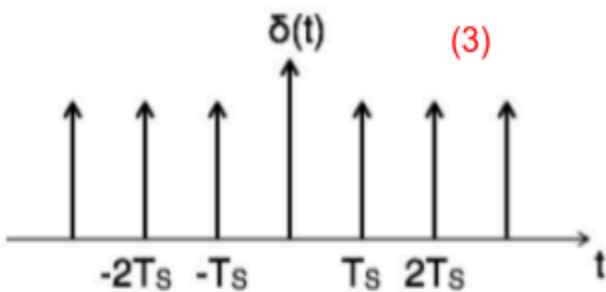
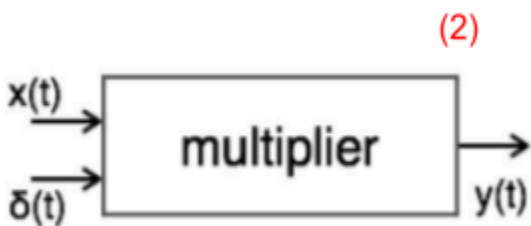
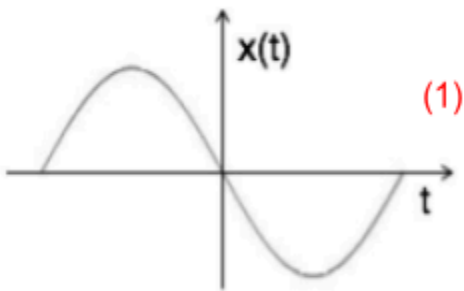
F_s is the **sampling frequency**.

F_m is the **maximum frequency** present in the signal.

Signal sampling theorem

Proof: consider a continus time signal x . The spectrum of $x(t)$ ⁽¹⁾ is a band limited to f_m Hz i.e. the spectrum of x is zero for $|w|>w_m$ ⁽⁵⁾.

Sampling of input signal $x(t)$ can be obtained by multiplying $x(t)$ ⁽¹⁾ with an impulse train $s(t)$ ⁽³⁾ of period T_s ⁽²⁾. the output of multiplier is a discrete signal called sampled signal which is represented with $y(t)$ ⁽⁴⁾.



Here you can observe that the sampled signal takes the period of impulse. The process of sampling can be explained by the following mathematical expression:

$$\text{Sampled signal } y(t) = x(t) \cdot s(t) \dots\dots(1)$$

The trigonometric Fourier series representation of $s(t)$ is given by

$$s(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_s t + b_n \sin n\omega_s t) \dots\dots(2)$$

$$\text{Where } a_0 = \frac{1}{T_s} \int_{-\frac{T}{2}}^{\frac{T}{2}} s(t) dt$$

$$\because s(t) = 0 \text{ except at } t = 0 \rightarrow \infty$$

Also, the area under $S(t)$

$$\because \int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} s(t) dt = 1$$

$$= \frac{1}{T_s} s(0) = \frac{1}{T_s}$$

$$\because s(t) = 0 \text{ except at } t = 0 \rightarrow \infty$$

$$\because \cos 0 = 1$$

$$a_n = \frac{2}{T_s} \int_{-\frac{T}{2}}^{\frac{T}{2}} s(t) \cos n\omega_s t dt = \frac{2}{T_s} s(0) \cos n\omega_s 0 = \frac{2}{T}$$

$$\because s(t) = 0 \text{ except at } t = 0 \rightarrow \infty$$

$$\therefore \sin 0 = 0$$

$$b_n = \frac{2}{T_s} \int_{\frac{-T}{2}}^{\frac{T}{2}} s(t) \sin n\omega_s t \, dt = \frac{2}{T_s} s(0) \sin n\omega_s 0 = 0$$

Substitute above values in equation 2.

$$\therefore s(t) = \frac{1}{T_s} + \sum_{n=1}^{\infty} \left(\frac{2}{T_s} \cos n\omega_s t + 0 \right)$$

Substitute st in equation 1.

$$\rightarrow y(t) = x(t).s(t)$$

$$= x(t) \left[\frac{1}{T_s} + \sum_{n=1}^{\infty} \left(\frac{2}{T_s} \cos n\omega_s t \right) \right]$$

$$= \left[x(t) \frac{1}{T_s} + \sum_{n=1}^{\infty} \left(\frac{2}{T_s} \cos n\omega_s t \right) x(t) \right]$$

$$= \frac{1}{T_s} [x(t) + 2 \sum_{n=1}^{\infty} (\cos n\omega_s t) x(t)] , \frac{1}{T_s} \text{ common factor}$$

$$y(t) = \frac{1}{T_s} [x(t) + 2 \cos \omega_s t \cdot x(t) + 2 \cos 2\omega_s t \cdot x(t) + \dots]$$

Take Fourier transform on both sides.

$$\therefore \cos(\omega_s t) = \frac{1}{2} s(\omega - \omega_s) + \frac{1}{2} S(\omega + \omega_s)$$

$$Y(\omega) = \frac{1}{T_s} [X(\omega) + X(\omega - \omega_s) + X(\omega + \omega_s) + X(\omega - 2\omega_s) + \dots]$$

$$\therefore Y(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s) \text{ where } n = 0, \pm 1, \pm 2,$$

...

Aliasing effect

- The overlapped region in case of under sampling represents aliasing effect which can be removed by:
- Considering $F_s \geq 2F_m$
- By using anti aliasing filters

Few points about Sampling Theorem

- **Nyquist Rate and Spectrum Repetition:** When the sampling frequency F_s is greater than twice the maximum signal frequency F_m (i.e., $F_s > 2F_m$), the spectrum of the sampled signal repeats periodically without overlapping. This ensures that each cycle of the signal is isolated in frequency.
- **Infinite Spectrum of Sampled Signals:** The spectrum of a sampled signal theoretically extends to infinity. This is due to the periodic repetition of the spectrum across the frequency domain when sampling is performed.
- **Recovery of Original Spectrum:** The original signal spectrum, centered at $\omega=0$ with a maximum frequency W_m , can be perfectly reconstructed by passing the sampled signal through an ideal low-pass filter with a cutoff frequency W_m .
- **Non-Overlapping Condition ($F_s > 2F_m$):** If the sampling frequency F_s is greater than $2F_m$, the successive cycles of the sampled signal's spectrum do not overlap, allowing perfect reconstruction of the original signal without aliasing.
- **Nyquist Rate ($F_s = 2F_m$):** When F_s is exactly $2F_m$, the successive cycles of the spectrum touch but do not overlap. This is the minimum sampling

rate required to avoid overlap and ensure reconstruction.

- **Aliasing ($F_s < 2F_m$):** If F_s is less than $2F_m$, the successive cycles of the sampled spectrum overlap, causing aliasing. In this case, the original spectrum cannot be extracted or reconstructed without distortion.
- **Condition for Distortion-Free Reconstruction:** For accurate and distortion-free reconstruction of the original signal, the sampling frequency must meet or exceed twice the maximum signal frequency, i.e., $F_s \geq 2F_m$ (known as the Nyquist Criterion).

Nyquist rate and Nyquist Interval

- Sampling rate is the number of samples per second
- Sampling interval is the distance in time between successive samples.
- Nyquist rate is the sampling frequency when $F_s = 2 F_m$ and is also called minimum sampling rate.
- Nyquist Interval is the interval of Nyquist sampling frequency and is also called maximum sampling Interval.
- $T_s = 1 / (2F_m)$

Example -1

- An analog signal is expressed by,

$$x(t) = 3 \cos 50\pi t + 10 \sin 300 \pi t - \cos 100 \pi t$$

Calculate the Nyquist rate and Nyquist Interval

solution: the three angular frequencies in the signal are:

$$w_1 = 50\pi, \quad w_2 = 300\pi, \quad w_3 = 100\pi \quad \text{with } w = 2\pi f, \quad f = w/2\pi$$

$$f_1 = 50\pi / 2\pi \quad f_2 = 300\pi / 2\pi \quad f_3 = 100\pi / 2\pi$$

$$f_1 = 25 \text{ Hz} \quad f_2 = 150 \text{ Hz} \quad f_3 = 50 \text{ Hz}$$

the maximum frequency, F_m is 150 Hz

$$\text{Nyquist rate } F_s = 2 F_m = 2 \times 150 = 300 \text{ Hz}$$

$$\text{Nyquist interval } T_s = 1/F_s = 1/300 = 3.333 \text{ msec.}$$

- Find the Nyquist rate and Nyquist Interval for the following signal

$$x(t) = 1/2 \pi \cos(4000 \pi t) \cdot \cos(1000 \pi t)$$

solution: it is better to convert product to adding

$$\text{with } 2 \cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$$

Example -2

$$\begin{aligned}x(t) &= \frac{1}{2}\pi\left[\frac{1}{2}\{\cos(4000\pi t + 1000\pi t)\right. \\&\quad \left.+ \cos(4000\pi t - 1000\pi t)\}\right] \\&= \frac{\pi}{4}\{\cos(5000\pi t)\} + \frac{\pi}{4}\{\cos(3000\pi t)\}\end{aligned}$$

$$\therefore f_s = 2f_m \rightarrow f_m = \frac{f_s}{2}$$

$$\rightarrow f_{m_1} = \frac{5000\pi t}{2} = 2500\text{Hz}, f_{m_2} = \frac{3000\pi t}{2} = 1500\text{Hz}$$

$$\therefore T_s = \frac{1}{f_s}$$

$$T_{s1} = \frac{1}{f_{s1}} \rightarrow T_{s1} = \frac{1}{2500\text{Hz}} = 4 \times 10^{-4}\text{sec}$$

$$T_{s2} = \frac{1}{f_{s2}} \rightarrow T_{s2} = \frac{1}{1500\text{Hz}} = 6.666 \times 10^{-4}\text{sec}$$

Example:

Suppose that an analog signal is given as

$x(t) = 5 \cos(2\pi \cdot 1000t)$, for $t > 0$, is sampled at the rate of 8,000Hz.

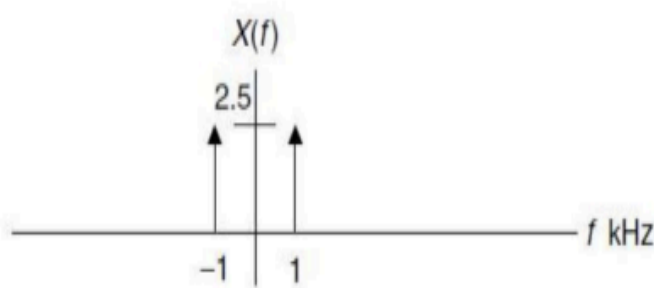
- Sketch the spectrum for the original signal.
- Sketch the spectrum for the sampled signal for 0 to 20kHz.

Sol:

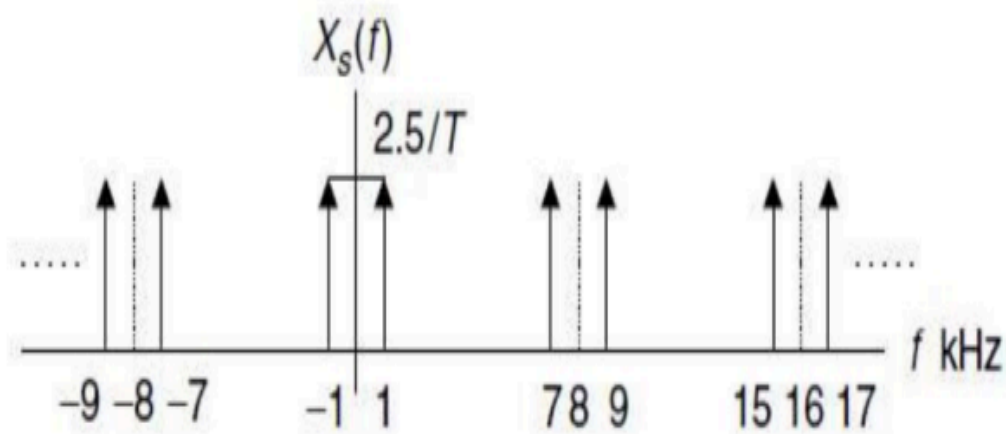
A. Since the analog signal is sinusoid with a peak value of 5 and frequency of 1,000 Hz, we can write the sine wave using Eulers identity:

$$\therefore \cos \Theta = \frac{e^{j\Theta} + e^{-j\Theta}}{2} =$$

$$5 \cos(2\pi \times 1000t) = 5 \cdot \left(\frac{e^{j2\pi \cdot 1000t} + e^{-j2\pi \cdot 1000t}}{2} \right)$$
$$2.5e^{j2\pi \cdot 1000t} + 2.5e^{-j2\pi \cdot 1000t}$$



B. After the analog signal is sampled at the rate of 8,000Hz, the sampled signal spectrum and its replicas centered at the frequencies $\pm nf_s$, each with the scaled samplitude being $2.5/T$, are as shown in figure below.



Notice that the spectrum of the sampled signal contains the images of the original spectrum;

That the images repeat at multiples of the sampling frequency f_s (for our example, 8kHz, 16kHz, 24kHz, ...); and that all images must be removed, since they convey no additional information.

For benefit about Eulers identity :

$$e^{j\Theta} = \cos \Theta + j \sin \Theta \text{ and } e^{-j\Theta} = \cos \Theta - j \sin \Theta$$

$$\text{Adding } 2 \cos \Theta = e^{j\Theta} + e^{-j\Theta}$$

$$\text{or } \cos \Theta = \frac{e^{j\Theta} + e^{-j\Theta}}{2} =$$

$$\text{Subtracting } 2 j \sin \Theta = e^{j\Theta} - e^{-j\Theta}$$

$$\text{or } \sin \Theta = \frac{e^{j\Theta} - e^{-j\Theta}}{2j}$$

Reconstruction Filter (Low Pass Filter)

- The low pass filter is that type of filter which passes only low frequencies up to a specified cut-off frequency and rejects all other frequencies above cut-off frequency.

- The ideal LPF has sharp – change in response at cut-off frequency
- The practical LPF has amplitude response decrease slowly to zero.
- It has transition band.

Effect of Under Sampling : ALIASING

- When a continuous - time band – limited signal is sampled at a rate **lower than Nyquist rate $F_s < 2F_m$** , then the successive cycles of the spectrum of the sampled signal **overlap** with each other.
- **Aliasing:** is the phenomenon in which a high frequency component in the frequency spectrum of the signal takes identity of a lower frequency
- It is clear that because of the overlap due to aliasing phenomenon, it is not possible to recover original signal from sampled signal by low pass filtering since the spectral components in the overlap regions add and hence the signal is distorted.
- **To decide a sampling frequency is problem.?**

Therefore , a signal is first passed through a low pass filter which blocks all the frequencies which are above F_m Hz. This process is known as band limiting of the original signal.

- low pass filter is called pre alias filter because it is used to prevent aliasing effect.

To avoid aliasing:

- - Prealias filter must be used to limit band of frequencies of frequencies of the signal to F_m Hz.
- - Sampling frequency “ F_s ” must be selected such that $F_s > 2 F_m$.

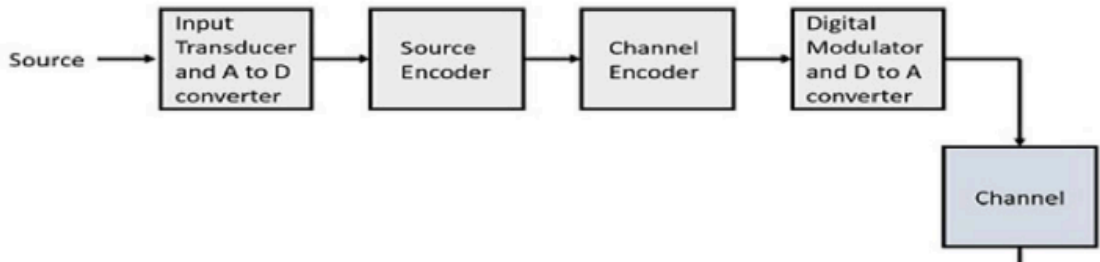
Sampling of Bandpass Signals

- The sampling theorem for bandpass signal $X(t)$ whose maximum bandwidth is $2W$ can be completely represented into and recovered from its samples if it is sampled at minimum rate of twice the bandwidth.

First month Question

a-

Q1/Draw block diagram for the transmitter part of the digital communication system and explain the function of one its components?



Source Encoder

The source compresses the data into minimum number of bits. This process helps in effective utilization of the bandwidth. It removes the redundant bits (unnecessary excess bits, .i.e., zeroes) encoder

Q2/ Explain the sampling process in time domain by equations and signal diagram?

Go to [Signal sampling theorem](#)

Q3/An analog signal, $x(t) = 10 \cos (2\pi 2000t)$ is to be sampled at Nyquist rate.

a. Sketch the spectrum of original signal

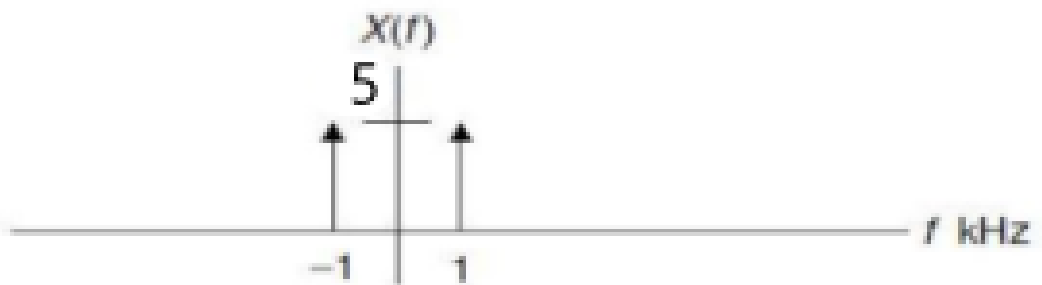
sol/ Since the analog signal is sinusoid with a peak value of 10 and frequency of 2,000 Hz, we can write the sine wave using Eulers identity:

$$\therefore \cos \Theta = \frac{e^{j\Theta} + e^{-j\Theta}}{2} =$$

$$10 \cos(2\pi \times 2000t) =$$

$$10 \cdot \left(\frac{e^{j2\pi \times 2000t} + e^{-j2\pi \times 2000t}}{2} \right)$$

$$5e^{j2\pi \cdot 2000t} + 5e^{-j2\pi \cdot 2000t}$$



b. What is its Nyquist rate and Nyquist interval

$$\text{sol/ } \therefore f_s = 2f_m \rightarrow f_m = \frac{f_s}{2}$$

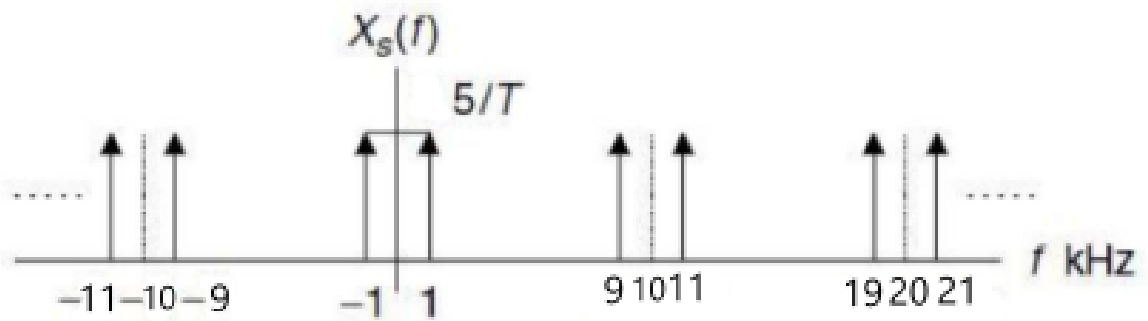
$$\rightarrow f_{m1} = \frac{2000 \pi t}{2} = 1000\text{Hz}$$

$$\therefore T_s = \frac{1}{f_s}$$

$$T_{s1} = \frac{1}{f_{s1}} \rightarrow T_{s1} = \frac{1}{1000\text{Hz}} = 1 \times 10^{-3}\text{sec}$$

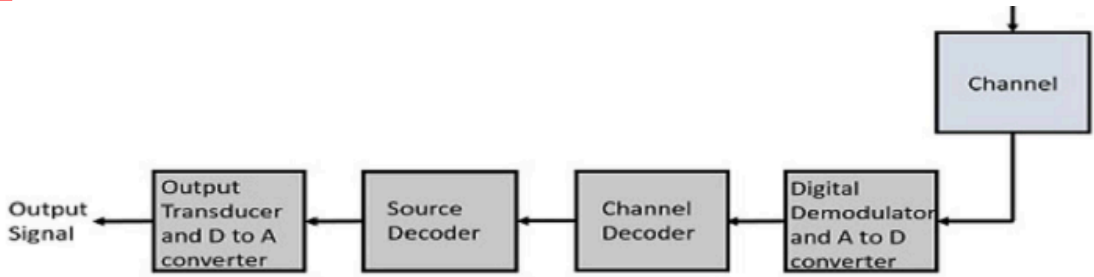
C. sketch the spectrum for the sampled signal from 0 to 10kHz

sol/ After the analog signal is sampled at the rate of 10,000Hz, the sampled signal spectrum and its replicas centered at the frequencies $\pm n f_s$, each with the scaled samplitude being $5/T$, are as shown in figure below.



b-

Q1/Draw block diagram for the receiver part of the digital communication system and explain the function of one of its components?



Output Signal

This is the output which is produced after the whole process. Example – The sound signal received.

Q2/ Explain the sampling process in frequency domain by equations and signal diagram?

Go to [Signal sampling theorem](#)

Q3/An analog signal, $x(t) = 10 \cos(2\pi 4000t)$ is to be sampled at Nyquist rate.

a. Sketch the spectrum of original signal?

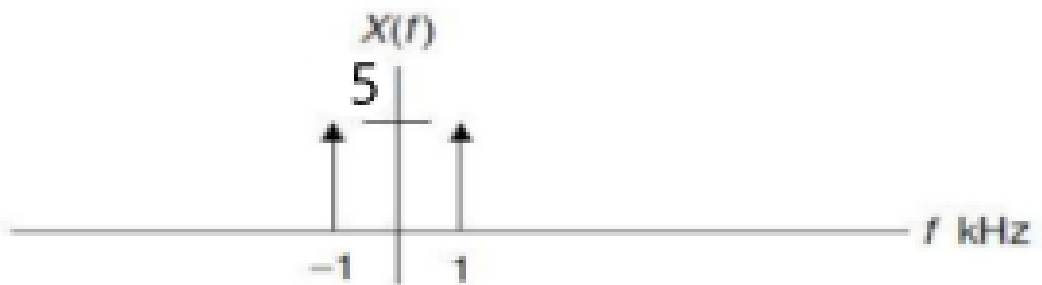
sol/ Since the analog signal is sinusoid with a peak value of 10 and frequency of 4,000 Hz, we can write the sine wave using Eulers identity:

$$\therefore \cos \Theta = \frac{e^{j\Theta} + e^{-j\Theta}}{2} =$$

$$10 \cos(2\pi \times 4000t) =$$

$$10 \left(\frac{e^{j2\pi \times 4000t} + e^{-j2\pi \times 4000t}}{2} \right)$$

$$5e^{j2\pi \cdot 4000t} + 5e^{-j2\pi \cdot 4000t}$$



b. What is its Nyquist rate and Nyquist interval?

$$\text{sol/ } \therefore f_s = 2f_m \rightarrow f_m = \frac{f_s}{2}$$

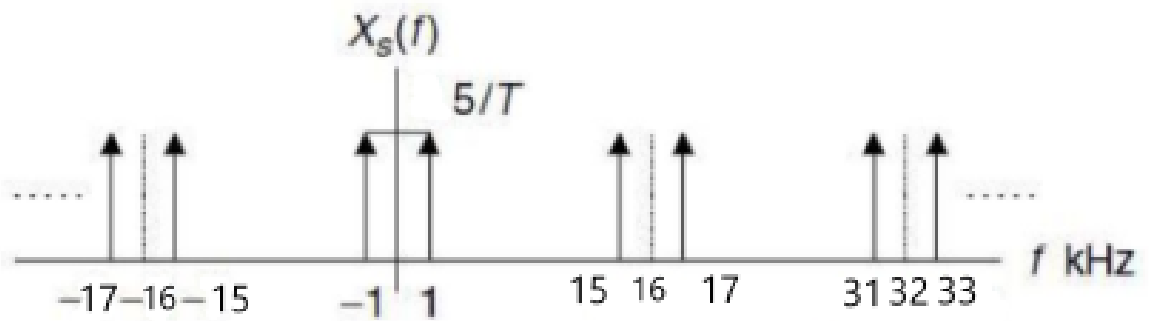
$$\rightarrow f_{m1} = \frac{4000 \pi t}{2} = 2000 \text{ Hz}$$

$$\therefore T_s = \frac{1}{f_s}$$

$$T_{s1} = \frac{1}{f_{s1}} \rightarrow T_{s1} = \frac{1}{2000 \text{ Hz}} = 5 \times 10^{-4} \text{ sec}$$

C. sketch the spectrum for the sampled signal from 0 to 16 kHz?

sol/ After the analog signal is sampled at the rate of 16,000Hz, the sampled signal spectrum and its replicas centered at the frequencies $\pm n f_s$, each with the scaled samplitude being $5/T$, are as shown in figure below.



Chapter 3: Sampling techniques

They are three types

1. **Ideal sampling or instantaneous sampling or impulse sampling** which is shown in study of sampling theorem. The sampling function is train of impulses $S(nT_s)$. The width of samples () is approaches zero. Hence the transmitted power is negligible and the method is impracticable. The CKT is known as the SWITCHING SAMPLER.

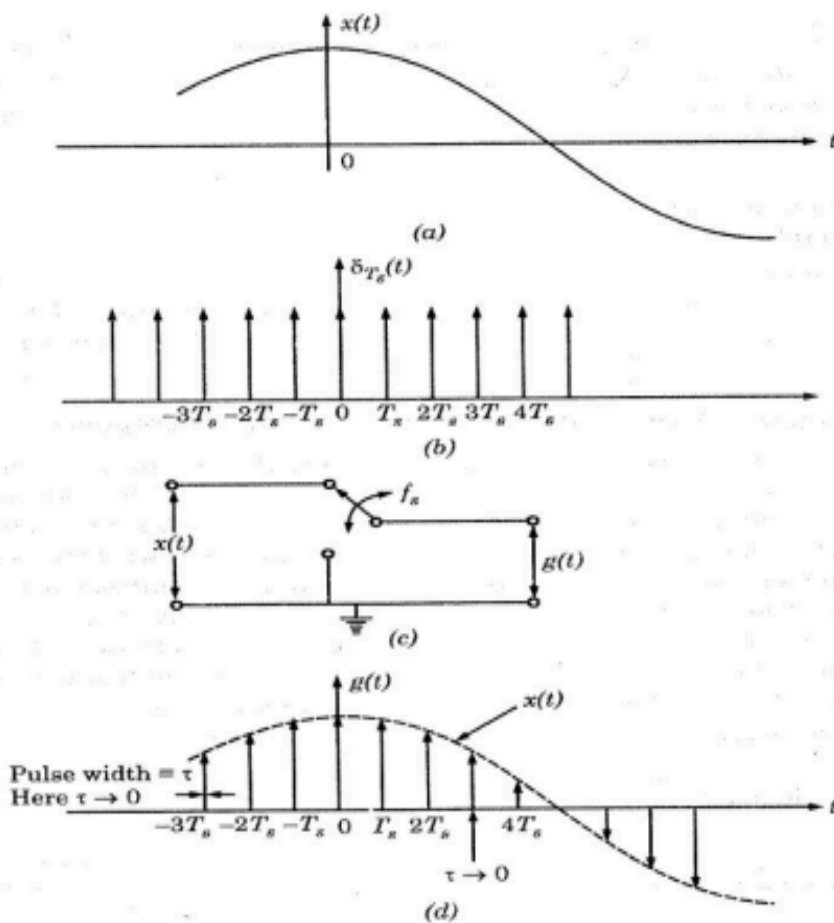


Fig. 5.11 (a) Baseband signal (b) impulse train (c) functional diagram of a switching sampler (d) sampled signal.

2. **Natural sampling or chopper sampling**, The sampling function in this type is a train of periodic pulses of width and frequency equal to F_s Hz. also should consider the pulse has a finite width, It is practical method but is hard also called practical sampling.

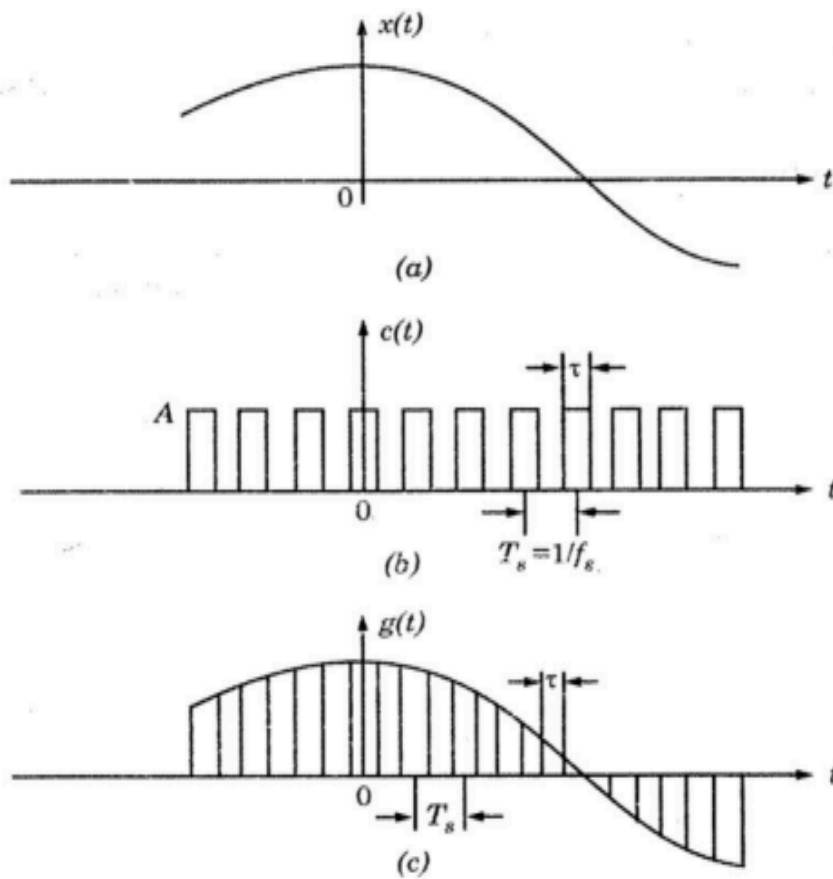


Fig. 5.13. (a) Continuous time signal $x(t)$.
 (b) Sampling function waveform i.e. periodic pulse train
 (c) Naturally sampled signal waveform $g(t)$.

3. **Flat Top Sampling or Rectangular pulse sampling** this is also a practically possible sampling method called practical sampling. it is very easy to get flat top samples the top of the samples remains constant and equal to instantaneous value of baseband signal $x(t)$ at the start of sampling. The duration of each sample is and sampling rate is equal to $F_s = 1/T_s$. The CKT used to generate this type is called **Sample and Hold CKT**.

Flat top sampled method:

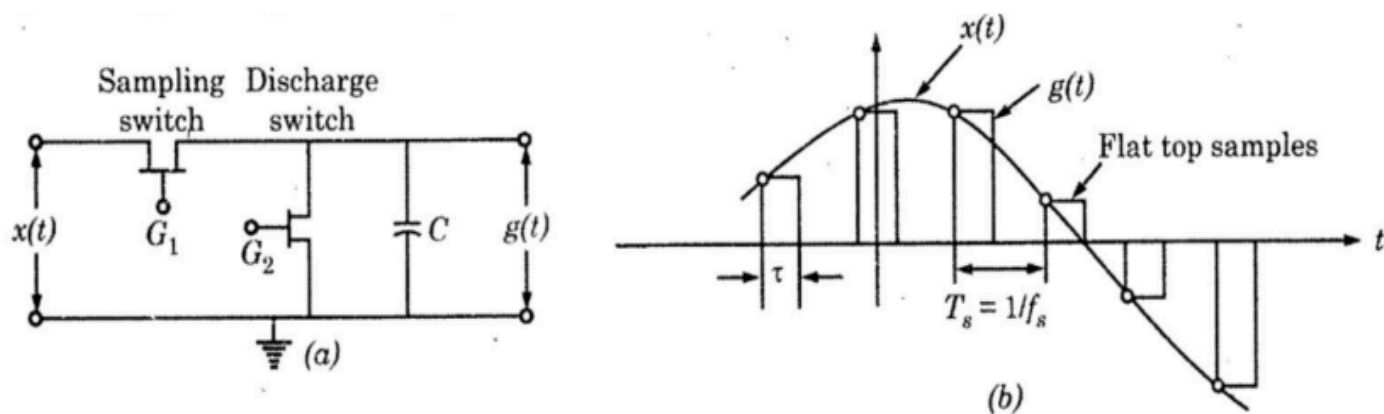


Fig. 5.15 (a) A sample and hold circuit to generate flat top samples.
(b) A general waveform of flat top sampling.

Flat sampling method:

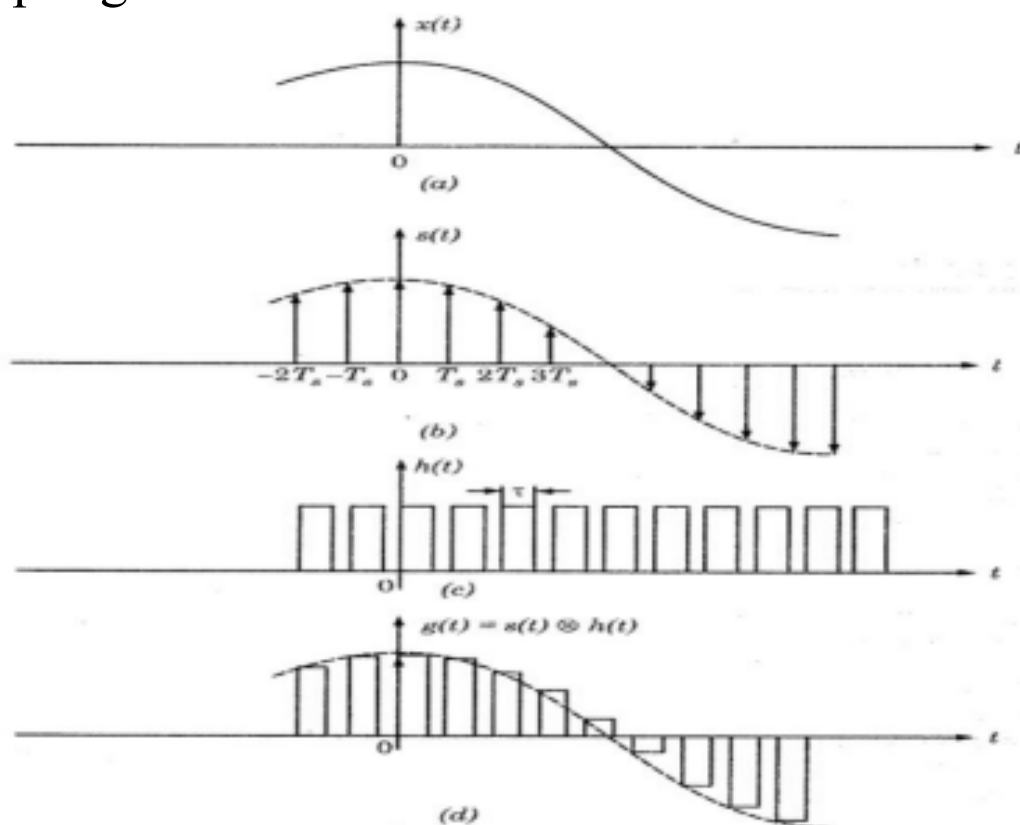
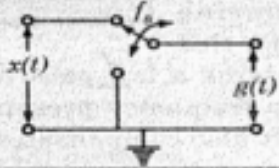
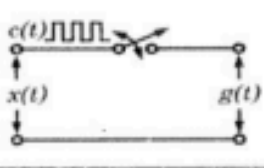
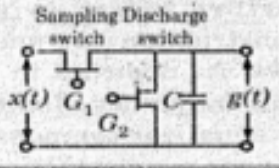
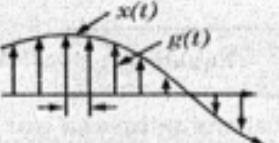
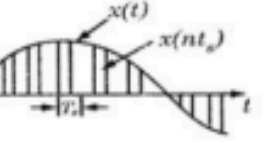
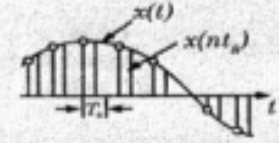


Fig. 5.17 (a) Baseband signal $x(t)$ (b) Instantaneously sample signal $s(t)$
(c) Constant pulse width function $h(t)$ (d) Flat top sampled signal $g(t)$ obtained through convolution of $h(t)$ and $s(t)$

Performance comparison of three sampling techniques

TABLE 5.1. Performance comparison of three sampling techniques				
S.No.	Parameter of comparison	Ideal or instantaneous sampling	Natural sampling	Flat top sampling
1.	Sampling principle	It uses multiplication	It uses chopping principle	It uses sample and hold circuit
5.	Generation circuit			
3.	Waveforms involved			
4.	Feasibility	This is not a practically possible method	This method is used practically	This method is also used practically
5.	Sampling rate	Sampling rate tends to infinity	Sampling rate satisfies Nyquist criteria	Sampling rate satisfies Nyquist criteria
6.	Noise interference	Noise interference is maximum	Noise interference is minimum	Noise interference is maximum
7.	Time domain representation	$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \delta(t - nT_s)$	$g(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t) \sin c(nf_s \tau) e^{j2\pi n f_s t}$	$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) h(t - nT_s)$
8.	Frequency domain representation	$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$	$G(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \sin c(nf_s \tau) X(f - nf_s)$	$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) H(f)$

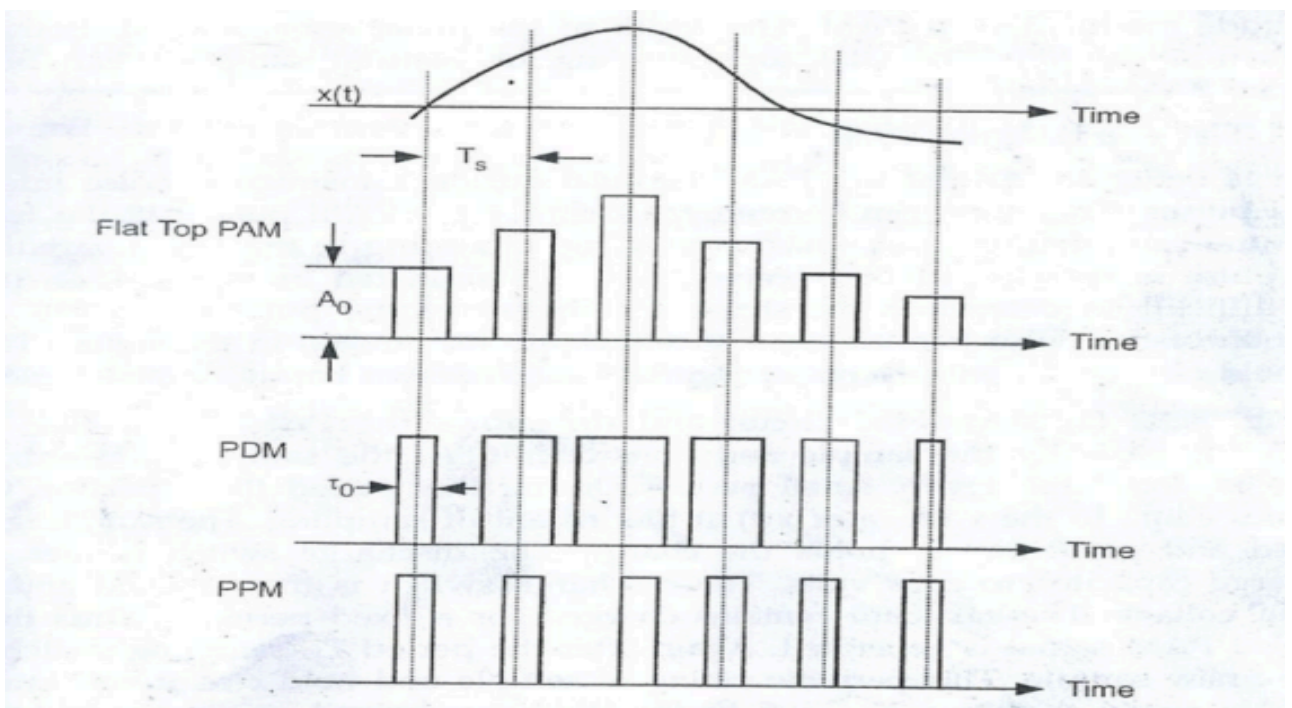
Chapter 4 :Analog pulse Modulation Methods

A continuous time signal $x(t)$ to be transmitted, is sampled by a pulse train at frequency F_s sufficiently above the highest frequency present in $X(t)$, The amplitude of the modulating signal $x(t)$ modulates one parameter of the pulse train.

The parameters are : **Amplitude, Duration (width) and position.**

- If the modulating signal modulate the **amplitude** of the pulse train then the modulation type is called **Pulse Amplitude Modulation PAM**
- If the modulating signal Modulate the **Duration** of the pulse train then the modulation type is called **Pulse Duration Modulation PDM**
- If the modulating signal Modulate the **position** of the pulse train then the modulation type is called **Pulse Position Modulation PPM**

Different types of analog pulse modulation techniques



Pulse Amplitude Modulation PAM

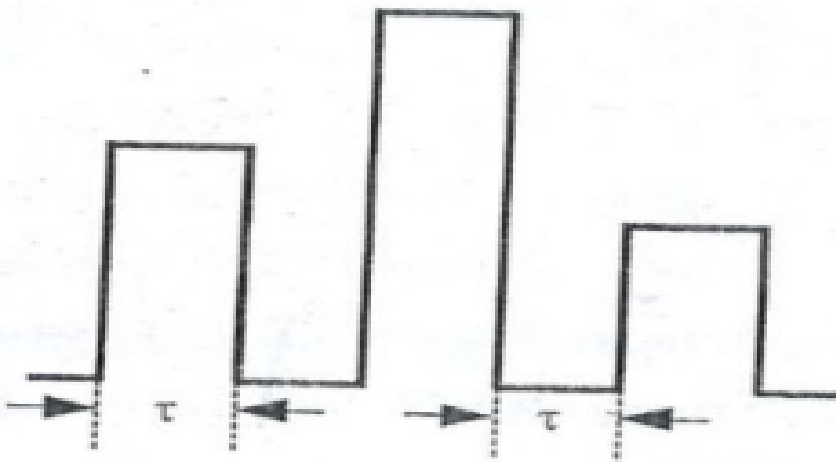
- the amplitudes of regularly spaced rectangular pulses vary with the instantaneously sample values of a continuous message signal in a one to one fashion.
- The width of the pulse remains fixed.
- Both types of practical sampling, flat top sampling or Natural sampling can be used to generate PAM signal.

Flat Top Sampled PAM

- It is better to use Flat Top PAM , because during transmission noise interferes the top of pulses. This noise interference can be removed easily if the PAM pulse has flat top.
- Sample and hold circuit is used to generate the flat top PAM

Transmission Bandwidth of PAM signal

- The pulse duration “ τ ” is supposed to be very small compared to time period T_s between the two samples.
- If the maximum frequency in the signal $x(t)$ is F_m then by sampling theorem:
 - $F_s \geq 2F_m$
 - $T_s \leq 1/(2F_m)$
 - $F_s = 1/T_s$
 - And $\tau = T_s = 1/(2F_m)$
- If ON and Off time of the pulse is same, then frequency of the PAM pulse Becomes:
 - $f_{max} = 1/(\tau + \tau) = 1/(2\tau)$



- Bandwidth required for transmission of PAM signal will be equal to maximum frequency F_{max} given by above equation
 - $BT \geq F_{max}$ hence $BT \geq 1/(2\tau)$
 - Since $\tau = 1/(2F_m)$
 - $BT \geq 1/(2\tau) = F_m$
 - Transmission bandwidth of PAM signal : $BT = F_m$

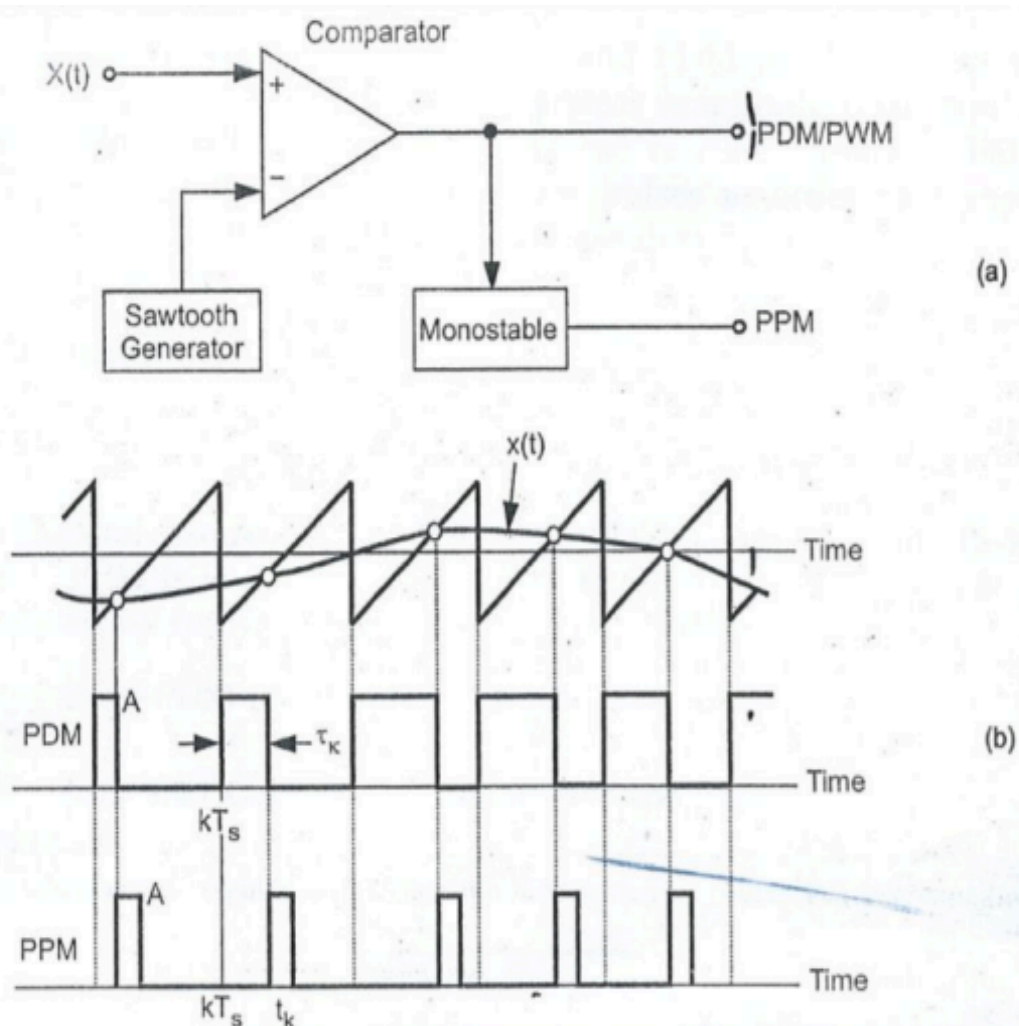
Disadvantages of PAM

- The bandwidth needed for transmission of PAM signal is very very large compared to its maximum frequency content.
- The amplitude of PAM pulses varies according to modulating signal, therefore interference of noise is maximum for the PAM signal and this noise cannot be removed very easily.
- Since amplitude of PAM signal varies , this also varies the peak power required by the transmitter with modulating signal.

Pulse Duration and Pulse Position Modulation

- Pulse Position Modulation (PPM) and Pulse Duration Modulation (PDM) or (PWM) both modulate the time parameter of the pulses.
- PPM has fixed width pulses where as width of PDM pulses varies.
- Both the methods are of constant amplitude.

Generator of PPM and PDM, Block Diagram and Waveform



Transmission bandwidth of PDM and PPM

- Both PPM and PWM needs a sharp rise time and fall time for pulses in order to preserve the message information.

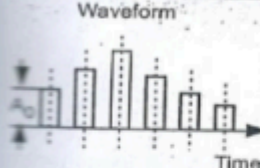
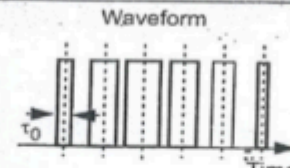
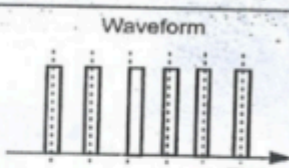
- Rise time should be very – very less than T_s
 $t_r \ll T_s$

And Transmission Bandwidth, BT should be,

$$BT \geq \frac{1}{2t_r}$$

Thus the transmission band width of PPM and PDM is higher than PAM

Table 5.3.1 Comparison of PAM, PPM and PDM

Pulse Amplitude Modulation	Pulse Width/Duration Modulation	Pulse Position Modulation
		
Amplitude of the pulse is proportional to amplitude of modulating signal	Width of the pulse is proportional to amplitude of modulating signal	The relative position of the pulse is proportional to the amplitude of modulating signal
The bandwidth of the transmission channel depends on width of the pulse	Bandwidth of transmission channel depends on rise time of the pulse	Bandwidth of transmission channel depends on rising time of the pulse
The instantaneous power of the transmitter varies.	The instantaneous power of the transmitter varies	The instantaneous power of the transmitter remains constant
Noise interference is high.	Noise interference is minimum	Noise interference is minimum
System is complex	Simple to implement	Simple to implement
Similar to amplitude modulation	Similar to frequency modulation	Similar to phase modulation

Examples

Example -1

- A voice signal of $F_{\max} = 3\text{KHz}$ is to be transmitted by Pulse amplitude modulation technique.

Calculate the required transmission bandwidth BT if the sampling frequency, $F_s = 8\text{KHz}$ and pulse duration $\tau = 0.1 T_s$

Solution:

Sampling interval, $T_s = 1/F_s = 1/(8 \times 10^3) \text{ sec}$

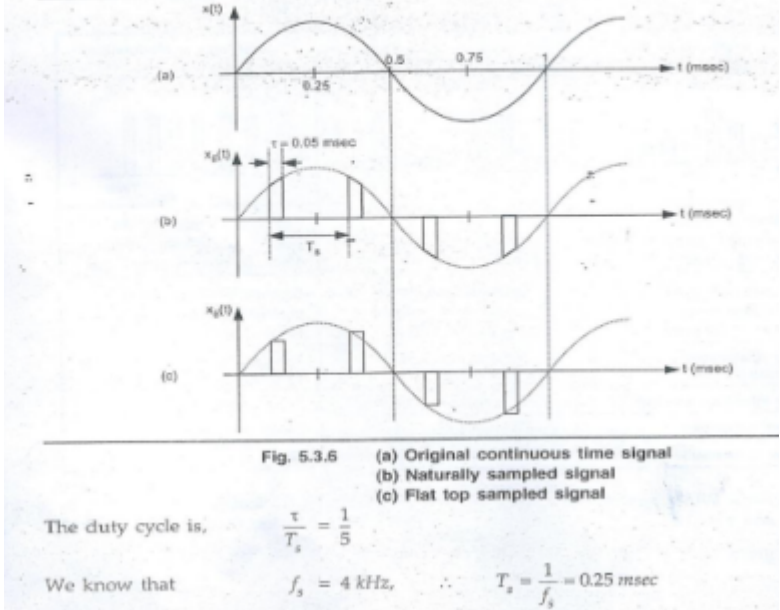
then $\tau = 0.1 T_s = \frac{0.1}{8000} \text{ sec.}$

$$BT \geq \frac{1}{2\tau} \geq \frac{1}{\frac{2 \times 0.1}{8 \times 10^3}} = 40 \text{ KHz}$$

EX2:

Sketch the naturally sampled pam waveform that results from sampling a 1 khz sine wave of peak value equal 5 at 4 khz rate. Also sketch the flat top sampled waveform.

Take duty cycle ($\frac{\tau}{T_s} = \frac{1}{5}$)



Solution :

The equation for given waveform $x(t)$,

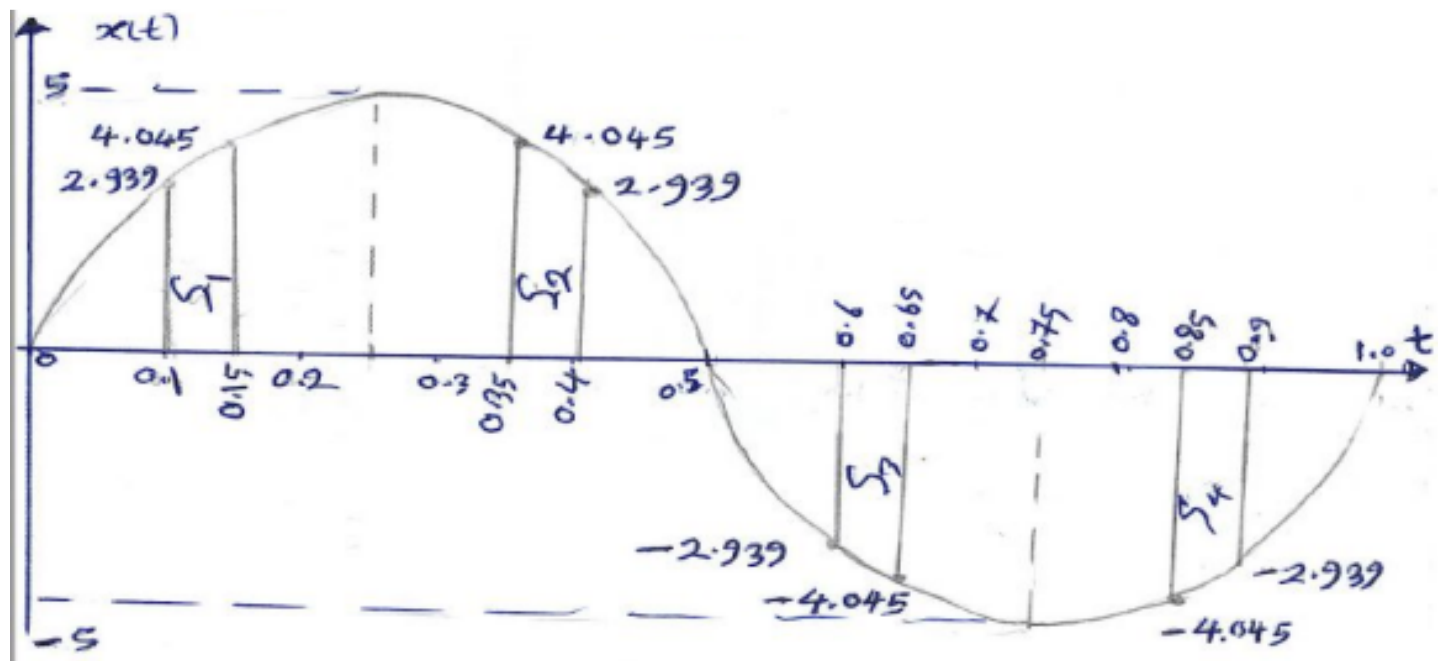
$$x(t) = 5 \sin(2\pi \cdot 1000t) = 5 \sin(2\pi \cdot 1 \cdot 10^3 t)$$

$$\therefore T_o = \frac{1}{f_o} = \frac{1}{1000} = 1 \times 10^{-3} \text{ sec} = 1 \text{ msec.}$$

We have sample duration = $t_o = 0.25 \text{ msec.}$

$$\text{Sample interval } T_s, \frac{\tau}{T_s} = \frac{1}{5}$$

$$\therefore \tau = T_s \times \frac{1}{5} = 0.25 \times \frac{1}{5} = \frac{0.25}{5} = 0.05 \text{ msec.}$$



sample	Ts.s msec	X(t)	□	x(t)
1	0.1	$x(t) =$ $5\sin(2\pi*10^3*0.1*10^{-3})$ $= 5\sin(2\pi*0.1)$ $= 5\sin(0.2\pi)$ $= 2.939$	$0.1+0.05$ $=0.15$	$x(t)=5\sin^*$ $(2\pi*0.15)$ $=5\sin(0.3\pi)$ $=4.045$
2	$0.1+0.25$ $= 0.35$	$x(t) = 5\sin(2\pi*0.35)$ $= 5\sin(0.7\pi)$ $=4.045$	$0.35+$ $0.05=$ 0.4	$x(t)=5\sin^*$ $(2\pi*0.4)$ $=2.939$
3	$0.1+0.5$ $= 0.6$	$x(t) = 5\sin(2\pi*0.6)$ $= 5\sin(1.2\pi)$ $=-2.939$	$0.6+0.05$ $=0.65$	$x(t)=5\sin^*$ $(2\pi*0.65)$ $=5\sin(1.3\pi)$ $=-4.045$
4	$0.1+0.75$ $= 0.85$	$x(t) = 5\sin(2\pi*0.85)$ $= 5\sin(1.7\pi)$ $=-4.045$	$0.85+$ 0.05 $=0.9$	$x(t)=5\sin^*$ $(2\pi*0.9)$ $=5\sin(1.8\pi)$ $=-2.939$

Example -3

- For the signal given in example – 1, if the rise time is 1% of the width of the pulse(τ)

Find out the minimum transmission bandwidth needed for transmission of signal by using PDM and PPM.

Solution: in example1, we obtain the pulse width $\tau = 0.1 Ts = \frac{0.1}{8000}$

The rise time, tr is given by 1% of τ

$$tr = \tau \times 0.01 = \frac{0.1}{8000} \times 0.01 = 1.25 \times 10^{-7} \text{ sec.}$$

$$\text{with } BT \geq \frac{1}{2tr} \geq \frac{1}{2 \times 1.25 \times 10^{-7}} \geq 4 \text{ MHZ}$$

which is very high compare with bandwidth needed for PAM (40KHZ)

Chapter 5 :MULTIPLEXING

Multiplexing may be defined as a **technique** which allows many signals (users) to share a communication channel simultaneously.

There are two major types of multiplexing:

- Frequency Division Multiplexing (**FDM**)
- Time Division Multiplexing (**TDM**)

Frequency Division Multiplexing (FDM)

- FDM : this technique permits a fixed frequency band to every user in the complete channel Band.
- It mainly used for modulated signal broadcasting, TV broadcasting

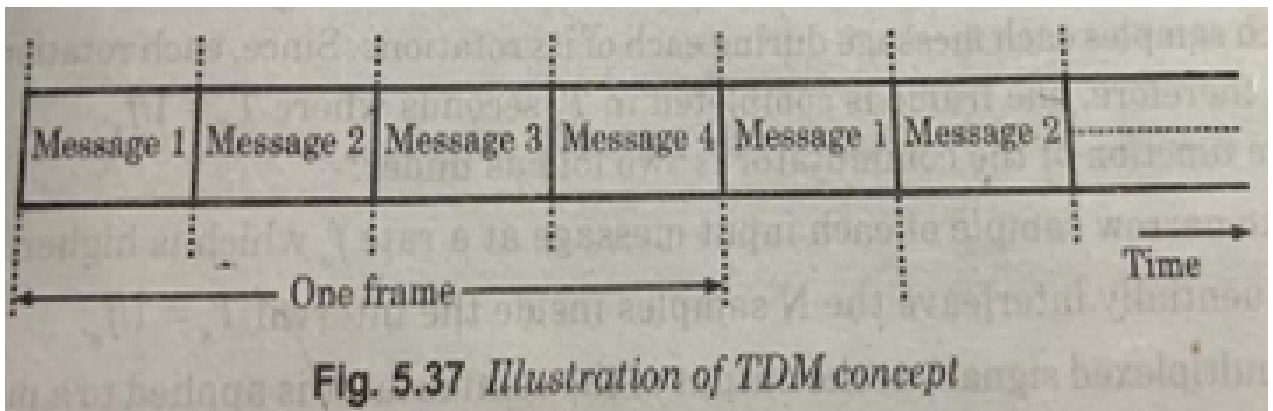
Time Division Multiplexing (TDM)

- In PAM, PPM and PDM, the pulse is present for a short duration and for most of the time between the two pulses, no signal is present.
- This free space between the pulses can be occupied by pulses from other channels.
- This is known as Time Division Multiplexing **TDM**
- Thus, TDM makes maximum utilization of the transmission channel.

Comparison TDM with FDM

- In FDM , All the signals are transmitted simultaneously over same communication channel, and the signals occupy frequency slots.
- In TDM, signals to be multiplexed are transmitted sequentially one after the other.
- Each signal occupy a short time slot.
- In TDM the complete bandwidth of the communication channel is available to each signal be transmitted.
- Conceptual Diagram
- Concept of Frame in TDM
 - Frame is the time period required to transmit all the signals once on the transmission channel.
 - Each frame is completed in T_s seconds
- The TDM system can be used to multiplex analog or digital signals.

Concepts - Conceptual Diagram -Frame



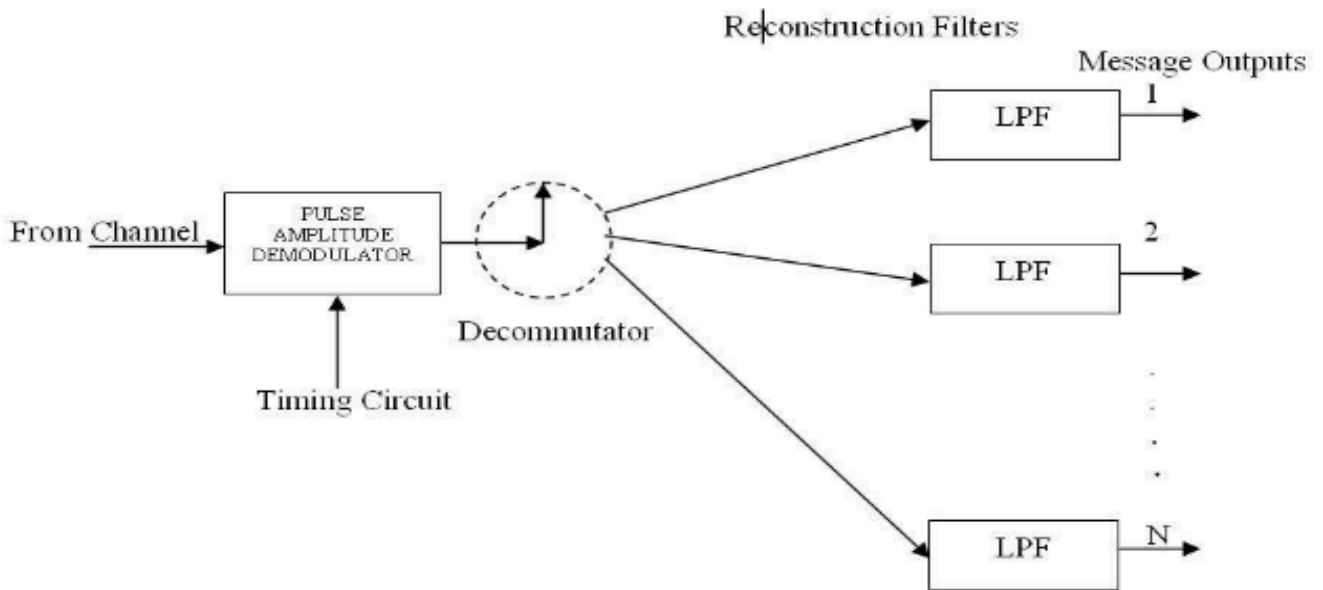
Concept of Frame in TDM

- Frame is the time period required to transmit all the signals once on the transmission channel.
- Each frame is completed in T_s seconds
- The TDM system can be used to multiplex analog or digital signals.

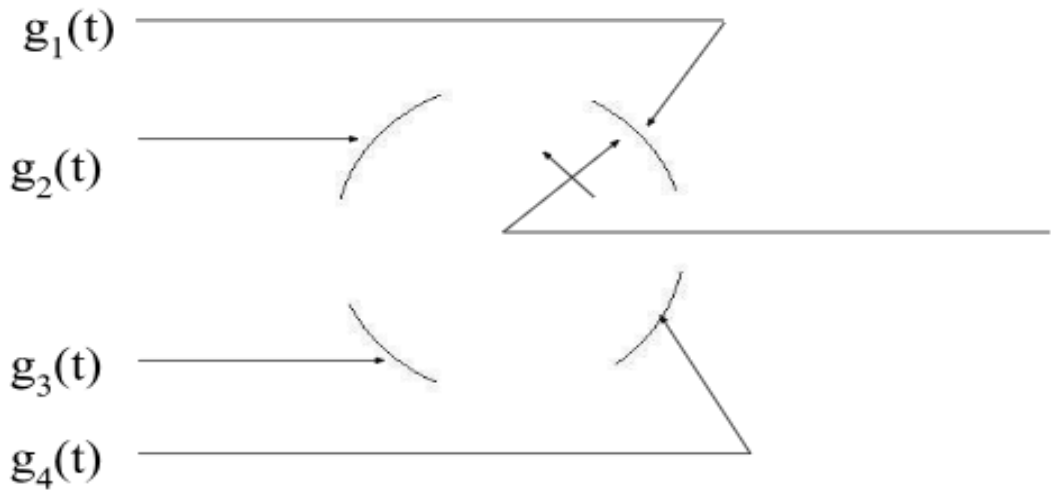
A PAM/TDM SYSTEM

THIS SYSTEM COMBINES THE CONCEPTS OF PAM
& TDM

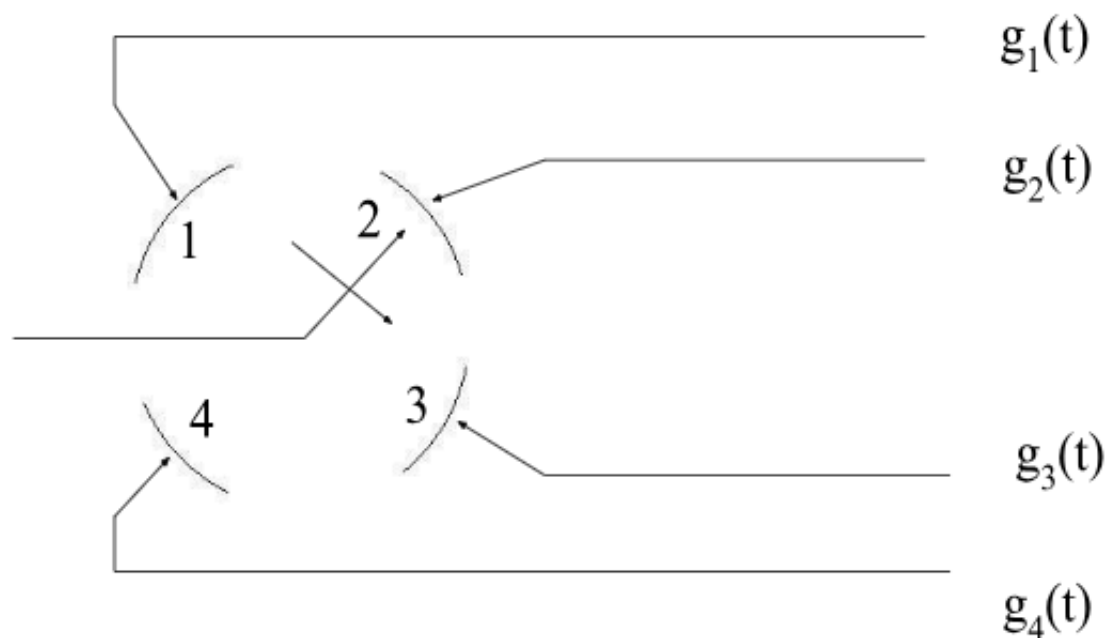
TDM-PAM : Receiver



Commutator Arrangement



Decommutator



5.17 A PAM/TDM SYSTEM

(i) Definition

Now, let us discuss a PAM/TDM system. Infact, this system combines the concepts of PAM and TDM both as shown in figure 5.38.

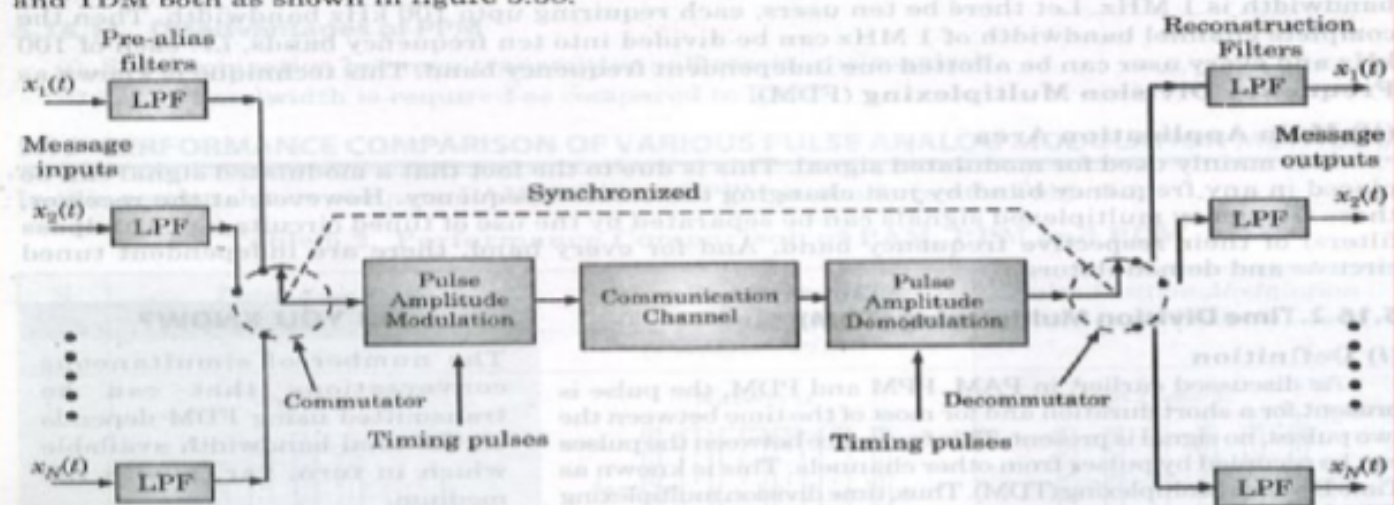
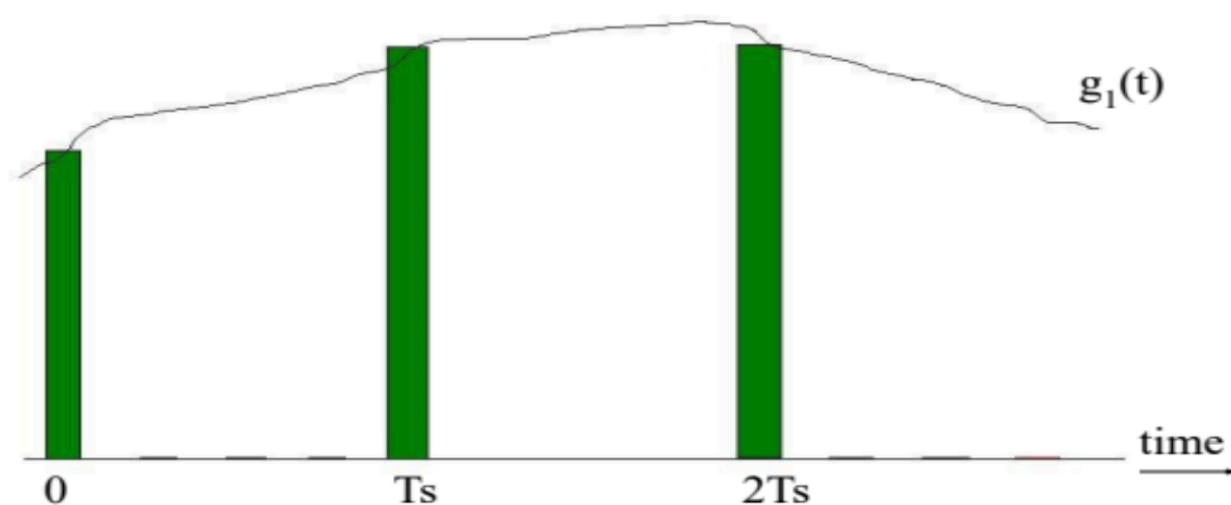


Fig. 5.38 Block diagram of a PAM/TDM system

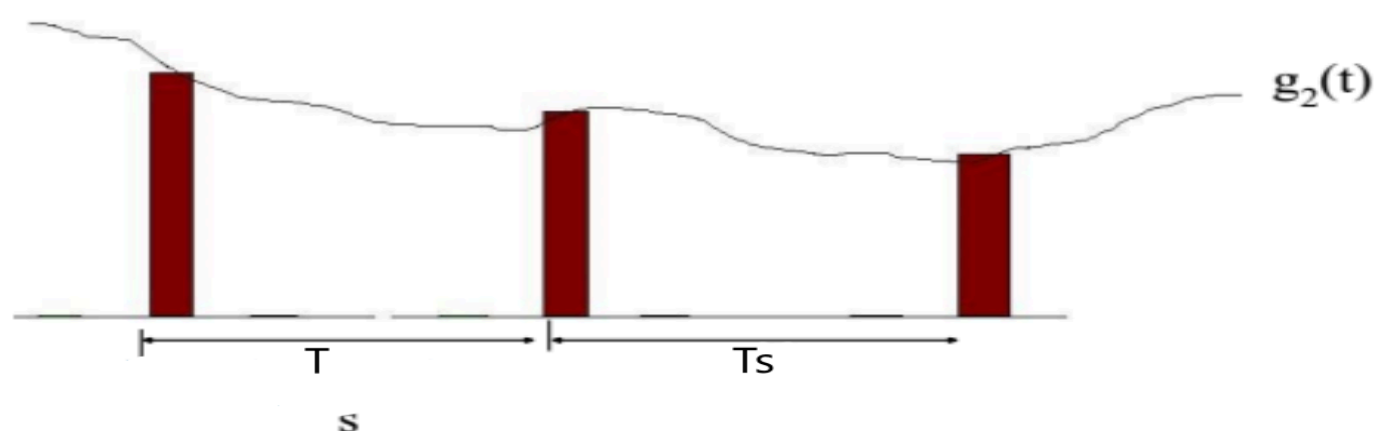
Working principle

- The multiplexer is a single pole rotating switch or commutator.
- It can be mechanical or electronic switch rotate at F_s rotation per second
- As the switch arm rotates, it is going to make contact with the points 1,2,3 or N for a short time
- If there are N signals connected to these contact points, hence the switch arm will connect these N input signals **one by one** to the communication channel.
- Hence, the rotating switch samples each message during each of its rotations
- Since each rotation corresponds to one frame, therefore, one frame is completed in T_s second where $T_s = 1/F_s$
- Hence the function of the commutator is two:
 - I. To take narrow sample of each input message at a rate F_s
 - II. To sequentially interleave the **N** samples inside the interval $T_s = 1/F_s$

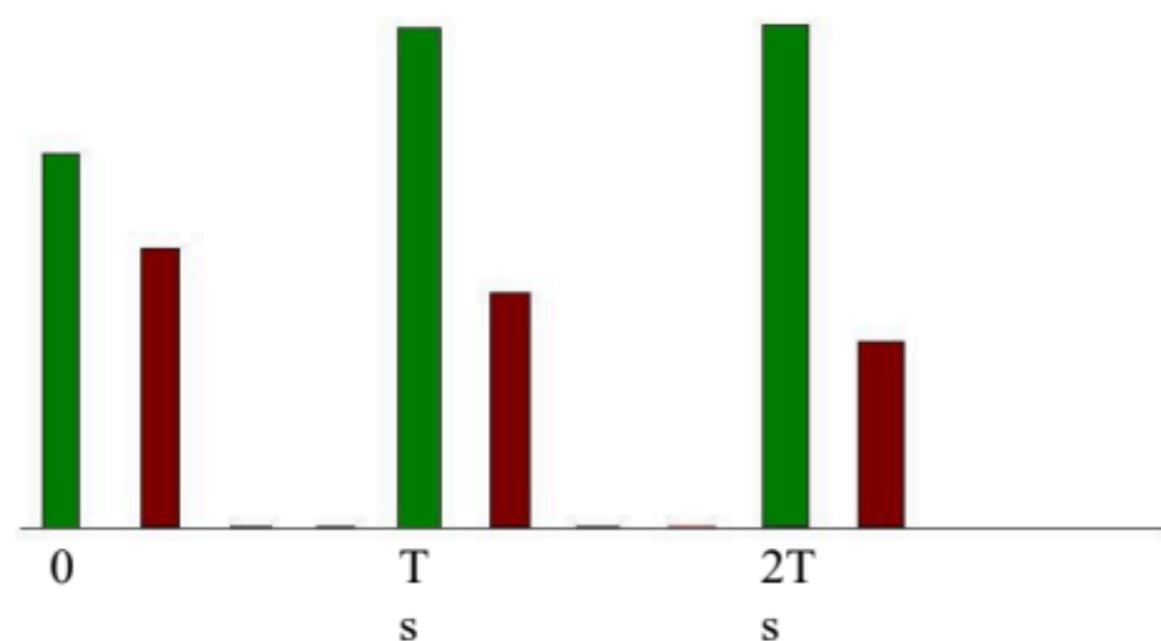
Samples of Signal - 1



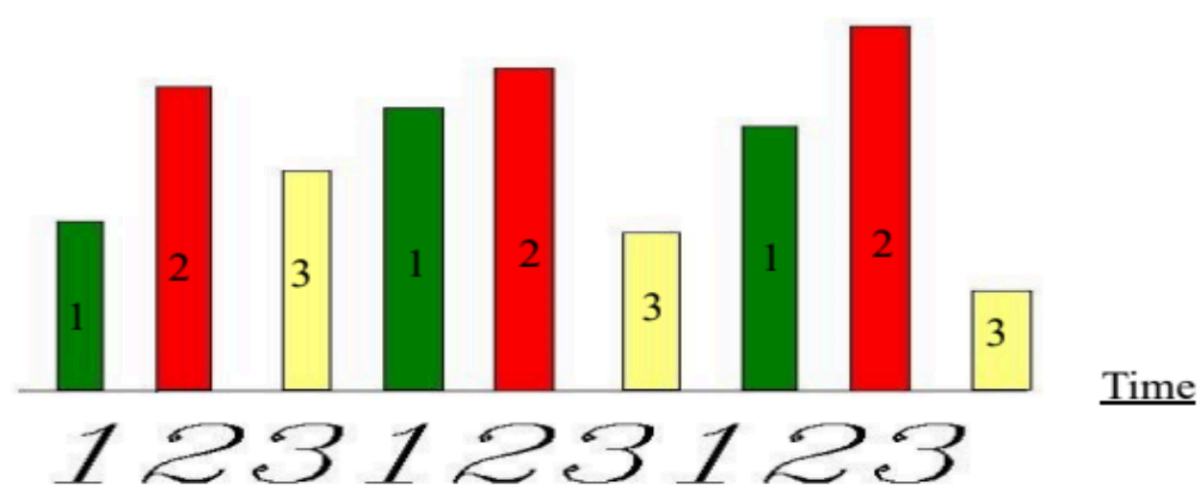
Samples of signal - 2



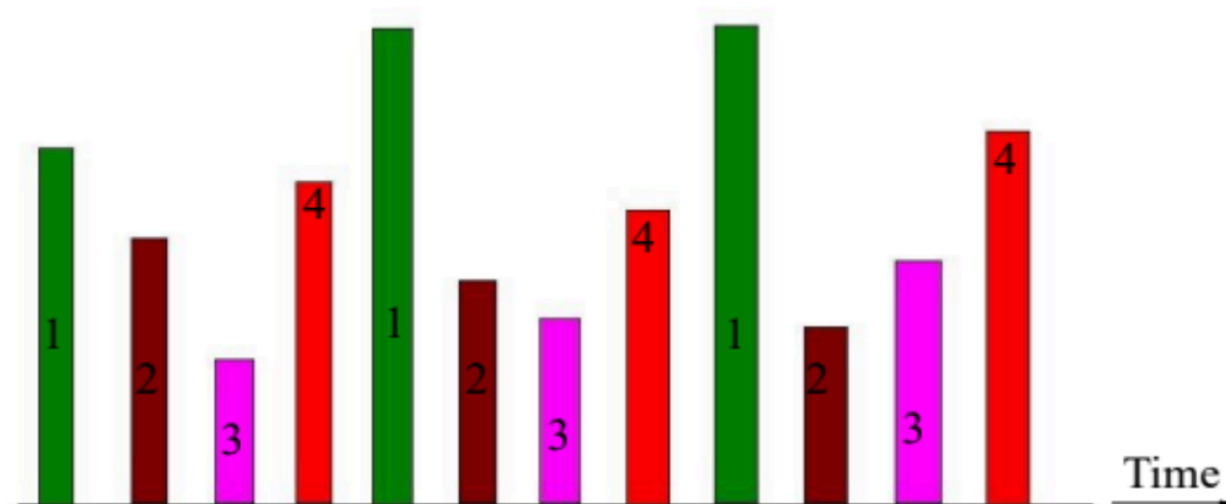
Multiplexing of TWO signals



TDM-PAM for 3 signals



TDM-PAM for 4 signals.



Signaling rate (r)

1. Signaling Rate (r):

- The signaling rate is the number of pulses transmitted per second.
- Given F_m as the maximum frequency of the input signals X_1 to X_n , the sampling frequency F_s must satisfy the Nyquist criterion, so $F_s > 2F_m$.
- The commutators rotate at F_s rotations per second. Each rotation corresponds to one complete frame, which includes samples from all N input signals.

2. Frame and Pulse Definitions:

- **1 Rotation of Commutator:** This corresponds to one frame that contains one sample from each of the N input signals.
- **1 Frame Period (T_s):** The duration of one frame is $T_s = \frac{1}{F_s}$ seconds.
- **Number of Pulses in One Frame:** Each frame contains N pulses, one from each input signal.

3. Pulse to Pulse Spacing:

- The pulse-to-pulse spacing within a frame is given by:

$$\text{Pulse to pulse spacing} = \frac{T_s}{N} = \frac{1}{NF_s}$$

4. Pulse Duration:

- The duration of each pulse is equal to the pulse-to-pulse

spacing:

$$\text{Pulse duration} = \frac{1}{NF_s}$$

5. Signaling Rate Calculation:

- The number of pulses transmitted per second can be calculated as:

$$\text{Signaling rate (r)} = \frac{1}{\text{Pulse duration}} = NF_s \text{ pulses per second}$$

6. Minimum Condition for $F_s \geq 2F_m$:

- Since $F_s \geq 2F_m$ (by Nyquist sampling theorem), the signaling rate becomes:

$$r \geq 2NF_m \text{ pulses per second}$$

7. Transmission Bandwidth of PAM/TDM Channel:

- The minimum transmission bandwidth BW for a PAM-TDM system is half the signaling rate:

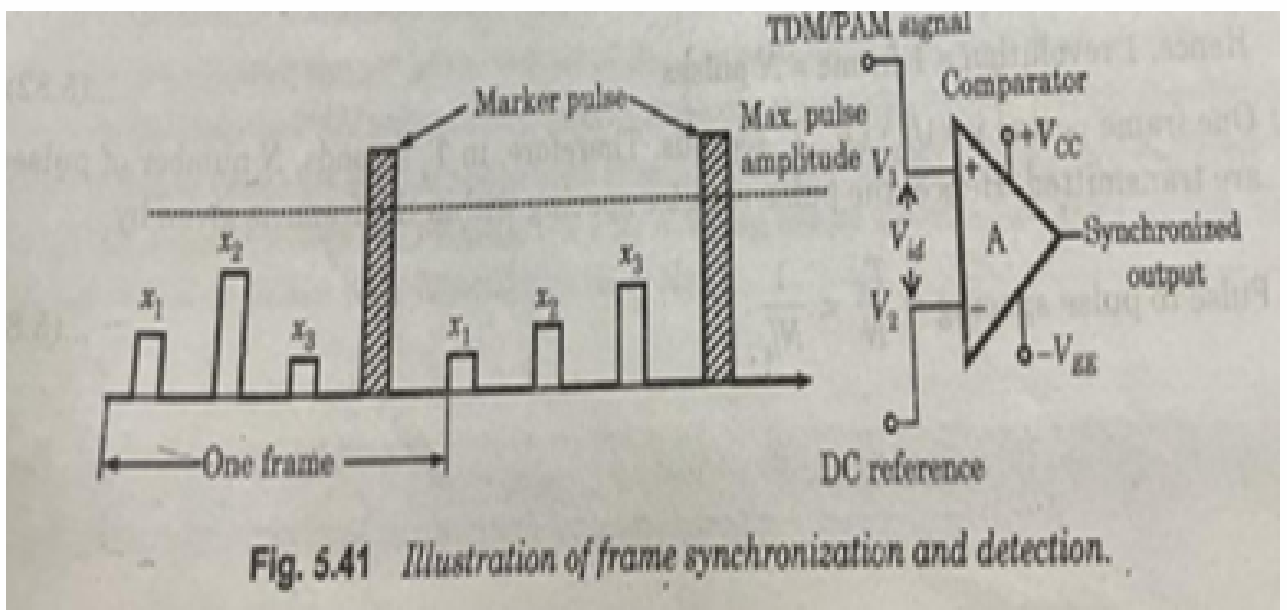
$$BW = \frac{1}{2}r$$

Substituting the minimum signaling rate r:

$$BW \geq \frac{1}{2}(2NF_m) = NF_m$$

Synchronization and its Signification in PAM/TDM system

- The multiplexed PAM signals can be received properly if and only if the transmitter and receiver commutators are synchronized to each other in terms of the speed and the position.
- In order to ensure synchronization, a marker pulse is introduced at the end of each frame in the transmitted signals.
- The amplitude of this pulse is kept higher than the maximum permissible amplitude of the multiplexed signals
- At the receiver, the received signal is compared with a DC reference level.
- The comparator responds to only the marker pulse to produce pulse.

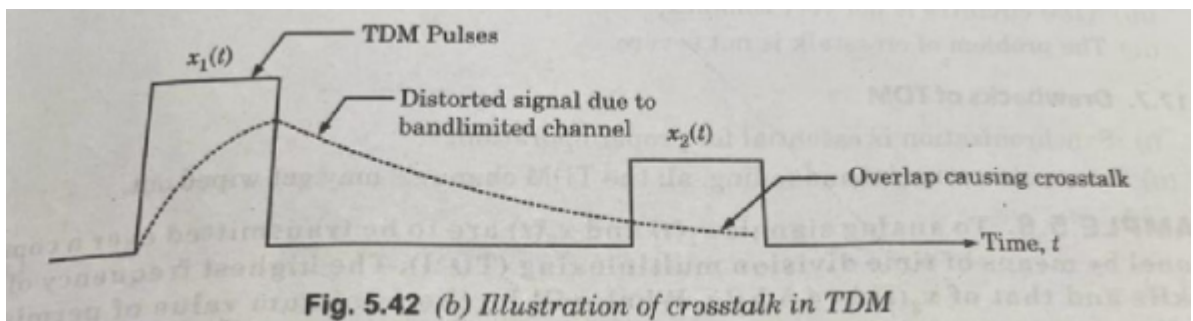
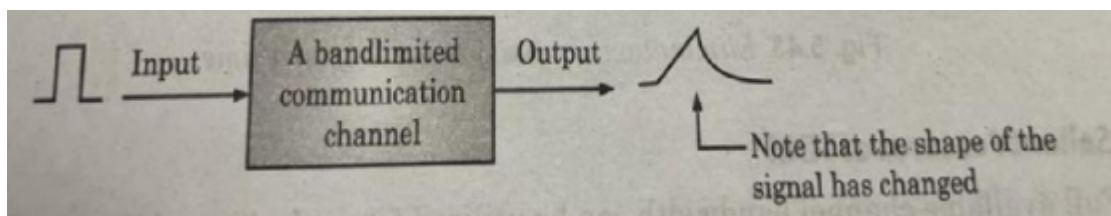


Crosstalk in a PAM/TDM system

- Crosstalk basically means interference between adjacent signals
- It is an unwanted coupling of information from one channel to other.
- To prevent crosstalk, a guard time T_g , is introduced between the adjacent TDM signals.

Let us note few important points about crosstalk as under

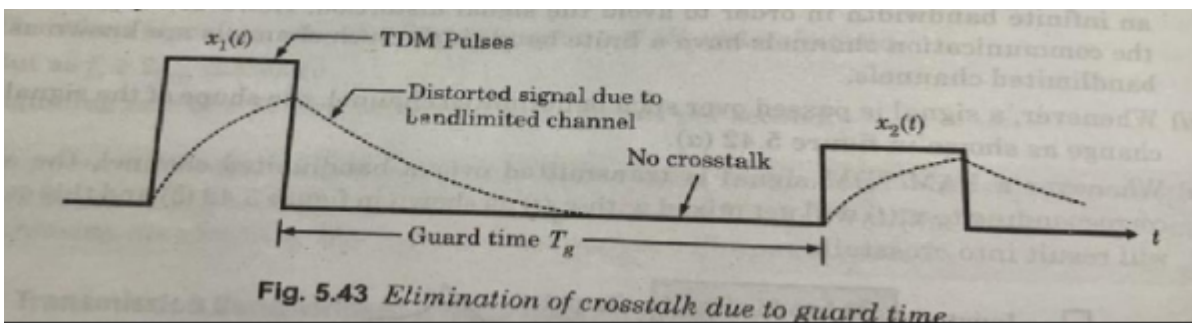
- (i) The communication channel over which the TDM signal is travelling should ideally have an infinite bandwidth in order to avoid the signal distortion. However, in practice, all the communication channels have a finite bandwidth. Such channels are known as the bandlimited channels.
- (ii) Whenever, a signal is passed over such bandlimited channel, the shape of the signal will change as shown in figure 5.42 (a).
- (iii) Whenever a PAM/TDM signal is transmitted over a bandlimited channel, the signal corresponding to $x_1(t)$ will get mixed with $x_2(t)$, as shown in figure 5.42 (b) and this overlap will result into crosstalk.



- (vi) One more reason for the crosstalk between the adjacent TDM signals is the use of bandlimiting filters. Because of these filters, the shapes of the TDM pulses are distorted and they get overlapped and crosstalk will take place.

Concept of Guard Time

- (i) **Definition:** The crosstalk resulting from the pulse overlap shown in figure 5.13(b) can be reduced by introducing guard time of sufficient duration between the adjacent TDM pulses as shown in figure 5.43. The guard time is denoted by T
- (ii) **Mathematical Expression** If the crosstalk is to be kept below -30 dB, then the value of guard time (T) that should be introduced between the adjacent TDM signals is given by. Guard time = $0.55 / B$ where B = Bandwidth of the channel. The equation (5.84) shows that the guard time required to avoid the crosstalk will decrease with increase in the channel bandwidth. As T reduces, we can increase the signaling rate of the PAM/TDM system.



Important points

Salient Features and Drawbacks of TDM

- Salient Features
 - Full available channel Bandwidth can be utilized for each signal
 - Intermodulation distortion is absent
 - TDM circuitry is not very complex
 - The problem of crosstalk is not severe
- Drawbacks
 - synchronization is essential for proper operation
 - Due to slow narrowband fading, all the TDM signals may get wiped out

Examples

Example 5.9: Two analog signals $x_1(t)$ and $x_2(t)$ are to be transmitted over a common channel by means of time division multiplexing (TDM). The highest frequency of $x_1(t)$ is 4 kHz and that of $x_2(t)$ is 4.5 kHz. What will be the minimum value of permissible sampling rate?

Solution: The highest frequency component of the composite signal consisting of $x_1(t)$ and $x_2(t)$ is 4.5 kHz. Therefore, the minimum value of permissible sampling rate will be,

$$f_{s(\min)} = 2 \times 4.5 \text{ kHz} = 9 \text{ kHz Ans.}$$

Example 5.10: a signal $x_1(t)$ is bandlimited to 3 kHz. There are three more signals $x_2(t)$, $x_3(t)$ And $x_4(t)$ which are bandlimited to 1 kHz each. These signals are to be transmitted by a TDM system.

1. Design a TDM scheme where each signal is sampled at its Nyquist rate
2. What must be the speed of the commutator
3. Calculate the minimum transmission bandwidth of the channel.

Solution:

(i) Table 5.3 shows different message signals with corresponding Nyquist rates.

S.No.	Message signal	Bandwidth	Nyquist rate
1.	$x_1(t)$	3 kHz	6 kHz
2.	$x_2(t)$	1 kHz	2 kHz
3.	$x_3(t)$	1 kHz	2 kHz
4.	$x_4(t)$	1 kHz	2 kHz

If the sampling commutator rotates at the rate of 2000 rotations per second then the signals $x_2(t)$, $x_3(t)$ and $x_4(t)$ will be sampled at their Nyquist rate. But, we have to sample $x_1(t)$ also at its Nyquist rate which is three times higher than that of the other three. In order to achieve this, we should sample $x_1(t)$ three times in one rotation of the commutator. Therefore, the commutator must have at least 6 poles connected to the signals as shown in figure 5.44.

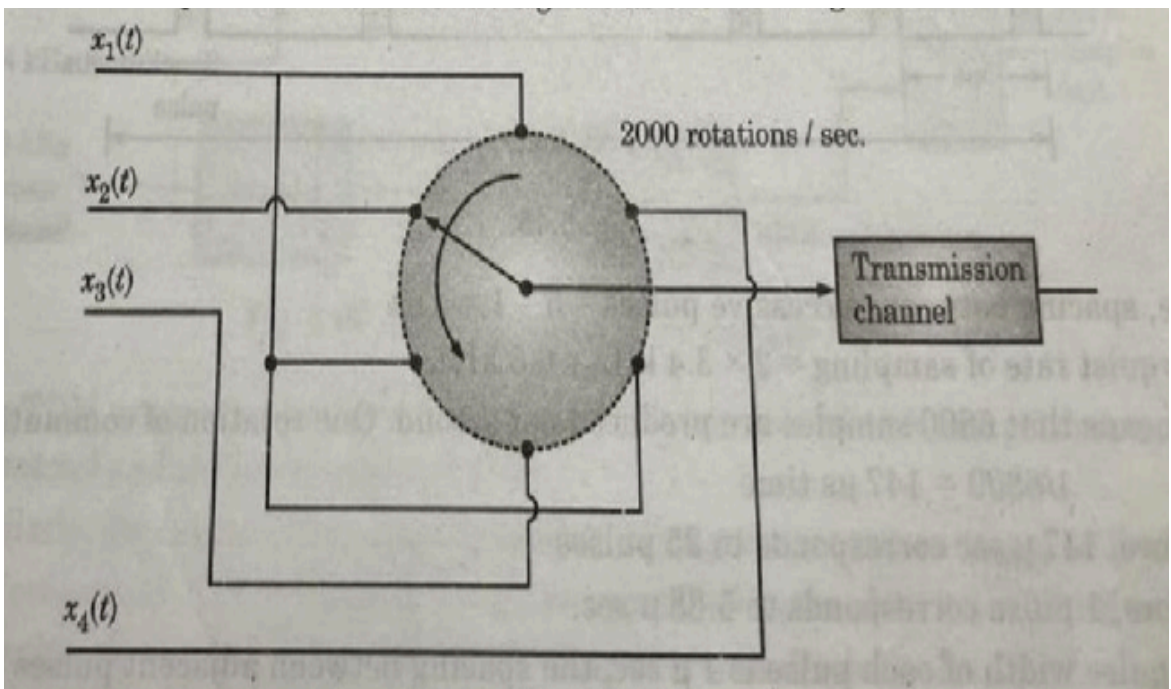


Fig. 5.44.

(ii) The speed of rotation of the commutator is 2000 rotation/sec.

(iii) Number of samples produced per second is calculated as under:

$x_1(t)$ produces $3 \times 2000 = 6000$ samples/sec.

$x_2(t)$, $x_3(t)$ and $x_4(t)$ produce 2000 samples/sec. Each.

Therefore, number of samples per second = $6000 + (3 \times 2000) = 12000$ samples/sec.

* Signaling rate = 12000 samples/sec.

(iv) The minimum channel bandwidth will be

$$BW = \frac{1}{2} (\text{signaling rate}) = 12000/2 = 6000\text{Hz Ans.}$$

Example

Twenty- four voice signals are sampled uniformly and then Time Division Multiplexed.

the sampling operation uses flat – top samples with 1 μ s. Duration. the multiplexing operation includes provision for synchronization by adding an extra pulse of appropriate amplitude and 1 μ s. Duration.

The highest frequency component of each voice signal is 3.4 KHz.

(i) Assuming a sampling rate of 8kHz, calculate the spacing between successive pulses of the multiplexed signal.

(ii) Repeat (i) assuming the use of Nyquist rate sampling.

Solution: (i) Given that

Sampling rate = 8kHz = 8000 samples/sec.

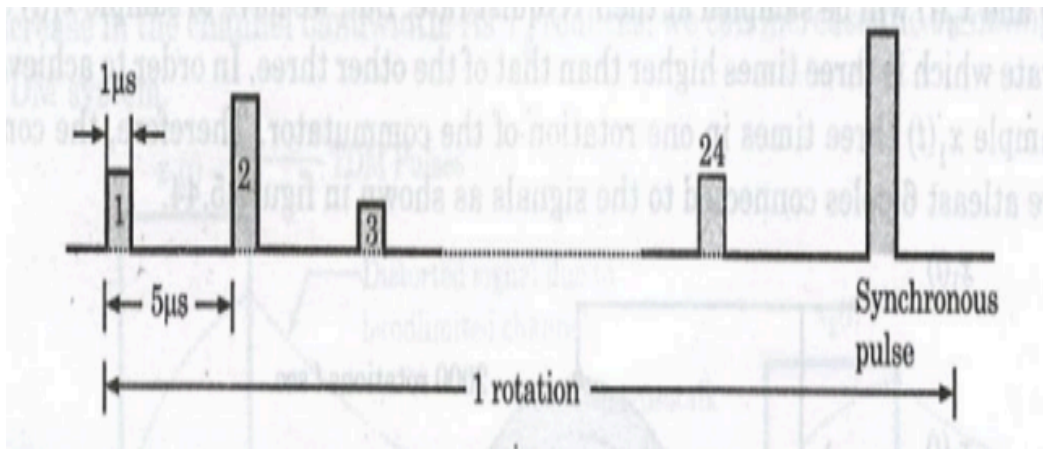
There are 24 voice signals + 1 synchronizing pulse.

Pulse width of each voice channel and synchronizing pulse is 1 μ s.

Now, time taken by the commutator for 1 rotation = $1/8000$
= 125 μ sec.

Number of pulses produced in 1 rotation = $24 + 1 = 25$

Therefore, the leading edges of the pulses are at $125/25 = 5 \mu$ seconds distance as shown in figure 5.45.



Hence, spacing between successive pulses = $5 - 1 = 4 \mu$ s

(ii) Nyquist rate of sampling = $2 \times 3.4 \text{ kHz} = 6.8 \text{ kHz}$.

This means that 6800 samples are produced per second. One rotation of commutator takes

$$1/6800 = 147 \mu\text{s time}$$

Therefore, 147 μ sec corresponds to 25 pulses

Therefore, 1 pulse corresponds to 5.88 μ sec.

As the pulse width of each pulse is 1 μ sec. The spacing between adjacent pulses will be 4.88 μ sec and if we assume $\square=0$ then the spacing between the adjacent pulses will be 5.88 μ sec.

Examples 5.12. Six message signals each of bandwidth 5 kHz are time division multiplexed and transmitted. Determine the signaling rate and the minimum channel bandwidth of the PAM/TDM channel.

Solution: The number of channels $N=6$.

Bandwidth of each channel, $f_m = 5 \text{ kHz}$

Therefore, minimum sampling rate $= 2 \times 5 \text{ kHz} = 10 \text{ kHz}$.

Signaling rate = Number of bits per second

$= 6 \times 10 \text{ kHz} = 60 \text{ K bits/sec}$. Ans.

Minimum channel bandwidth to avoid cross talk in PAM/TDM is,

$BW = Nf_m = 6 \times 5 \text{ kHz} = 30 \text{ kHz}$ Ans.

Example 1:

The following signals are sampled each at its Nyquist rate and multiplexed and transmitted by using TDM:

$S_1(t)$ bandlimited to 1.8 KHz

$S_2(t)$ bandlimited to 3.6 KHz

$S_3(t)$ bandlimited to 0.6 KHz

$S_4(t)$ bandlimited to 0.6 KHz

$S_5(t)$ bandlimited to 0.6 KHz

Solution:

number of pieces

F_s for $S_1(t) = 2 \cdot f_{\max} = 2 \cdot 1.8 = 3.6 \text{ KS/sec}$ 3

F_s for $S_2(t) = 2 \cdot f_{\max} = 2 \cdot 3.6 = 7.2 \text{ KS/sec}$ 6

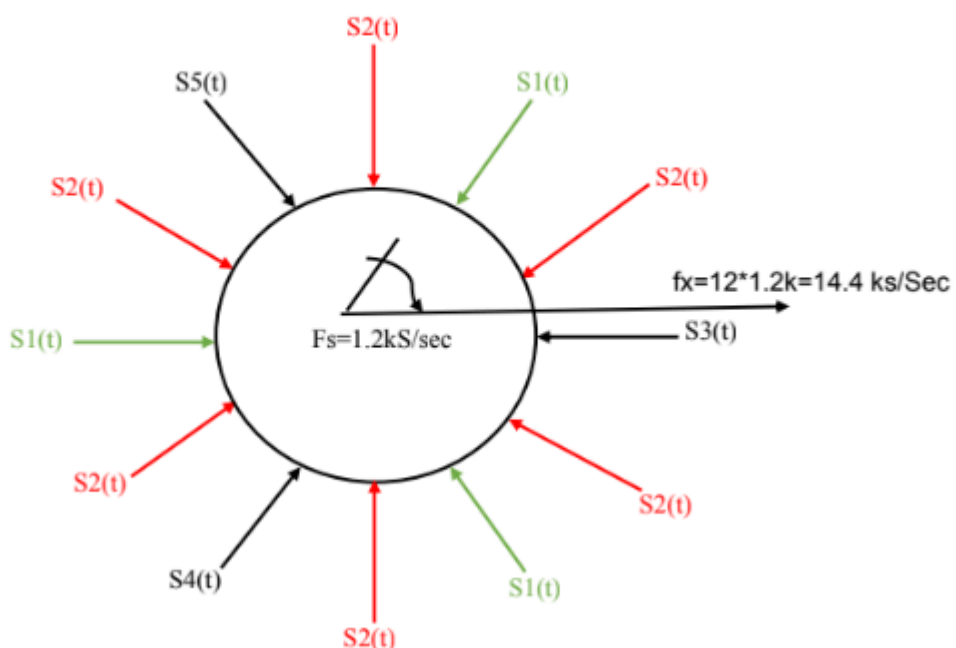
F_s for $S_3(t) = 2 \cdot f_{\max} = 2 \cdot 0.6 = 1.2 \text{ KS/sec}$ 1

F_s for $S_4(t) = 2 \cdot f_{\max} = 2 \cdot 0.6 = 1.2 \text{ KS/sec}$ 1

F_s for $S_5(t) = 2 \cdot f_{\max} = 2 \cdot 0.6 = 1.2 \text{ KS/sec}$ 1

The sum of pieces = 12

$12/3=4, 12/6=2, 12/1=12$



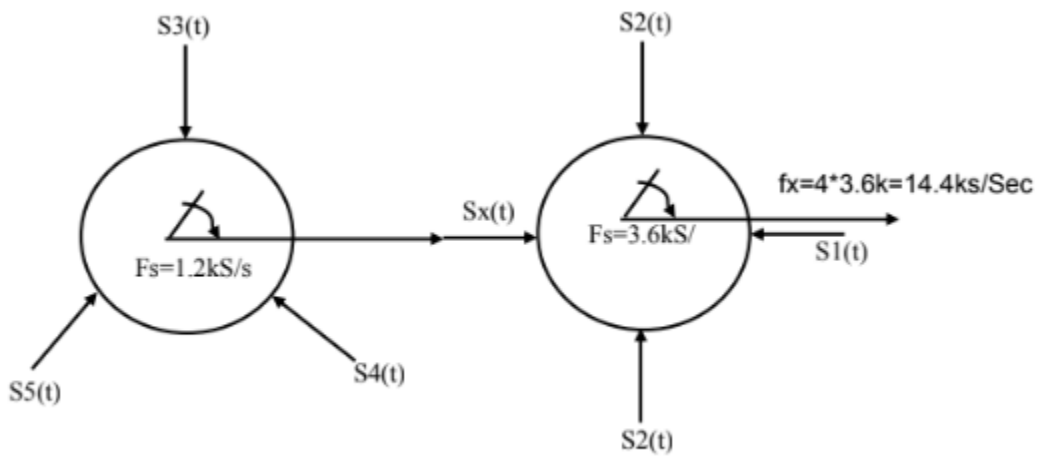
$$T_s = 1/1.2$$

$$T_x = (1/1.2)/12 = 1/14.4 \text{ msec}$$

$$F_x = 14.4 \text{ K S/sec}$$

$$BW = 1/2 f_x = 1/2 * 14.4 = 7.2 \text{ KHz}$$

Other solution:



$$T_s = 1/3.6$$

$$T_x = (1/3.6)/4 = 1/14.4 \text{ msec}$$

$$F_x = 14.4 \text{ K S/sec (Or } F_x = F_s * \text{ number of pieces} = 3.6 * 4 = 14.4 \text{ KS/sec)}$$

$$BW = 1/2 f_x = 1/2 * 14.4 = 7.2 \text{ KHz}$$

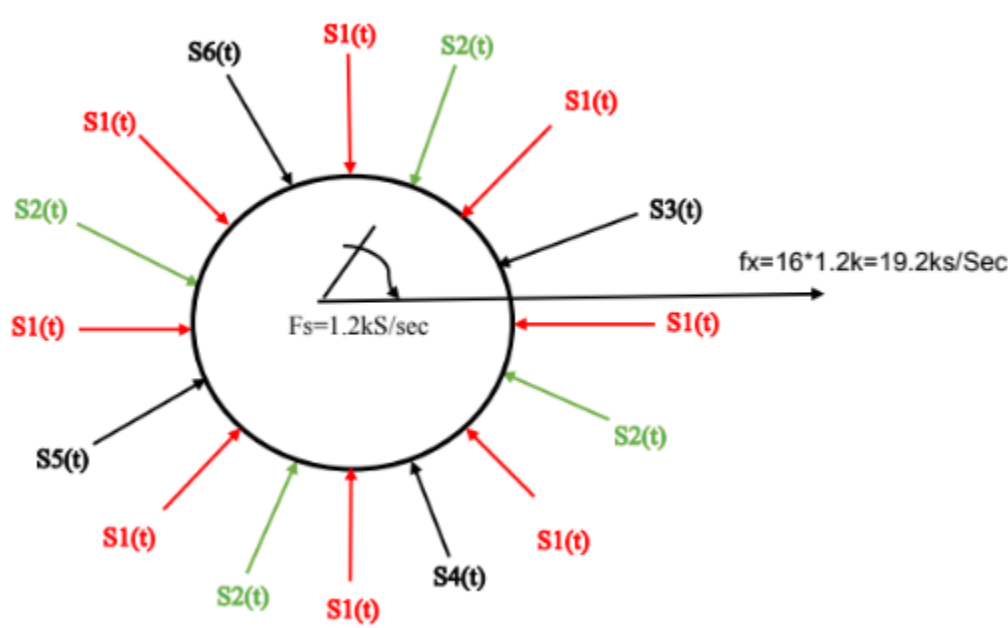
Example 2:

The following signals are sampled each at its Nyquist rate and multiplexed and transmitted by using TDM:

- S1(t) bandlimited to 4.8 KHz
- S2(t) bandlimited to 2.4 KHz
- S3(t) bandlimited to 0.6 KHz
- S4(t) bandlimited to 0.6 KHz
- S5(t) bandlimited to 0.6 KHz
- S6(t) bandlimited to 0.6 KHz

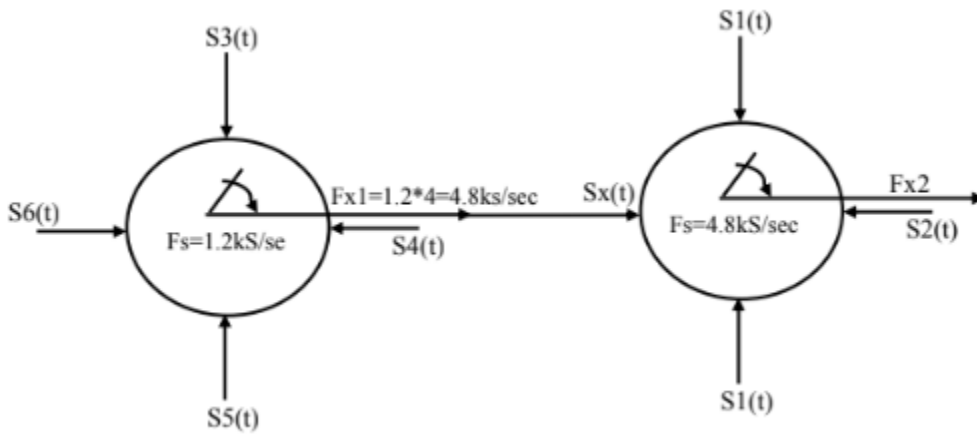
Solution:

	number of pieces
Fs for S1(t)= 2*fmax =2*4.8=9.6KS/sec	8
Fs for S2(t)= 2*fmax =2*2.4=4.8KS/sec	4
Fs for S3(t)= 2*fmax =2*0.6=1.2KS/sec	1
Fs for S4(t)= 2*fmax =2*0.6=1.2KS/sec	1
Fs for S5(t)= 2*fmax =2*0.6=1.2KS/sec	1
Fs for S6(t)= 2*fmax =2*0.6=1.2KS/sec	1
The sum of pieces= 16	
16/8=2, 16/4=4, 16/1=16	



$F_x = F_s \cdot \text{number of pieces} = 1.2 \cdot 16 = 19.2 \text{ KS/sec}$
 $BW = \frac{1}{2} f_x = \frac{1}{2} \cdot 19.2 = 9.6 \text{ KHz}$

Other solution



$$F_{x2} = 4.8 * 4 = 19.2 \text{ KS/sec}$$

$$BW = \frac{1}{2} f_{x2} = \frac{1}{2} * 19.2 = 9.6 \text{ KHz}$$

Example 3:

The following 36 channels are to be multiplexed using TDM:

1 channel sampled at 8KHz

1 channel sampled at 4KHz

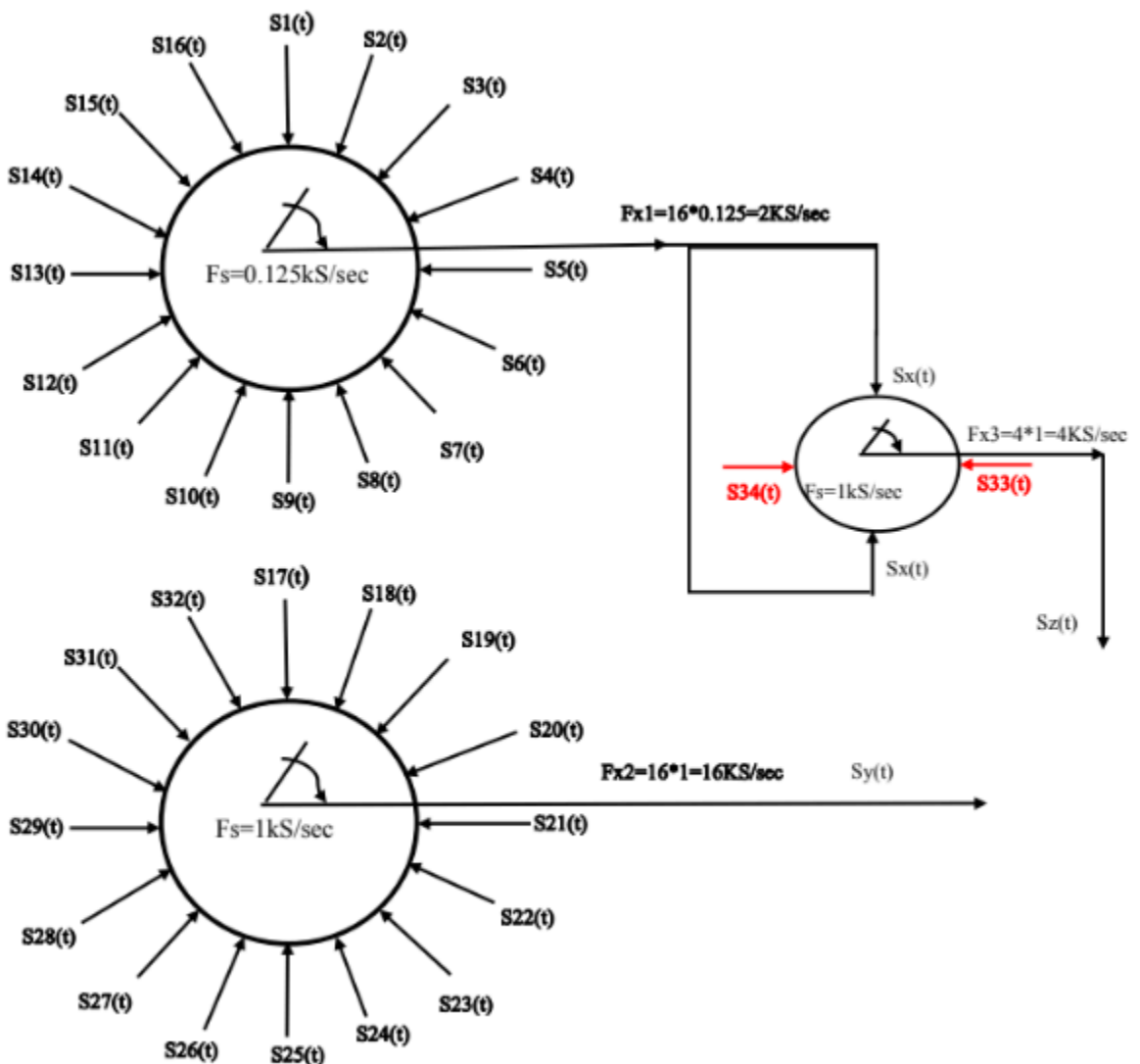
18 channel sampled at 1KHz

16 channel sampled at 0.125KHz

Solution:

First, we notice that every channel was sampled which mean that the above frequencies given are F_s .

To solve this example, we have 16 channels sampled at 0.125KHz will sub-commutation firstly.

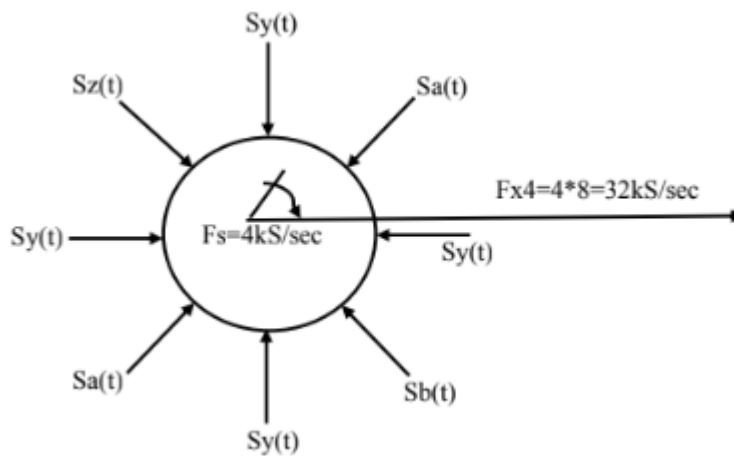


If we looking at the remaining signals and the produced signals from the above TDM system we have:

	number of pieces
1 channel sampled at 8KHz [Sa(t)]	2
1 channel sampled at 4KHz [Sb(t)]	1
1 channel sampled at 16KHz [Sy(t)]	4
1 channel sampled at 4KHz [Sz(t)]	1

The sum of pieces = 8

$$8/2=4, 8/1=8, 8/4=2$$



$$BW= 1/2 f_{x4} = 1/2 * 32= 16 \text{ KHz}$$

If the above example was modified to be, Design TDM system to transmit these channels:

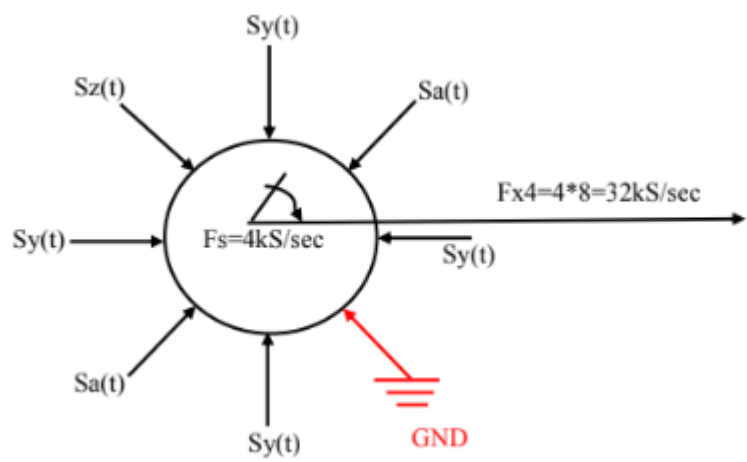
- 1 channel sampled at 8KHz
- 18 channel sampled at 1KHz
- 16 channel sampled at 0.125KHz

The solution would be the as the first part of the previous example, the difference would be in the second part:

	number of pieces
1 channel sampled at 8KHz [Sa(t)]	2
1 channel sampled at 16KHz [Sy(t)]	4
1 channel sampled at 4KHz [Sz(t)]	1
The sum of pieces = 7	

We notice that the summation of pieces divisible with remainder. To solve this problem, we add an additional channel which called ground (GND) to the other channels so the sum of pieces =8

	number of pieces
1 channel sampled at 8KHz [Sa(t)]	2
1 channel sampled at 16KHz [Sy(t)]	4
1 channel sampled at 4KHz [Sz(t)]	1
1 channel GND	1
The sum of pieces = 8	



$BW= 1/2 f_{x4} = 1/2 * 32= 16 \text{ KHz}$

Sampling and TDM Quiz

Four (4) sinusoidal signals (in phase):

$$x_1(t) = 1 \sin(2\pi 1000t), \quad x_2(t) = 3 \sin(2\pi 1000t)$$

$$x_3(t) = 6 \sin(2\pi 1000t), \quad x_4(t) = 9 \sin(2\pi 1000t)$$

are sampled and Time Division Multiplexed at $F_s = 4\text{KHZ}$

The four signal are connected to a commutator with the sequence that:

The first one at 45° , the second at 135° , the third at 225° and the fourth at 315°

The synchronization is done by adding a mark pulse at 345.6°

The contact time for each signal and mark generator is 0.01 msec.

Solve for:

1. Sketch the TDM system scheme
2. Find the value of each sample (at beginning and end)
3. Draw the time diagram of sampled signal
4. Find the signaling rate and bandwidth of the multiplexed signal.

sol:

$$T_s = \frac{1}{F_s} = \frac{1}{4 \cdot 10^3} = \frac{1}{4} * 10^{-3}$$

$$T = 0.25 \text{ msec}$$

$$S_1 = \frac{45}{360} * 0.25\text{m} = 0.03125\text{m}$$

$$X_1(t) = 1 \sin(2\pi * 10^3 * 0.03125 * 10^{-3})$$
$$= 1 \sin(2\pi * 0.03125)$$

$$X_1(t) = 0.19499 \rightarrow S_1$$

$$S_2 = \frac{135}{360} * 0.25\text{m} = 0.09375\text{msec}$$

$$X_2(t) = 3 \sin(2\pi * 10^3 * 0.09375 * 10^{-3})$$

$$X_2(t) = 3 \sin(2\pi * 0.09375)$$

$$X_2(t) = 1.665 \rightarrow S_2$$

$$S_3 = \frac{225}{360} * 0.25\text{m} = 0.15625\text{msec}$$

$$X_3(t) = 6 \sin(2\pi * 10^3 * 0.15625 * 10^{-3})$$

$$X_3(t) = 6 \sin(2\pi * 0.15625)$$

$$X_3(t) = 4.987 \rightarrow S_3$$

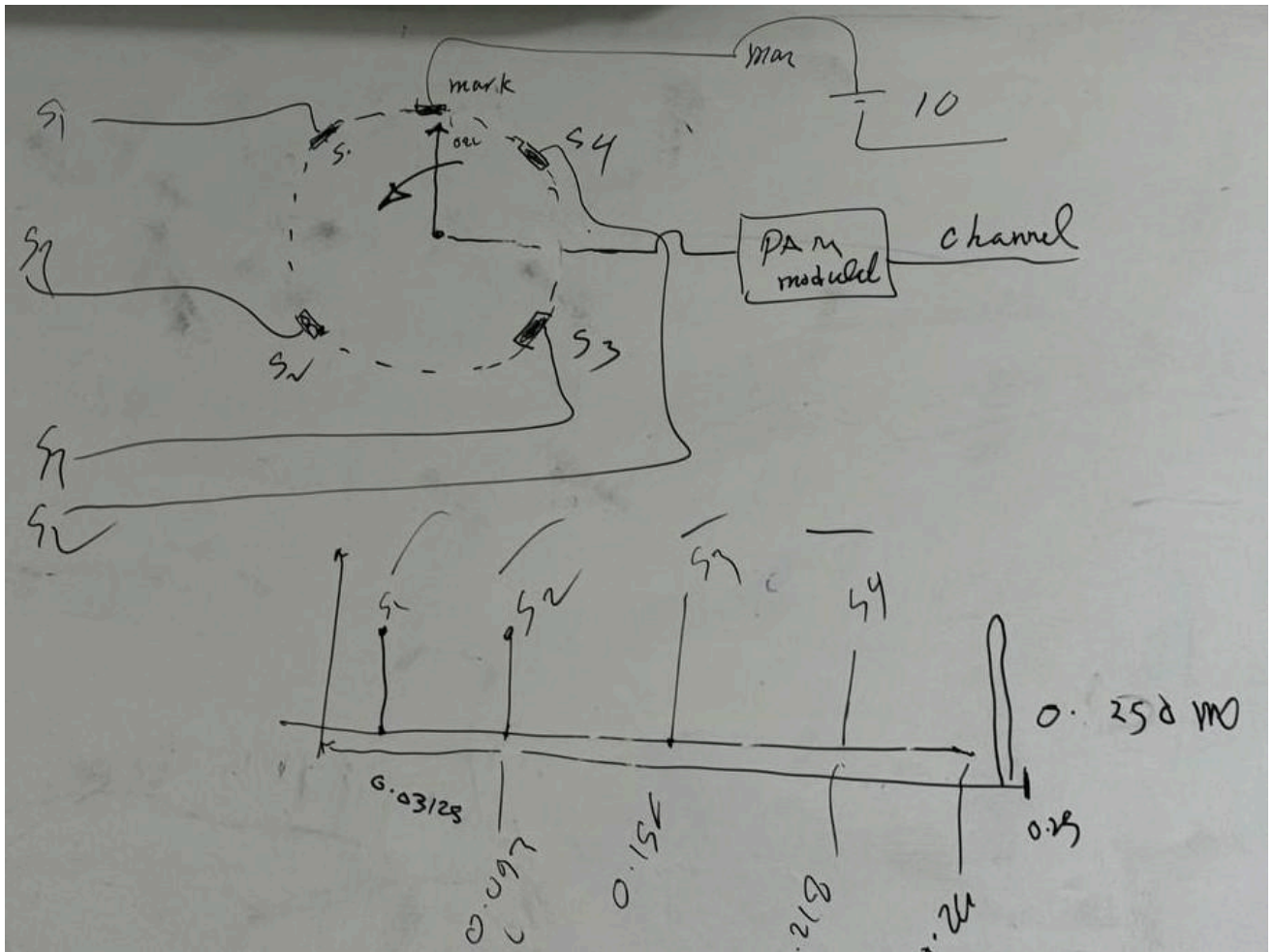
$$S_4 = \frac{315}{360} * 0.25\text{m} = 0.20625\text{msec}$$

$$X_4(t) = 9 \sin(2\pi * 10^3 * 0.20625 * 10^{-3})$$

$$X_4(t) = 9 \sin(2\pi * 0.20625)$$

$$X_4(t) = 8.66 \rightarrow S_4$$

$$S_5 \text{ marker} = \frac{345.6}{360} * 0.25\text{m} = 0.25\text{msec}$$



$$T_s = 0.25\text{m}$$

$$T_x = \frac{T_s}{N} = \frac{0.25 * 10^{-3}}{5} = 0.05\text{m}$$

$$r = \frac{1}{T_x} = \frac{1}{0.05 * 10^{-3}} = 20\text{k}$$

$$\text{Bus} = \frac{1}{2} r = \frac{1}{2} 20\text{k} = 10\text{khz}$$

Sampling and TDM Quiz

Four (4) sinusoidal signals (in phase):

$$x_1(t) = 2 \sin(2\pi 1000t), x_2(t) = 4 \sin(2\pi 1000t)$$

$$x_3(t) = 6 \sin(2\pi 1000t), x_4(t) = 8 \sin(2\pi 1000t)$$

are sampled and Time Division Multiplexed at

$$F_s = 4\text{KHZ}$$

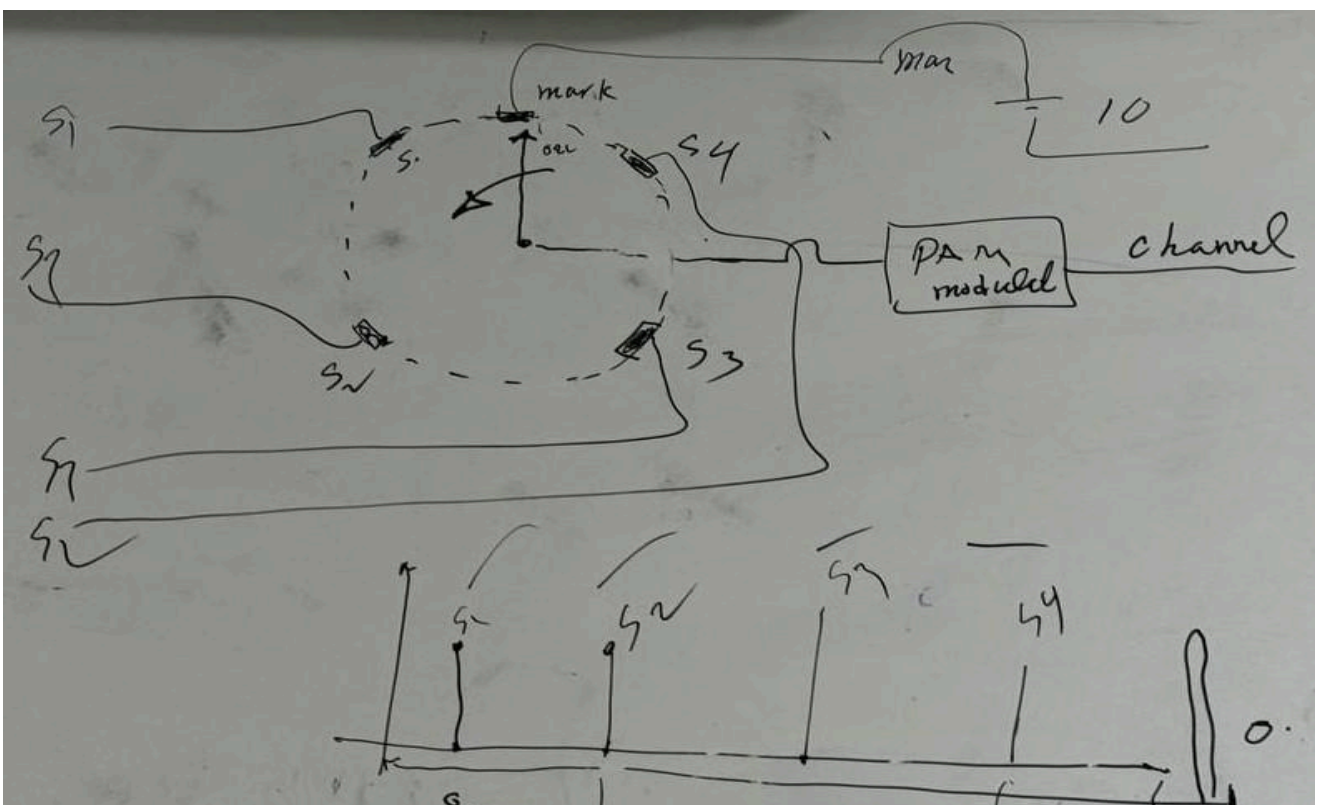
The four signal are connected to a commutator with the sequence that:

The first one at 45° , the second at 135° , the third at 225° and the fourth at 315° . The synchronization is done by adding a mark pulse at 345.6°

The contact time for each signal and mark generator is 0.01 msec.

Solve for:

1. Sketch the TDM system scheme
2. Find the value of each sample (at beginning and end)
3. Draw the time diagram of sampled signal
4. Find the signaling rate and bandwidth of the multiplexed signal.



Since $F_s = 4\text{KHZ}$ then $T_s = 1/4 \times 10^3 \text{ sec.} = 0.25 \text{ msec.}$
 The commutator complete 1 revolution within this time.
 (360° in 0.25 msec.)

The commutator start contact $x_1(t)$ at $t = (45/360) * 0.25 = 0.03125 \text{ msec.}$

The time for each degree = $0.25 / 360 \text{ msec.}$

The commutator end contact $x_1(t)$ at $t = 0.03125 + 0.01 = 0.04125 \text{ msec.}$

The same for $x_2(t)$, $x_3(t)$ and $x_4(t)$

For mark pule, it starts at $t = (345.6/360) \times 0.25 = 0.24 \text{ msec.}$ It ends at $t = 0.24 + 0.01 = 0.25$

Signa l	Start time (msec.)	End time (msec.)
X1(t)	0.03125	0.04125
X2(t)	0.21875	0.22875
X3(t)	0.15625	0.16625
X4(t)	0.21875	0.22875
Mark pulse	0.24	0.25

Chapter 6 : PULSE DIGITAL MODULATION or PULSE CODE MODULATION

To transport an information signal from one point to another over a communication channel, we can use digital or analog techniques.

The use of digital communication offers several advantages as compared with to analog communication. such as:

- ruggedness to channel noise**
- flexible operation of the system**
- common format for all information source**
- security of information during the transmission.**

To transmit analog signal by digital communication system , the signal has to undergo an analog to digital conversion.

In PAM the analog signal is converted from analog form(analog in amplitude and time) to signal discrete in time and analog in amplitude.

To completely conversion of analog signal to digital one, addition process called waveform coding technique is required.

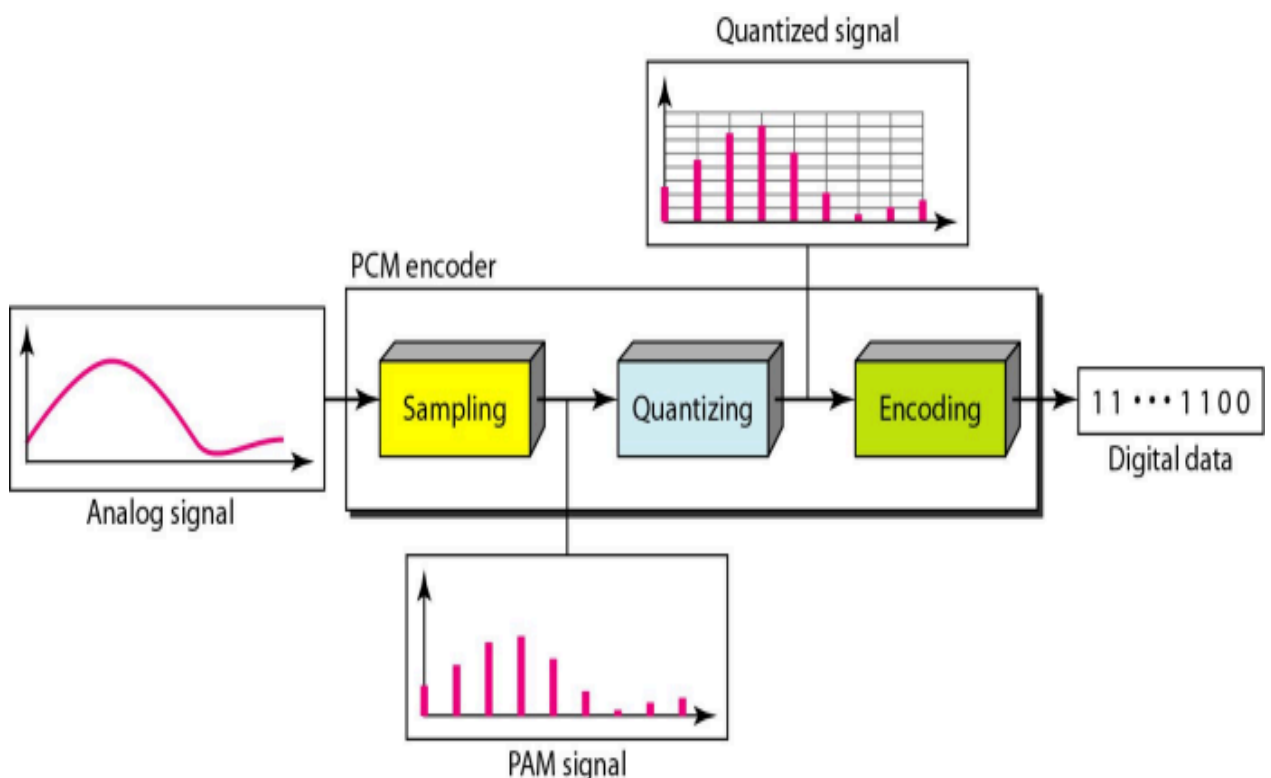
Pulse Code Modulation **PCM**

The process of coding sampled signal (conversion to digital form) can be done by Pulse Code Modulation (**PCM**)

In, PCM the message signal pass through three process:

- a. **sampling** - conversion of analog signal (in both amplitude and time) to a signal discrete in time analog in amplitude
- b. **Quantization** - conversion of signal analog amplitude to discrete by rounding off the analog amplitude to nearest one of a finite set of allowable values known as quantization levels.
- c. **Encoding** - conversion of discrete amplitudes(quantized) to digital form (0,1)

Figure 4.21 *Components of PCM encoder*



Sampling

is the process of converting a continuous-time analog signal into a discrete-time signal by taking measurements at specific intervals. According to the sampling theorem, the sampling frequency F_s must be at least twice the maximum frequency F_m of the signal ($F_s \geq 2F_m$) to accurately capture the signal without losing information.

- **Sampling Interval (T_s):** The time between each sample, calculated as $T_s = \frac{1}{F_s}$. This is the interval at which the analog signal is sampled.
- **Sampling Frequency (F_s):** The rate at which samples are taken, defined as $F_s = \frac{1}{T_s}$.
- **Sample Amplitude:** The amplitude of each sample depends on the signal's value at the exact time of sampling. While the signal becomes discrete in time, its amplitude remains analog (continuous values).

Types of Sampling:

1. **Ideal Sampling:** Uses an impulse at each sampling instant to measure the signal.
2. **Natural Sampling:** A short-width pulse captures the amplitude of the signal, varying with time.
3. **Flat-top Sampling:** Similar to natural sampling but holds the sampled value constant, resulting in a "step" effect.

Pulse Amplitude Modulation (PAM):

The process of sampling and holding the amplitude is known as PAM, where the signal retains analog (non-integer) amplitude values.

Quantization

- **Sampling:** Creates a series of pulses with varying amplitude values that lie between a minimum and a maximum. The amplitude can take any value within this range, making it continuous.
- **Need for Quantization:** To represent these amplitude values digitally, they must be mapped to a finite set of known values, since infinite values cannot be handled by digital systems.
- **Dividing the Range:** The amplitude range from the minimum to the maximum is divided into L zones, each with a height Δ given by:

$$\Delta = \frac{\text{max} - \text{min}}{L}$$

- **Quantization Levels:** Each zone is assigned a midpoint, which serves as the quantized value. A sample that falls within a particular zone is approximated to the midpoint of that zone.

Encoding

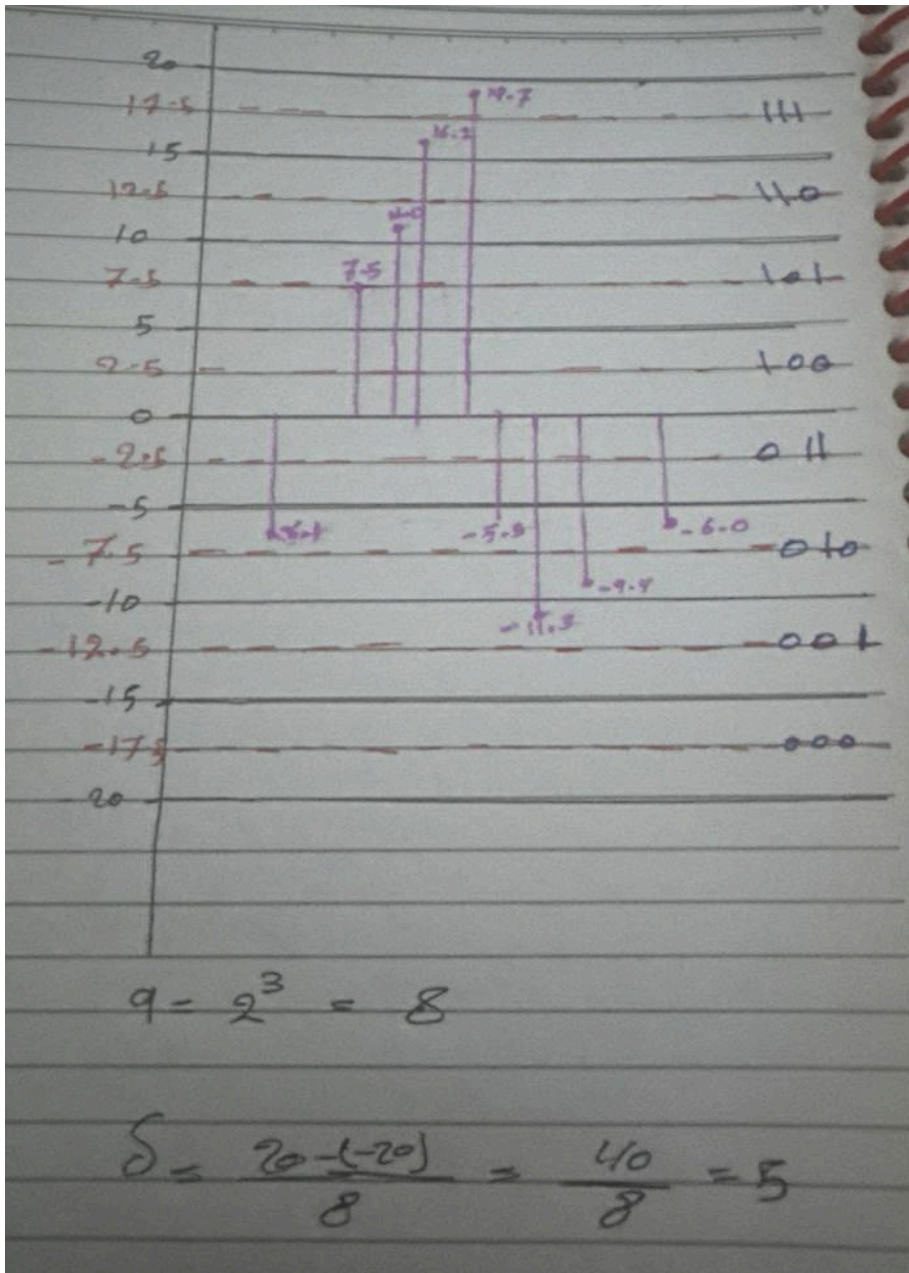
Converting of the quantized samples amplitude to binary.

$$q = 2^v$$

V : Bits per sample (Encoding word length).

q : The number of bits per samples determine the number of quantization levels.

Example:



Sample	Quantization	Quantization error	Coding
7.5	7.5	$7.5 - 7.5 = 0$	101
11.0	12.5	$11.0 - 12.5 = -1.5$	110
16.2	17.5	$16.2 - 17.5 = -1.3$	111
19.7	20	$19.7 - 20 = -0.3$	111
-5.5	-7.5	$-5.5 - (-7.5) = 2$	010
-6.0	-7.5	$-6.0 - (-7.5) = 1.5$	010
-6.1	-7.5	$-6.1 - (-7.5) = 1.4$	010
-9.4	-12.5	$-9.4 - (-12.5) = 3.1$	010
-11.3	-12.5	$-11.3 - (-12.5) = 1.2$	001

There are some applications of PCM (pulse code modulation) :

- satellite **transmission** system.
- It is used in space **communication**.
- It is used in telephony.
- The compact disc (CD) is a recent application of PCM.

Chapter 7 : Pulse code generator/receiver

PCM generator

What is PCM?

Pulse Code Modulation (PCM) is a method used to convert analog signals (like audio or video) into digital signals. This digital format makes it easier to transmit, store, and process information.

How does a PCM generator work?

1. Bandlimiting:

- The analog signal $x(t)$ is first passed through a low-pass filter.
- This filter limits the signal's frequency range to a maximum of 'W' Hertz.
- This step is crucial to prevent aliasing, a distortion that can occur during sampling.

2. Sampling:

- A sample-and-hold circuit takes snapshots of the filtered signal at regular intervals.
- The sampling rate, F_s , must be at least twice the maximum frequency 'W' (Nyquist-Shannon sampling theorem).
- This ensures that the original signal can be accurately reconstructed from the sampled values.

3. Quantization:

- The sampled values, which are still analog, are converted into digital values.
- A quantizer divides the signal's amplitude range into 'Q' discrete levels.
- Each sample is assigned to the nearest quantization level, introducing a small amount of error called quantization error.

4. Encoding:

- The quantized values are then encoded into binary code.
- Each binary code represents a specific quantization level.

The Output:

- The output of the PCM generator is a digital bitstream.
- This bitstream can be transmitted over various channels or stored digitally.

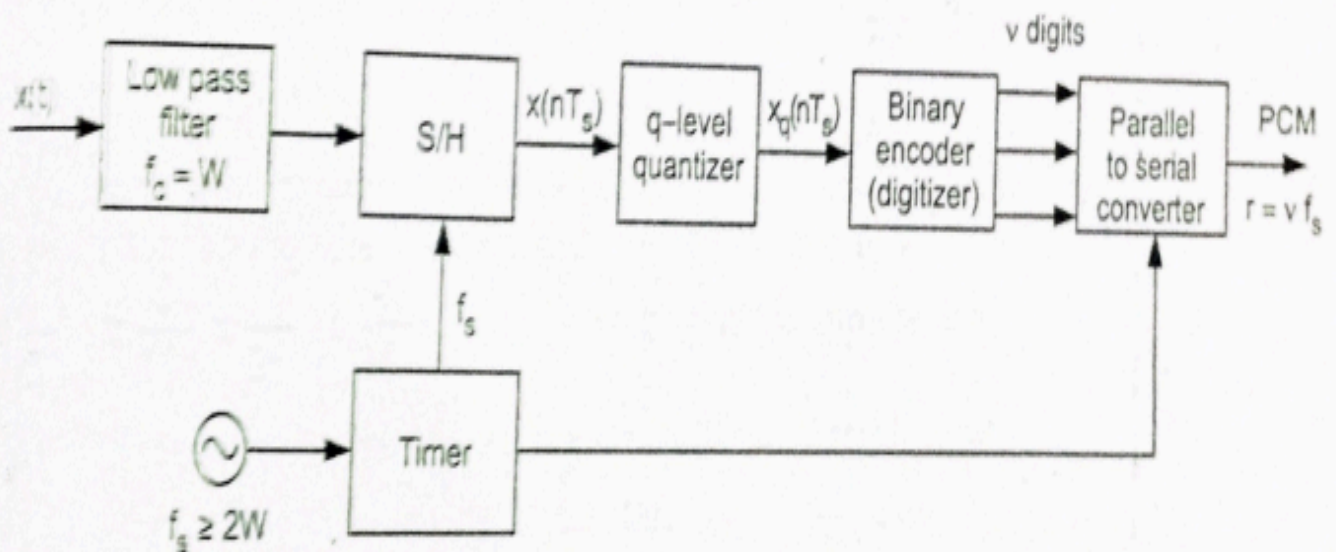


Fig. 6.1.1 PCM generator

Quantization Error in PCM

- Quantization error occurs because the quantizer can only represent a finite number of levels.
- The error is the difference between the original analog sample and the quantized value assigned to it.
- In the given example, the input signal $x(nT_s)$ can take any value between -45 and +45.
- The quantizer has 8 fixed levels: $\pm\delta/2$, $\pm3\delta/2$, $\pm5\delta/2$, and $\pm7\delta/2$.
- The step size δ determines the distance between adjacent levels.
- If the input sample falls exactly on one of the quantization levels, there is no error.
- However, if the input sample lies between two levels, it is assigned to the nearest level, resulting in quantization error.

Maximum Quantization Error:

- The maximum quantization error occurs when the input sample is halfway between two levels.
- In this case, the error is half the step size $\delta/2$.
- So, the maximum quantization error is $\pm\delta/2$.

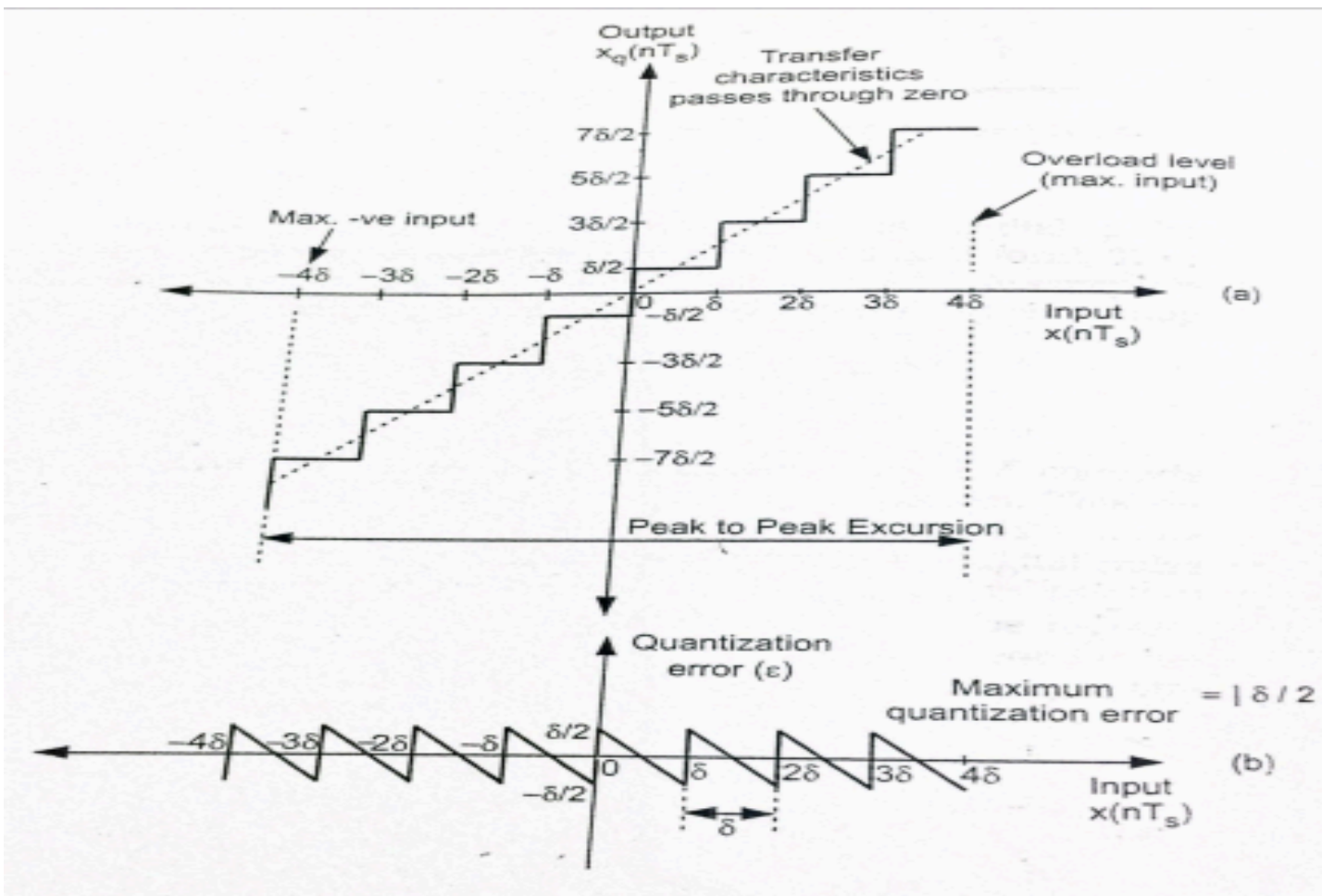
Illustration from the Image:

- If $x(nT_s) = 45$, the quantizer assigns it to the nearest level, which is $7\delta/2$. The error is $\delta/2$.
- If $x(nT_s) = -45$, the quantizer assigns it to the nearest level, which is $-7\delta/2$. Again, the error is $\delta/2$.

Reducing Quantization Error:

- To reduce quantization error, we can increase the number of quantization levels. This makes the step size smaller, resulting in finer granularity and less error.

- However, increasing the number of levels also increases the number of bits required to represent each level, which can lead to higher data rates.



Maximum quantization error = $\epsilon_{\max} = \left\lceil \frac{\delta}{2} \right\rceil$

Transfer characteristic of a quantizer and variation of quantization error with input

Calculating Signaling Rate:

- Bits per Sample: Each sample is represented by 'v' bits.
- Bits per Second (Signaling Rate):
 - To calculate the total number of bits transmitted per second, we multiply the number of samples per second (F_s) by the number of bits per sample (v).
 - **Signaling Rate (r) = $F_s * v$**

PCM Transmission Bandwidth:

This is the minimum bandwidth required to transmit the PCM signal without significant distortion.

Relationship between BT and r:

the transmission bandwidth (BT) should be at least half the signaling rate (r). This is because the bandwidth of a signal is related to its highest frequency component.

Deriving the Formula:

1. **Transmission Bandwidth: $BT \geq 1/2 * r$**
2. **Substituting $r = F_s * v \Rightarrow BT \geq 1/2 * (F_s * v)$**
3. **Applying the Sampling Theorem ($F_s \geq 2W$)**
 $\Rightarrow BT \geq 1/2 * (2W * v)$
4. **Simplifying: $BT \geq W * v$**

PCM Receiver

A PCM receiver is designed to convert a digital bitstream back into an analog signal. Here's a breakdown of the receiver's components and their functions:

1. Regenerator:

- Reshapes the received signal to its original form.
- Removes noise and interference that may have been added during transmission.

2. Serial-to-Parallel Converter:

- Converts the serial bitstream into parallel data.
- Each parallel word represents a quantized sample.

3. Digital-to-Analog Converter (DAC):

- Converts the digital code representing each sample into an analog voltage.
- A sample-and-hold circuit is often used to maintain the voltage level until the next sample is converted.

4. Low-Pass Filter:

- Removes high-frequency components introduced by the quantization process.
- Reconstructs the original analog signal from the discrete samples.

Limitations and Considerations:

- **Quantization Error:**

- The quantization process at the transmitter introduces a fundamental limitation.
- It's impossible to perfectly reconstruct the original analog signal due to the loss of information during quantization.
- The quantization error can be reduced by increasing the number of quantization levels (Q).

- **Trade-offs:**

- Increasing the number of quantization levels (Q) requires more bits per sample (v).
- More bits per sample lead to a higher signaling rate (r) and wider transmission bandwidth (BT).
- This trade-off between signal quality and bandwidth must be carefully considered.

PCM receiver and reconstruction waveform

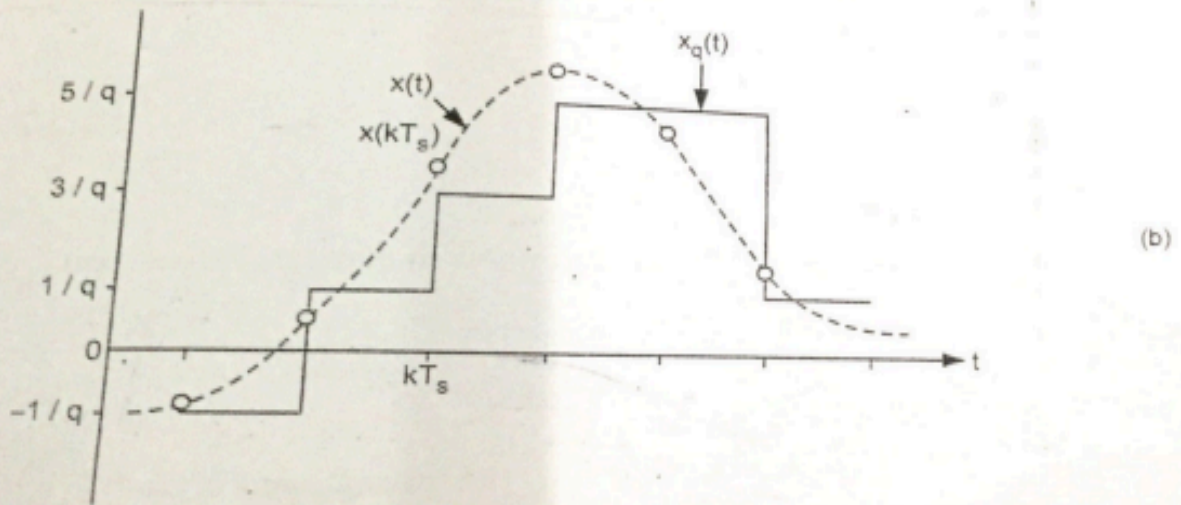
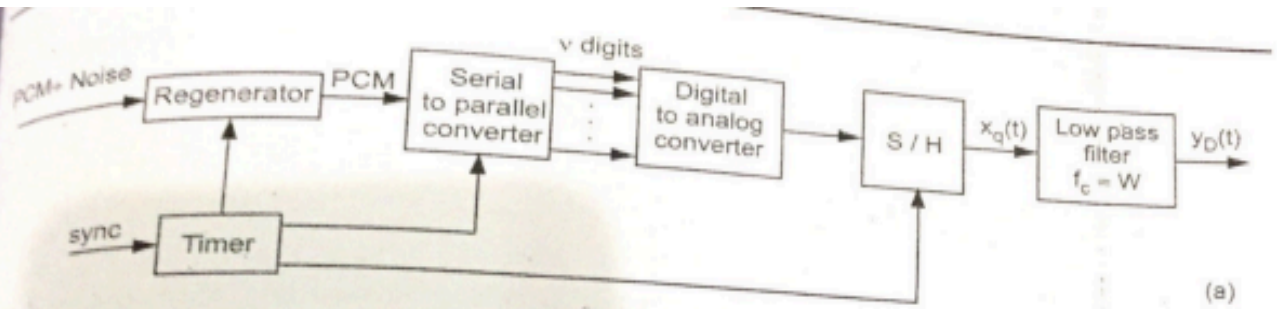


Fig. 6.1.4 (a) PCM receiver
(b) Reconstructed waveform

Quantization in PCM with example

The PCM signal is generated by carrying out following basic operations:

1. Band limiting (using LPF)
2. Sampling
3. Quantizing
4. Encoding

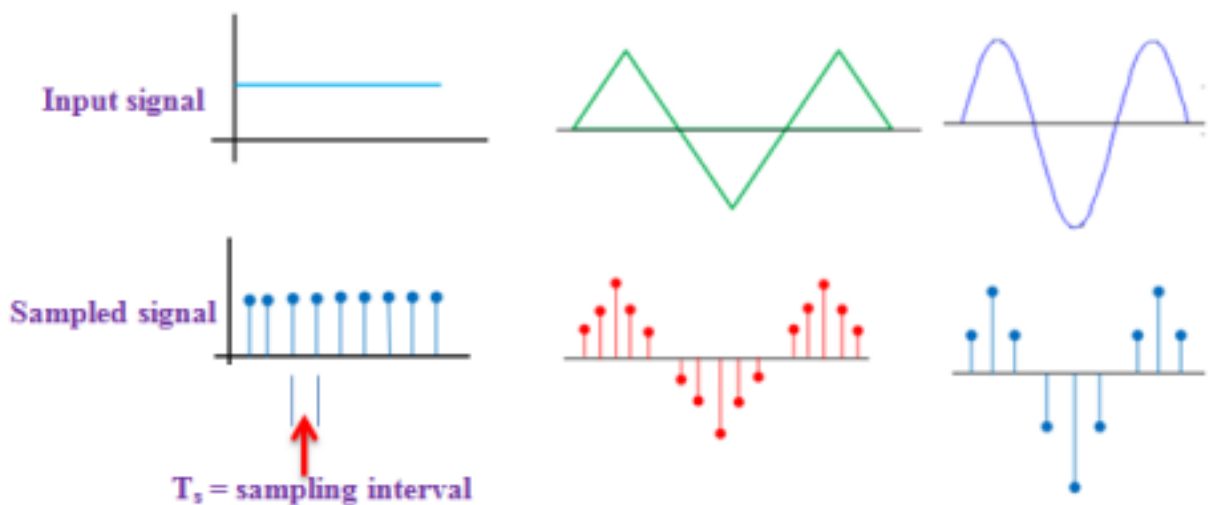
Two fundamental processes are involved in the generation of a PCM signal: sampling , quantization and encoding.

Sampling is **Time discretization** and quantization is **Amplitude discretization**.

In PCM, conversion of analog signal to digital signal is done in Three steps

- Sampling
- Quantization and encoding

Below figure shows sampling step:



Sampling theorem

$$T_s = 1/F_s \text{ with } F_s \geq 2 F_m$$

Quantization is the process of **rounding** of the sample value to the nearest **quantization level**.

Remember that number of quantization levels is **predefined** according to allocated bit per sample (n or v)

If n = number of bits used to represent the sample

Then, **q** = number of quantization levels = 2^n

$$\text{and, } \log q = \log (2^n) = n \log 2$$

$$n = \log q / \log 2$$

suppose, if $n = 2$, then $q = 2^n = 2^2 = 4$ quantization levels

if $n = 8$, then $q = 2^n = 2^8 = 256$ quantization levels exist

STEPS IN QUANTIZATION

1. Total voltage range is divided into q equal intervals of step size S

$$S = \frac{V_H - V_L}{q} = \frac{V_H - V_L}{2^n}$$

where V_H = Max. voltage value

V_L = Min. voltage value

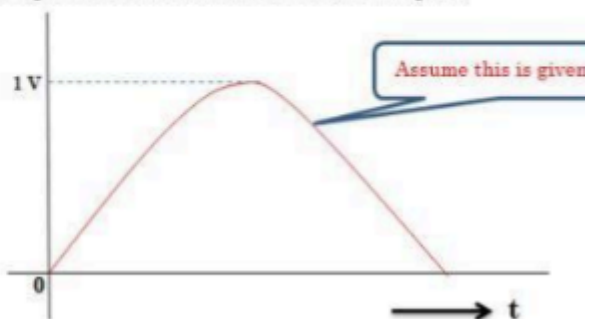
2. Draw mid lines representing quantization levels
3. Assign binary codes (pre-defined) to each quantization level
4. Calculate quantization error

Quantization Example

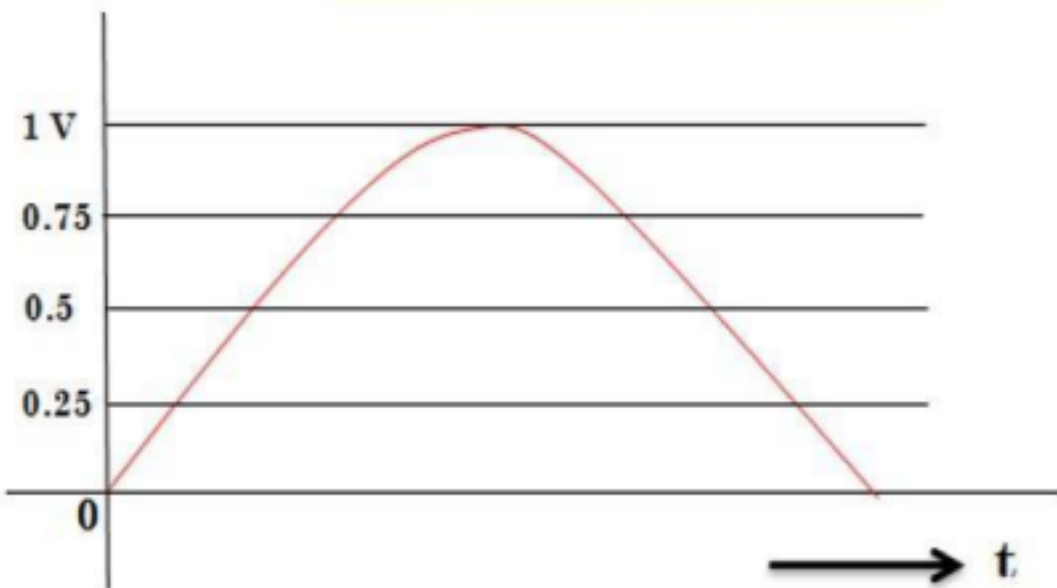
Q. A 2-bit PCM modulator is used with a 0-1 V signal. What is the binary digital value that will occur for the following inputs: 0.4 V, 0.78 V. What is the quantization error for these two samples?

Sol:

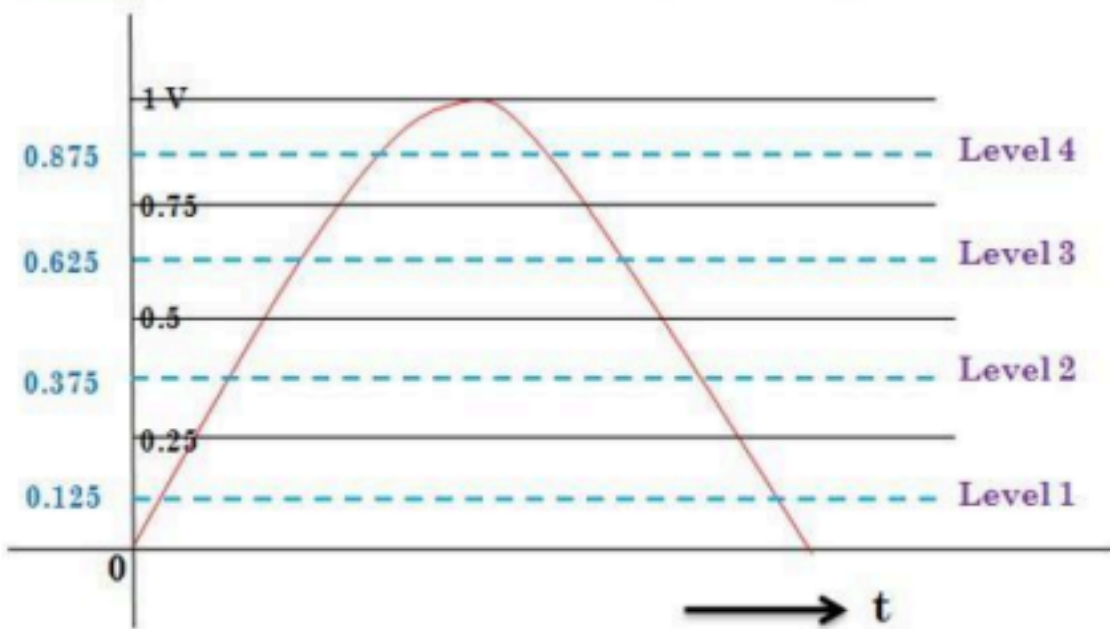
n = number of bits = 2
q = No. of quantization levels
= 2^n
= 4



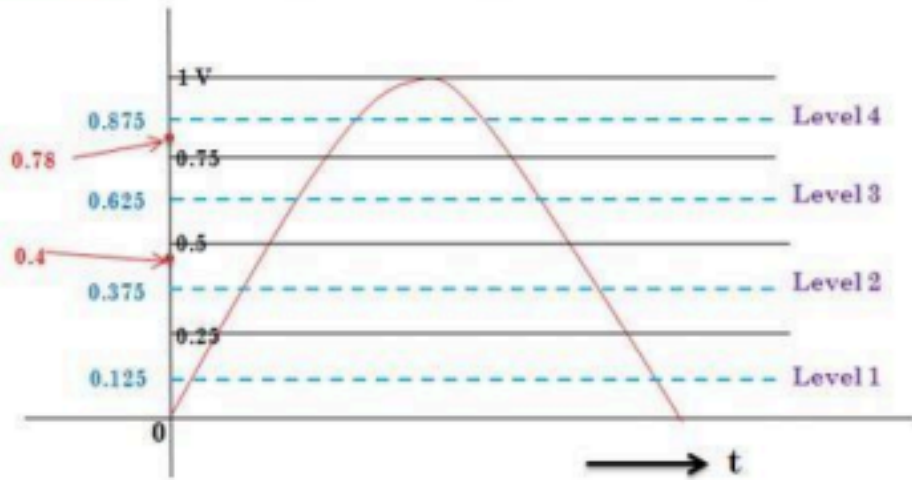
Step 1: Step size $S = \frac{V_H - V_L}{2^n} = \frac{1-0}{4} = \frac{1}{4} = 0.25$



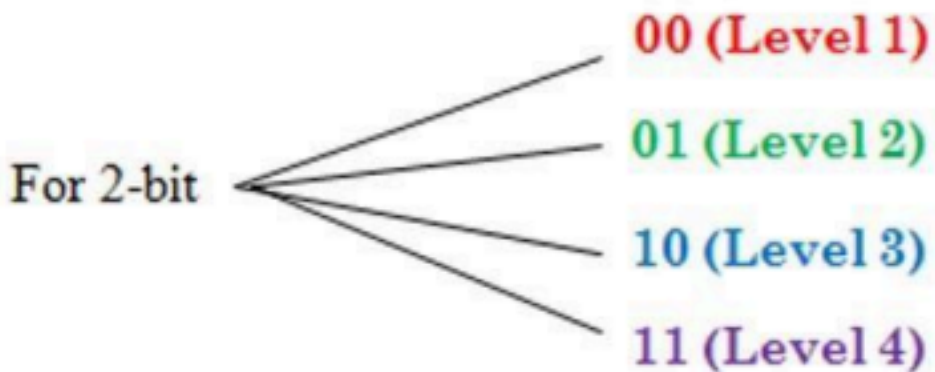
Step 2: Draw mid-lines, which represents quantization levels



Step 3: Denote given sample values i.e., 0.4 V & 0.78 V



Step 4: Represent each quantization level with predefined binary code



Quantizing 0.4 V sample value:

See that level 2 is near to 0.4 V. So, for sample voltage 0.4 V, 01 code is transmitted.

$$\text{Quantization error } e = 0.4 - 0.375 = 0.025 \text{ V}$$

Quantizing 0.78 V sample: Level 4 is nearest to 0.78 V. So, digital code 11 is transmitted.

$$\text{Quantization error } e = 0.875 - 0.78 = 0.095 \text{ V}$$

NOTE:

Quantization process introduces a certain amount of **error** or distortion.

This error known as **quantization noise** and is minimized by increasing the number of **quantization levels**. But increasing number of quantization levels increases number of bits to represents each sample and hence increases bit rate and cost of transmission.

Chapter 8: QUANTIZATION NOISE AND SIGNAL TO NOISE RATIO

What is Quantization?

Quantization involves mapping a continuous range of signal amplitudes into discrete levels during signal digitization. This introduces a small error known as **quantization error** (denoted as ϵ).

General equation for Signal to Noise ratio = $\frac{\text{signal power}}{\text{noise power}}$

Signal to Quantization Noise ratio = $\frac{S}{N} = \frac{\text{Signal power}}{\text{Error Noise power}}$

- To find above ratio, It is required to find a mathematical expression for both signal power and quantization – error power
- If the Signal power, S denoted by P ,
- then it required to find a mathematical expression for the quantization Noise power,

Amplitude Range of the Signal ($X(nT_s)$):

The original continuous signal has an amplitude range from $-X_{\max}$ to $+X_{\max}$.

- **Total range:** $2X_{\max}$.

Step Size (δ) Derivation:

- The **step size (δ)** between consecutive quantization levels is derived as:

$$\delta = \frac{X_{\max} - (-X_{\max})}{q}$$

Uniform Distribution Assumption:

if the step size (δ) is sufficiently small, it is reasonable to assume that the quantization error (ε) will be uniformly distributed.

Maximum Quantization Error:

the maximum quantization error ($\varepsilon_{\max}$) as half the step size:

$$\varepsilon_{\max} = \delta/2$$

This means that the error will never exceed this value in magnitude.

Error Range:

The error will lie within the range:

$$-\delta/2 \leq \varepsilon \leq \delta/2$$

Uniform Distribution Over the Range:

This means that the probability of the error taking on any value within this range is the same.

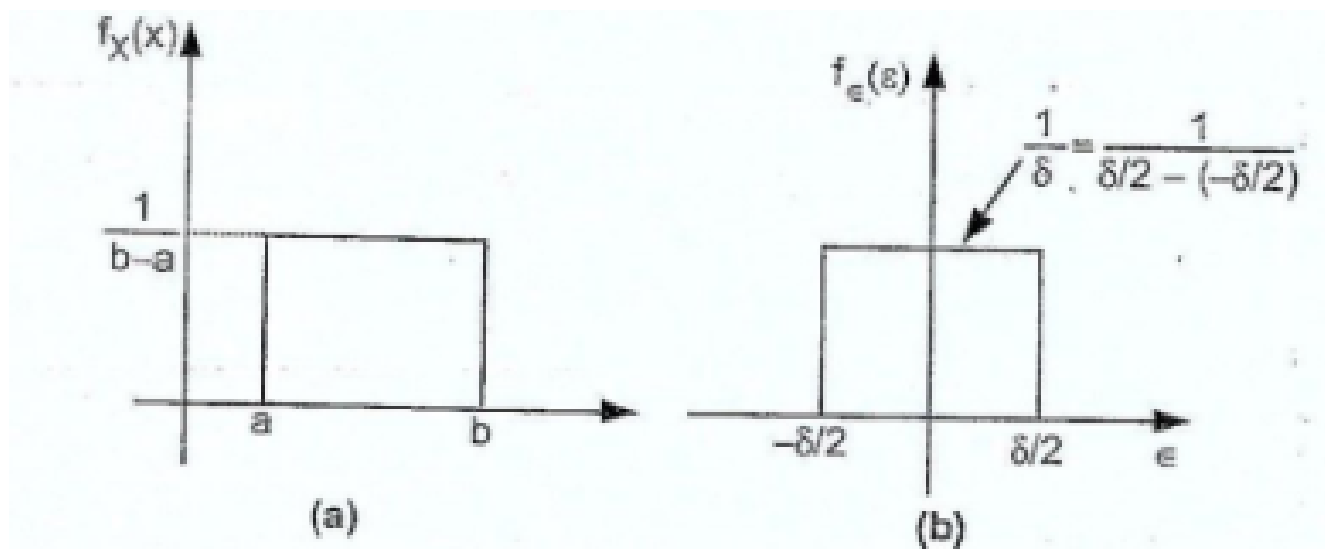


Fig. 6.1.5 (a) Uniform distribution

(b) Uniform distribution for quantization error

In fig. 6.1.5 (a), a random variable is said to be uniformly distributed over an interval (a,b) . Then pdf of 'x' is given by, (from equation of uniform pdf)

$$f_X(x)=0 \text{ for } x \leq a$$

$$= \frac{1}{b-a} \text{ for } a < x \leq b$$

$$= 0 \text{ for } x > b$$

The probability density function for quantization error ' ϵ ' as,

$$f_\epsilon(\epsilon) = 0 \text{ for } \epsilon \leq -\frac{\delta}{2}$$

$$= \frac{1}{\delta} \text{ for } -\frac{\delta}{2} < \epsilon < \frac{\delta}{2}$$

$$= 0 \text{ for } \epsilon \geq \frac{\delta}{2}$$

Quantization error ' ϵ ' has zero average value, that is mean ' m_ϵ ' of the quantization error is zero.

Signal to Quantization Noise Ratio :

$S/N = \text{Signal power} / \text{Noise power}$

Calculating Noise Power:

The noise power is given by:

$$\text{Noise power} = \frac{V_{\text{noise}}^2}{R}$$

where V_{noise}^2 is the mean square value of the noise voltage and R is the resistance.

Mean Square Value of Noise:

The mean square value of the noise V_{noise}^2 is calculated using the probability density function (PDF) of the quantization error (ϵ):

$$V_{\text{noise}}^2 = E[\epsilon^2] = \int (\epsilon^2 * f_{\epsilon}(\epsilon)) d\epsilon$$

where $f_{\epsilon}(\epsilon)$ is the PDF of the quantization error.

Mean Square Value of a Random Variable:

The mean square value of a random variable X is defined as:

$$X^2 = E[X^2] = \int (x^2 * f_x(x)) dx$$

where $f_x(x)$ is the PDF of the random variable X .

From equation above write equation below:

$$E[\epsilon^2] = \int_{-\delta/2}^{\delta/2} \epsilon^2 \times \frac{1}{\delta} d\epsilon$$

$$= \frac{1}{\delta} \left[\frac{(\delta/2)^3}{3} + \frac{(\delta/2)^3}{3} \right]$$

$$= \frac{1}{3\delta} \left[\frac{\delta^3}{8} + \frac{\delta^3}{8} \right] = \frac{\delta^2}{12}$$

The mean square value of noise voltage is,

$$V_{\text{noise}}^2 = \text{mean square value} = \frac{\delta^2}{12}$$

When load resistance, $r = 1$ ohm, then the noise power is normalized,

$$\begin{aligned} \text{Noise power (normalized)} &= \frac{V_{\text{noise}}^2}{1} \\ &= \frac{\frac{\delta^2}{12}}{1} \\ &= \frac{\delta^2}{12} \end{aligned}$$

Normalized noise power or quantization noise power

$$= \frac{\delta^2}{12}$$

for linear quantization or quantization error (in terms of power).

$$\frac{S}{N} = \frac{\text{Normalized Signal power}}{\text{Normalized Noise power}} = \frac{\text{Normalized Signal power}}{(\delta^2/12)}$$

Number of bits ‘v’ and quantization levels are related as,

$$Q = 2^v$$

Putting this value in equation step size:

$$\delta = \frac{2x_{\text{max}}}{2^v}$$

Putting this value in equation signal to noise ratio:

$$\frac{S}{N} = \frac{\text{Normalized Signal power}}{\left(\left(\frac{2x_{\text{max}}}{2^v}\right)^2 / 12\right)}$$

Let normalized singal power be denoted as ‘P’:

$$\frac{S}{N} = \frac{P}{\left(\frac{4x_{max}^2}{2^{2v}} x \frac{1}{12}\right)} = \frac{3P}{x_{max}^2} \cdot 2^{2v}$$

Maximum signal to quantization noise ration : $\frac{S}{N} = \frac{3P}{x_{max}^2} \cdot 2^{2v}$

This equation shows that signal to noise power ratio of quantizer increases exponentially with increasing bits per sample.

If we assume that input x(t) is normalized, i.e.,

$$x_{max} = 1$$

Then signal to quantization noise ratio will be,

$$\frac{S}{N} = 3 \times 2^{2v} \times P$$

$$P \leq 1$$

Then the signal to noise ratio is given as,

$$\frac{S}{N} \leq 3 \times 2^{2v}$$

$x_{max} = 1$ and $P \leq 1$, the singal to noise ratio given by above equation is normalized.

Expressing the singal to noise ratio in decibels,

$$\begin{aligned} \left(\frac{S}{N}\right)_{dB} &= 10 \log_{10} \left(\frac{S}{N}\right)_{dB} \quad \text{since power ratio} \\ &\leq 10 \log_{10}[3 \times 2^{2v}] \\ &\leq (4.8 + 6v) \text{ dB} \end{aligned}$$

Thus,

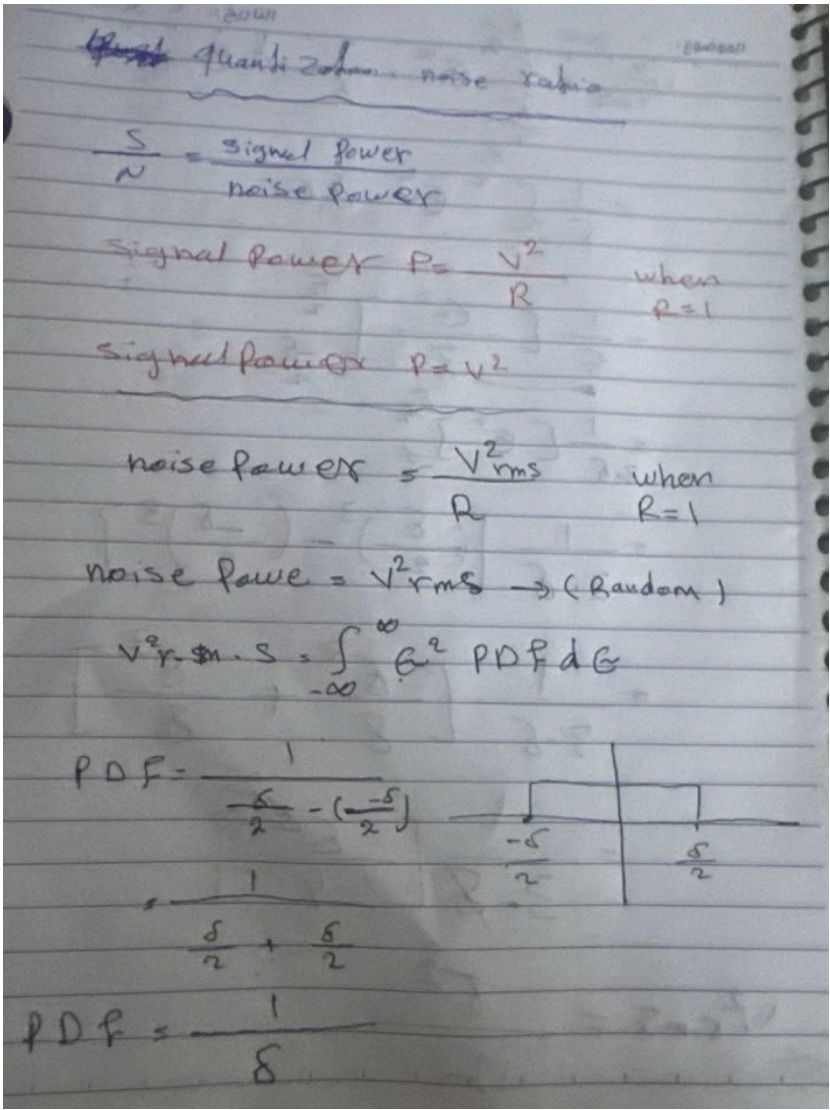
Signal to quantization noise ratio for normalized values of power ‘P’ and amplitude of input x (t)

$$: \left(\frac{S}{N}\right)_{dB} \leq (4.8) + 6v \text{ dB}$$

Signal to noise Ratio in PCM - Equation

case	Normal equation	dB Equation
General	$\frac{3P}{x_{max}^2} 2^{2v}$	
Normalized values of power P and amplitude of input x(t)	$\frac{S}{N} \leq 3 \times 2^{2v}$	$\leq (4.8 + 6v)$ dB
Sinusoidal input signal	1.5×2^{2v}	$= 1.8 + 6v$ db

اشتقاق مهم



$$V_{rms}^2 = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \omega^2 \cdot \frac{1}{\delta} d\omega$$

$$= \frac{1}{\delta} \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \omega^2 d\omega$$

$$= \frac{1}{\delta} \left[\frac{\omega^3}{3} \right]_{-\frac{\delta}{2}}^{\frac{\delta}{2}}$$

$$= \frac{1}{3 \cdot \delta} \left[\omega^3 \right]_{-\frac{\delta}{2}}^{\frac{\delta}{2}}$$

$$= \frac{1}{3 \cdot \delta} \left[\left(\frac{\delta}{2} \right)^3 - \left(-\frac{\delta}{2} \right)^3 \right]$$

$$= \frac{1}{3 \cdot \delta} \left[\frac{\delta^3}{8} + \frac{\delta^3}{8} \right]$$

$$= \frac{1}{3 \cdot \delta} \left[\frac{\delta^3}{4} \right]$$

$$V_{rms}^2 = \frac{\delta^2}{12}$$

$$\frac{S}{N} = \frac{P}{S^2/12}$$

$$S = \frac{2X_{max}}{9} = \frac{2X_{max}}{9V}$$

$$\frac{S^2}{12} = \left(\frac{2X_{max}}{9V} \right)^2 \div 12$$

$$S^2 = \frac{1}{12} \left(\frac{4X_{max}^2}{9V^2} \right)$$

$$S^2 = \frac{X_{max}^2}{3 \cdot 9V^2}$$

$$\frac{S}{N} = \frac{P}{\frac{X_{max}^2}{3 \cdot 9V^2}}$$

$$\frac{S}{N} = \frac{P}{1} \cdot \frac{3 \cdot 9V^2}{X_{max}^2}$$

$$\boxed{\frac{S}{N} = \frac{3 P \cdot 9V^2}{X_{max}^2}}$$

نتیجه

Chapter 9: PCM Quantization noise and S/N

Ex.6.1.1: Derive the expression for signal to quantization noise ratio for PCM system that employs linear quantization technique. Assume that input to the PCM system is a sinusoidal signal.

Or

A PCM system uses a uniform quantizer followed by a v bit encoder. Show that rms signal to quantization noise ratio is approximately given by $(1.8 + 6v)$ dB.

Sol: Assume that the modulation signal be a sinusoidal voltage, having peak amplitude A_m . Let this signal cover the complete excursion of representation levels.

The power of this signal will be,

$$P = \frac{V^2}{R} \quad \text{Here } V = \text{rms value}$$

$$= [A_m / \sqrt{2}]^2$$

When $R=1$, the power P is normalized, i.e.,

Normalized power: $P = \frac{A_m^2}{2}$ with $R = 1$ in above equation.

* signal to quantization noise ratio is given by equation 6.1.27 as,

$$\frac{S}{N} = \frac{3P}{x_{\max}^2} \times 2^{2v}$$

$$\text{Here } P = \frac{A_m^2}{2} \text{ and } x_{\max} = A_m$$

Putting these values in the above equation,

$$\frac{S}{N} = \frac{3 \times \frac{A_m^2}{2}}{A_m^2} \times 2^{2v} = \frac{3}{2} \times 2^{2v} = 1.5 \times 2^{2v}$$

Expressing signal to noise power ratio in dB,

$$\begin{aligned} \left(\frac{S}{N}\right)_{\text{dB}} &= 10 \log_{10} \left(\frac{S}{N}\right) = 10 \log_{10} (1.5 \times 2^{2v}) \\ &= 10 \log_{10} (1.5) + 10 \log_{10} 2^{2v} \\ &= 1.76 + 2v \times 10 \times 0.3 \end{aligned}$$

Thus,

$$\left(\frac{S}{N}\right)_{\text{dB}} \text{ in PCM} : \left(\frac{S}{N}\right)_{\text{dB}} = 1.8 + 6v; \text{ for sinusoidal signal}$$

Ex.6.1.2: A Television signal with a bandwidth of 4.2 MHz is transmitted using binary PCM. The number of quantization levels is 512.

Calculate,

- i) Code word length
- ii) Transmission bandwidth
- iii) Final bit rate
- iv) output signal to quantization noise ratio.

Sol.: The bandwidth is 4,2 MHZ, means highest frequency component will have frequency of 4.2 MHZ,
 $w = 4.2 \text{ MHz}$

Quantization levels $q = 512$

i) number of bits and quantization levels are related in binary PCM as,

$$Q = 2^v$$

$$512 = 2^v$$

$$\log 512 = v \log 2$$

$$\text{Or } v = \frac{\log 512}{\log 2}$$

$$= 9 \text{ bits}$$

Thus the code word length is 9 bits

ii) from equation 6.1.8 the transmission channel bandwidth is given as,

$$B_T \geq 37.8 \text{ MHz}$$

iii) The final bit rate will equal to signaling rate. From equation 6.1.5 signaling rate is given as,

$$R = v f_s$$

Sampling frequency $f_s \geq 2W$ by sampling theorem.

$$f_s \geq 2 \times 4.2 \text{ MHz since } W = 4.2 \text{ MHz}$$

$$f_s \geq 8.4 \text{ MHz}$$

Putting this value of ' f_s ' in equation for signaling rate,

$$r = 9 \times 8.4 \times 10^6$$

$$= 75.6 \times 10^6 \text{ bits/sec}$$

From equation 6.1.6 transmission bandwidth is also obtained as,

$$B_T \geq \frac{1}{2} r$$

$$\geq \frac{1}{2} \times 75.6 \times 10^6 \text{ bits/sec}$$

Or $B_T \geq 37.8 \text{ MHz}$ which is same as the value obtained earlier.

iv) The signal to noise ratio

$$\left(\frac{S}{N}\right)_{\text{dB}} \leq (4.8 + 6v) \text{ dB by equation}$$

$$\leq 4.8 + 6v \times 9$$

$$\leq 58.8 \text{ dB}$$

Ex.6.1.3: The bandwidth of signal input to the PCM is restricted to 4 kHz. The input varies from -3.8 V to + 3.8 V and has the average power of 30 mW. The required signal to noise ratio is 20 dB. The modulator produces binary output. Assume uniform quantization.

- i) Calculate the number of bits required per sample.
- ii) outputs of 30 such PCM coders are time multiplexed. What is the minimum required transmission bandwidth for the multiplexed signal?

Sol.: The given value of signal to noise ratio is 20 dB.

$$\left(\frac{S}{N}\right)\text{dB} = 10 \log_{10} \left(\frac{S}{N}\right) = 20\text{dB}$$

$$\frac{S}{N} = 100$$

- i) From equation 6.1.27, the signal to quantization noise ratio is given as,

$$\frac{S}{N} = \frac{3P}{x_{\max}^2} \times 2^{2v}$$

Here $x_{\max} = 3.8\text{V}$, $P = 30\text{mW}$ and $\frac{S}{N} = 100$

$$100 = \frac{3 \times 30 \times 10^{-3} \times 2^{2v}}{(3.8)^2}$$

$$V = 6.98 \text{ bits}$$

$$= 7 \text{ bits}$$

- ii) The maximum frequency is,

$$W = 4 \text{ kHz}$$

The transmission bandwidth is given by equation 6.1.8 as, $B_T \geq vW$

Since there are 30 PCM coders which are time multiplexed, the transmission bandwidth will be,

$$B_T \geq 30 \times v.W$$

$$\geq 30 \times 7 \times 4 \text{ kHz}$$

$$\geq 840 \text{ kHz}$$

Signaling rate is two times the transmission bandwidth as given by equation:

$$\text{Signaling rate } r = 840 \times 2 \text{ bits/sec} = 1680 \text{ bits/sec}$$

Ex.6.1.4: The information in an analog signal voltage waveform is to be transmitted over PCM system with an accuracy of $\pm 0.1\%$ (full scale). The analog voltage waveform has a bandwidth of 100 Hz and an amplitude range of -10 to +10 volts.

- a) Determine the maximum sampling rate required.
- b) Determine the number of bits in each PCM word.
- c) Determine minimum bit rate required in the PCM signal.
- d) Determine the minimum absolute channel bandwidth required for the transmission of the PCM signal.

Sol.: Here an accuracy is given as $\pm 0.1\%$. That is quantization error should be $\pm 0.1\%$, or the maximum quantization error should be $\pm 0.1\%$.

Or $\epsilon_{\max} = \pm 0.1\% = \pm 0.001$

From equation 6.1.2 the maximum quantization error for an uniform quantizer is given as,

$$\epsilon_{\max} = \left| \frac{\Delta}{2} \right|$$

Or $\left| \frac{\Delta}{2} \right| = 0.001$

That is Step size $\Delta = 2 \times 0.001 = 0.002$

From equation 6.1.10, the step size, number of levels and maximum value of the signal are related as,

$$\Delta = \frac{2x_{\max}}{q} \text{ here } |x_{\max}| = 10 \text{ volts}$$

Putting values of Δ and $X_{\max}$,

$$0.002 = \frac{2 \times 10}{q} \text{ or } q = \frac{20}{0.002} = 10,000$$

That is the number of levels are 10,000.

a) The maximum frequency in the signal is 100Hz

$$W = 100\text{Hz}$$

By sampling theorem, minimum sampling frequency should be, $f_s \geq 2W$

$$\geq 2 \times 100 \geq 200\text{Hz}$$

b) we know that minimum 10,000 levels should be used to quantize the signal. If binary PCM is used, then number of bits for each sample can be calculated as,

$$q = 2^v$$

Here, q = number of levels

V = bits in PCM,

$$10,000 = 2^v$$

$$\log_{10} 10,000 = v \log_{10} 2$$

$$\text{Or } v = \frac{\log 10,000}{\log 2} = 13.288$$

Or $v = 14$ bits

c) from equation 6.1.5 the bit rate or signaling rate is given as,

$$R = v f_s$$

$$= 14 \times 200$$

$$\geq 2800 \text{ bits per second.}$$

d) the transmission channel for PCM is given by equation 6.1.6 as,

$$B_T \geq \frac{1}{2} R$$

$$\geq \frac{1}{2} \times 2800$$

$$\geq 1400 \text{ Hz}$$

Ex.6.1.5: A PCM system uses a uniform quantizer followed by a 7-bit binary encoder. The bit rate of the system is equal to 50×10^6 bit/sec.

- a) What is the maximum message bandwidth for which the system operates satisfactorily?
- b) Determine the output signal to quantization noise ratio when a full load sinusoidal modulation wave of frequency 1 MHz is applied to the input.

sol.: a) let us assume that the message bandwidth be W Hz. Therefore sampling frequency should be,

$$f_s \geq 2W$$

The number of bits $v = 7$ bits

From equation 6.1.5 the signaling rate is given as,

$$r = v.f_s$$

$$r \geq 7 \times 2W$$

$$50 \times 10^6 \geq 14W$$

$$W \leq 3.57 \text{ MHz}$$

Thus the maximum message bandwidth is 3.57MHz

- b) The modulating wave is sinusoidal. For such signal, the signal to quantization noise ratio is given by equation 6.1.34 as,

$$\left(\frac{S}{N}\right)_{\text{dB}} = 4.8 + 6v$$

$$= 4.8 + 6 \times 7 \text{ (putting for } v = 7)$$

$$= 43.8 \text{ dB}$$

Ex.6.1.6: The information in an analog waveform with maximum frequency $f_m = 3\text{KHz}$ is to be transmitted over an M-level PCM system where the number of pulse levels is $M=16$. The quantization distortion is specified not to exceed 1% of peak to peak analog signal.

i) what is the maximum number of bits per sample that should be used in this PCM system?

ii) what is the minimum sampling rate and what is the resulting bit transmission rate?

Sol.: i) Since the number of levels given here are $M=16$,
 $q = M = 16$

Then bits and levels in binary PCM are related as,

$$q = 2^v \quad v = \text{bits}$$

$$16 = 2^v$$

$$\text{Or } v = 4$$

ii) since $f_m = W = 3 \text{ kHz}$

Minimum $f_s \geq 2W$ by sampling theorem

$$f_s \geq 2 \times 3 \text{ kHz}$$

$$\geq 6 \text{ kHz}$$

Bit transmission rate or signaling rate is given by equation 6.1.5 as,

$$r = v f_s$$

$$\geq 4 \times 6 \times 10^3$$

$$\geq 24 \times 10^3 \text{ bits per second}$$

Ex.6.1.7: A signal of bandwidth 3.5kHz is sampled quantized and coded by a PCM system. The Coded signal is then transmitted over a transmission channel of supporting a transmission rate of 50 k bits/sec. Calculate the maximum signal to noise ratio that can be obtained by this system.

The input signal has peak to peak value of 4 volts and rms value of 0.2v.

Sol.: The maximum frequency of the signal is 3.5 kHz,
 $W = 3,5\text{kHz}$

Therefore sampling frequency will be

$$\begin{aligned} f_s &\geq 2W \\ &\geq 2 \times 3.5 \text{ kHz} \\ &\geq 7 \text{ kHz} \end{aligned}$$

The signaling rate is given by equation 6.1.5 as,

$$r = v f_s$$

Putting values of $r = 50 \times 10^3$ bits/sec and $f_s = 7 \times 10^3$ Hz in above equation

$$50 \times 10^3 \leq v \cdot 7 \times 10^3$$

$$v \geq 7.142 \text{ bits}$$

$$v = 8 \text{ bits}$$

The rms value of the signal is 0.2V. Therefore the normalized signal power will be,

$$\text{Normalized signal power} = \frac{(0.2)^2}{1}$$

[$R = 1$ for normalized power]

$$P = 0.04\text{w}$$

The maximum signal to noise ratio is given by equation 6.1.27 as,

$$\frac{S}{N} = \frac{3P}{x_{\max}^2} \times 2^{2v}$$

Putting the values of $p = 0.04$, $v = 8$ and $x_{\max} = 2$ in above equation,

$$\frac{S}{N} = \frac{3 \times 0.04 \times 2^{2 \times 8}}{4} = 1966.08 = 33\text{dB}$$

Ex.6.1.10: A 7 bit PCM system employing uniform quantization has an overall signaling rate of 56 k bits per second. Calculate the signal to quantization noise ratio that would result when its input is a sine wave with peak to peak amplitude equal to 5. Calculate the dynamic range for the sine wave inputs in order that the signal to quantization noise ratio may be less than 30 dBs. What is the theoretical maximum frequency that this system can handle?

Sol.: The number of bits in the PCM system are
 $v = 7$ bits

Assume that 5 V peak to peak voltage utilizes complete range of quantizer. Then from equation 6.1.34 we can find the signal to quantization noise ratio as,

$$\begin{aligned} \left(\frac{S}{N}\right) \text{ dB} &= 1.8 + 6v \text{ dB} \\ &= 1.8 + 6 \times 7 \\ &= 43.8 \text{ dB} \end{aligned}$$

By equation 6.1.5 signaling rate is given as,

$$r = v f_s$$

Putting $r = 56 \times 10^3$ bits/second and $v = 7$ bits in above equation we get,

$$56 \times 10^3 = 7 \cdot f_s$$

Sampling frequency, $f_s = 8 \times 10^3$ Hz

By sampling theorem, $f_s \geq 2W$

Maximum frequency that can be handled is given as,

$$W \leq \frac{f_s}{2}$$

$$\leq \frac{8000}{2}$$

$$W \leq 4000 \text{ Hz}$$

Ex.6.1.11: The bandwidth of TV video plus audio signal is 4.5MHz. If the signal is converted to PCM bit stream with 1024 quantization levels, determine the number of bits/sec generated by the PCM system. Assume that the signal is sampled at the rate of 20% above nyquist rate. If above linear PCM system is converted to companded PCM, will the output bit rate change?justify.

Sol.:The given data is,

$$W = 4.5\text{MHz}$$

$$q = 1024 \text{ levels}$$

$$\text{The nyquist rate} = 2W = 2 \times 4.5 = 9 \text{ MHz}$$

The sampling rate is 20% above the nyquist rate.

$$\text{Sampling rate, } f_s = 1.2 \times 9 = 10.8 \text{ MHz}$$

We know that quantization levels q and number of bits v are related as,

$$q = 2^v$$

$$1024 = 2^v$$

$$v = 10 \text{ bits}$$

The number of bits/sec generated by PCM system is called bit rate or signaling rate.

$$\begin{aligned} \text{Signaling rate, } r &= v f_s \\ &= 10 \times 10.8 \times 10^6 \text{ bits/sec} \\ &= 108 \times 10^6 \text{ bits/sec} \end{aligned}$$

The output bit rate doesnot change if liner PCM is converted into companded PCM. companded PCM is used to improve the signal to noise ratio.

Second month exam

A-

Q1/ a 3-bit PCM is used to convert a sinusoidal signal given by:

$x(t) = 2\sin(2\pi 2000t)$. In to a binary form

The sampling is done at twice (2) Nyquist rate and first sample is taken at $t=0.01\text{msec}$ Draw the signal and its samples and for each sample, find the value of:

1. Samples value 2. quantized value
3. quantization error 4. sample code

sol/

$$f_o = 2k, A_m = 2, T_o = \frac{1}{f_o} \Rightarrow \frac{1}{2} k = 0.5\text{msec}, f_m = 2000t,$$

$$X_{\max} = 2$$

$$f_s = 2F_N \Rightarrow 2(2f_m) \Rightarrow f_s = 4f_m \Rightarrow f_s = 4 \times 2 = 8k \text{ sample/sec}$$

$$T_s = \frac{1}{f_s} \Rightarrow T_s = \frac{1}{8} = 0.125\text{msec}$$

$$T_s = 0.01 \longrightarrow s_1 = 2\sin(2\pi 2000 \cdot 0.01 \cdot 10^{-3}) = 0.250$$

$$T_s = 0.01 + 0.125 = 0.135 \longrightarrow s_2 = 2\sin(2\pi 2000 \cdot 0.135 \cdot 10^{-3}) = 1.984$$

$$T_s = 0.135 + 0.125 = 0.260 \longrightarrow$$

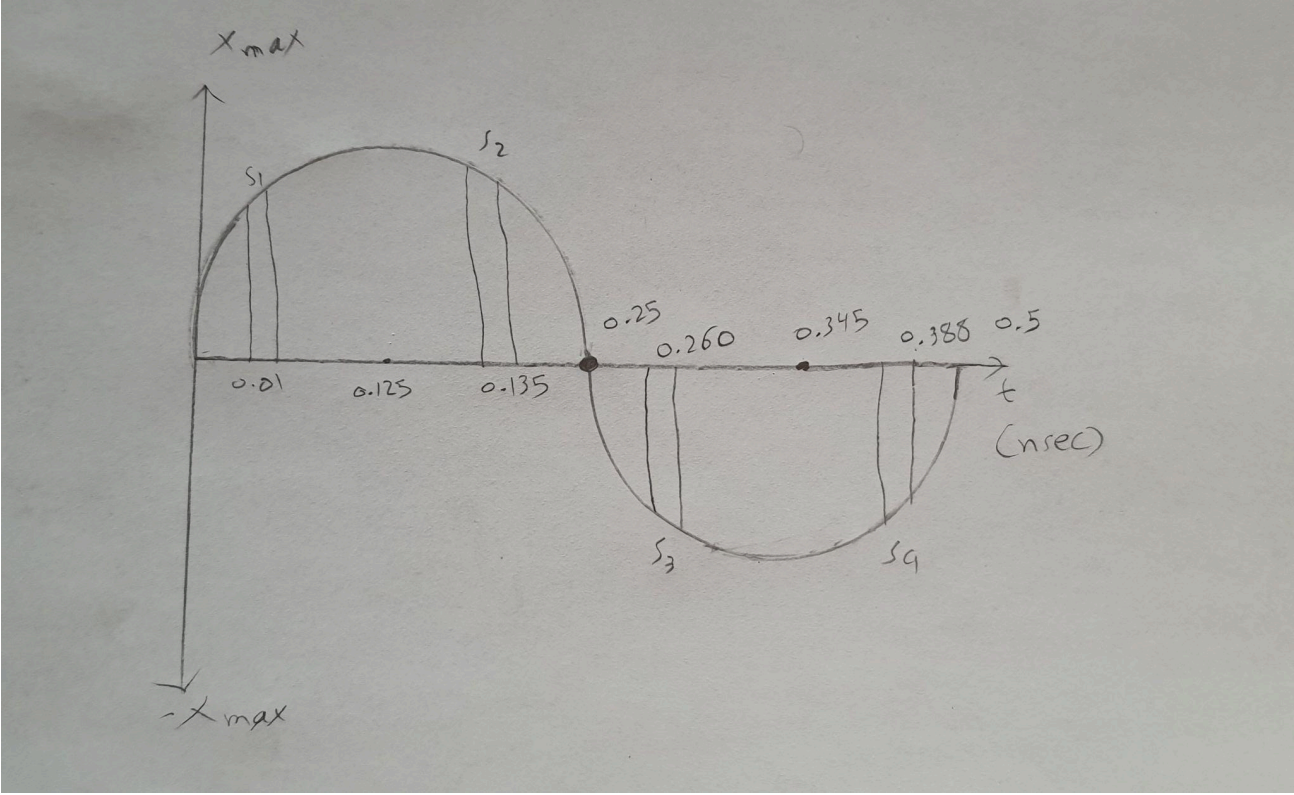
$$s_3 = 2\sin(2\pi 2000 \cdot 0.260 \cdot 10^{-3}) = -0.250$$

$$T_s = 0.260 + 0.125 = 0.385 \longrightarrow$$

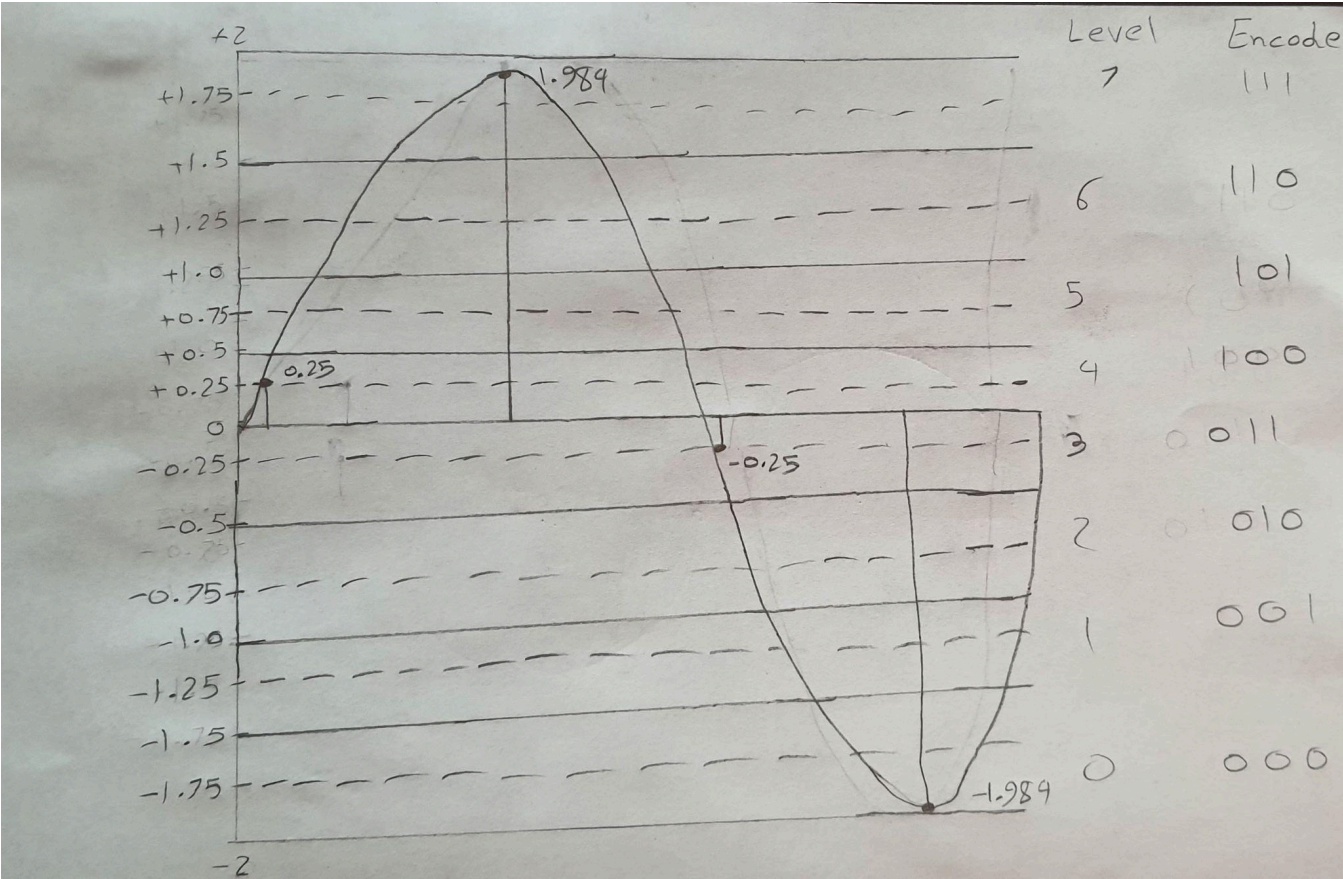
$$s_4 = 2\sin(2\pi 2000 \cdot 0.385 \cdot 10^{-3}) = -1.984$$

$$\ast v=3, q=2^v \Rightarrow q=2^3 \Rightarrow q=8$$

$$\square = \frac{2x_{\max}}{q} \Rightarrow \square = \frac{2 \times 2}{8} = 0.5v$$



Sample value	Quantized value	Quantized error	Sample code
$S_1=0.250$	$+0.25$	$0.25-0.25=0$	100
$S_2=1.984$	$+1.75$	$1.984-1.75 = 0.234$	111
$S_3=-0.250$	-0.25	$-0.25+0.25=0$	011
$S_4=-1.984$	-1.75	$-1.984+1.75 = 0.234$	000



Q2/an analog signal of bandwidth 4kHz is converted to digital form by a PCM system and then transmitted over a transmission channel of supporting a transmission rate of 60bits/sec.

If the input analog signal has peak value of 2 volts and rms of 0.2v calculate the signal to noise raio in dB

Sol/

The maximum frequency of the signal is 4 kHz,

$$W = 4\text{kHz}$$

Therefore sampling frequency will be

$$f_s \geq 2W$$

$$\geq 2 \times 4 \text{ kHz}$$

$$\geq 8 \text{ kHz}$$

The signaling rate is given by equation,

$$r = v f_s$$

Putting values of $r = 60 \times 10^3$ bits/sec and $f_s = 8 \times 10^3$ Hz in above equation

$$60 \times 10^3 \leq v \times 8 \times 10^3$$

$$v \geq 7.5 \text{ bits}$$

$$v = 8 \text{ bits}$$

The rms value of the signal is 0.2V. Therefore the normalized signal power will be,

$$\text{Normalized signal power} = \frac{(0.2)^2}{1}$$

[$R = 1$ for normalized power]

$$P = 0.04\text{w}$$

The maximum signal to noise ratio is given by equation,

$$\frac{S}{N} = \frac{3P}{x_{\max}^2} \times 2^{2v}$$

Putting the values of $p = 0.04$, $v = 8$ and $x_{\max} = 1$ in above equation,

$$\frac{S}{N} = \frac{3 \times 0.04 \times 2^{2 \times 8}}{2} = 3,932.16 = 49.8\text{dB}$$

B-

Q1/ a 3-bit PCM is used to convert a sinusoidal signal given by:

$x(t) = 2\sin(2\pi 2500t)$. In to a binary form

The sampling is done at twice (2) Nyquist rate and first sample is taken at $t=0.015\text{msec}$ Draw the signal and its samples and for each sample, find the value of:

2. Samples value 2.quantized value

3.quantization error 4.sample code

sol/

$$f_o = 2k, A_m = 2, T_o = \frac{1}{f_o} \Rightarrow \frac{1}{2} k = 0.5\text{msec}, f_m = 2500\text{Hz}$$

$$X_{\max} = 2$$

$$f_s = 2F_N \Rightarrow 2(2f_m) \Rightarrow f_s = 4f_m \Rightarrow f_s = 4 \times 2.5 = 10\text{k sample/sec}$$

$$T_s = \frac{1}{f_s} \Rightarrow T_s = \frac{1}{10} = 0.1\text{msec}$$

$$T_s = 0.015 \longrightarrow s_1 = 2\sin(2\pi 2500 \times 0.015 \times 10^{-3}) = 0.466$$

$$T_s = 0.015 + 0.1 = 0.115 \longrightarrow s_2 = 2\sin(2\pi 2500 \times 0.115 \times 10^{-3}) = 1.944$$

$$T_s = 0.115 + 0.115 = 0.230 \longrightarrow$$

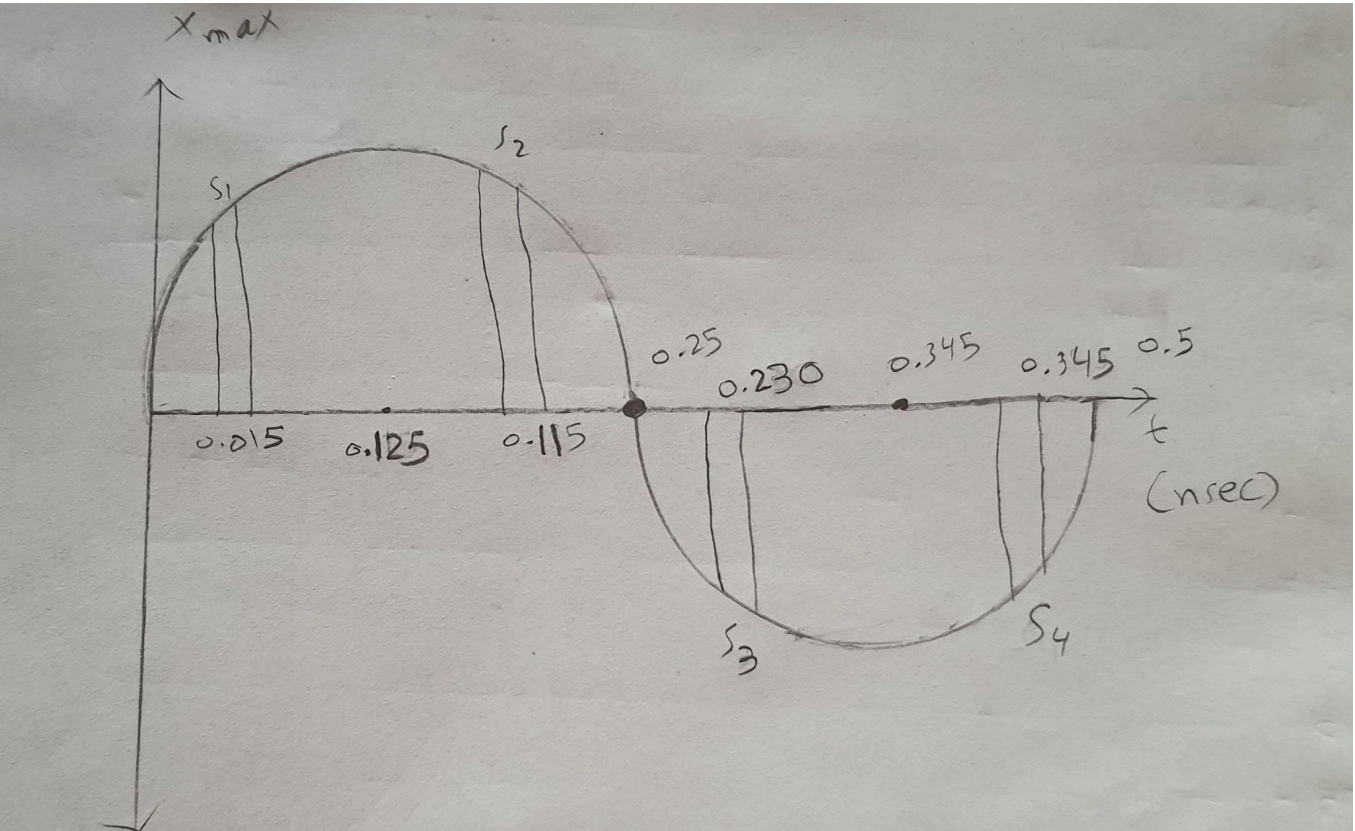
$$s_3 = 2\sin(2\pi 2500 \times 0.230 \times 10^{-3}) = -0.907$$

$$T_s = 0.230 + 0.115 = 0.345 \longrightarrow$$

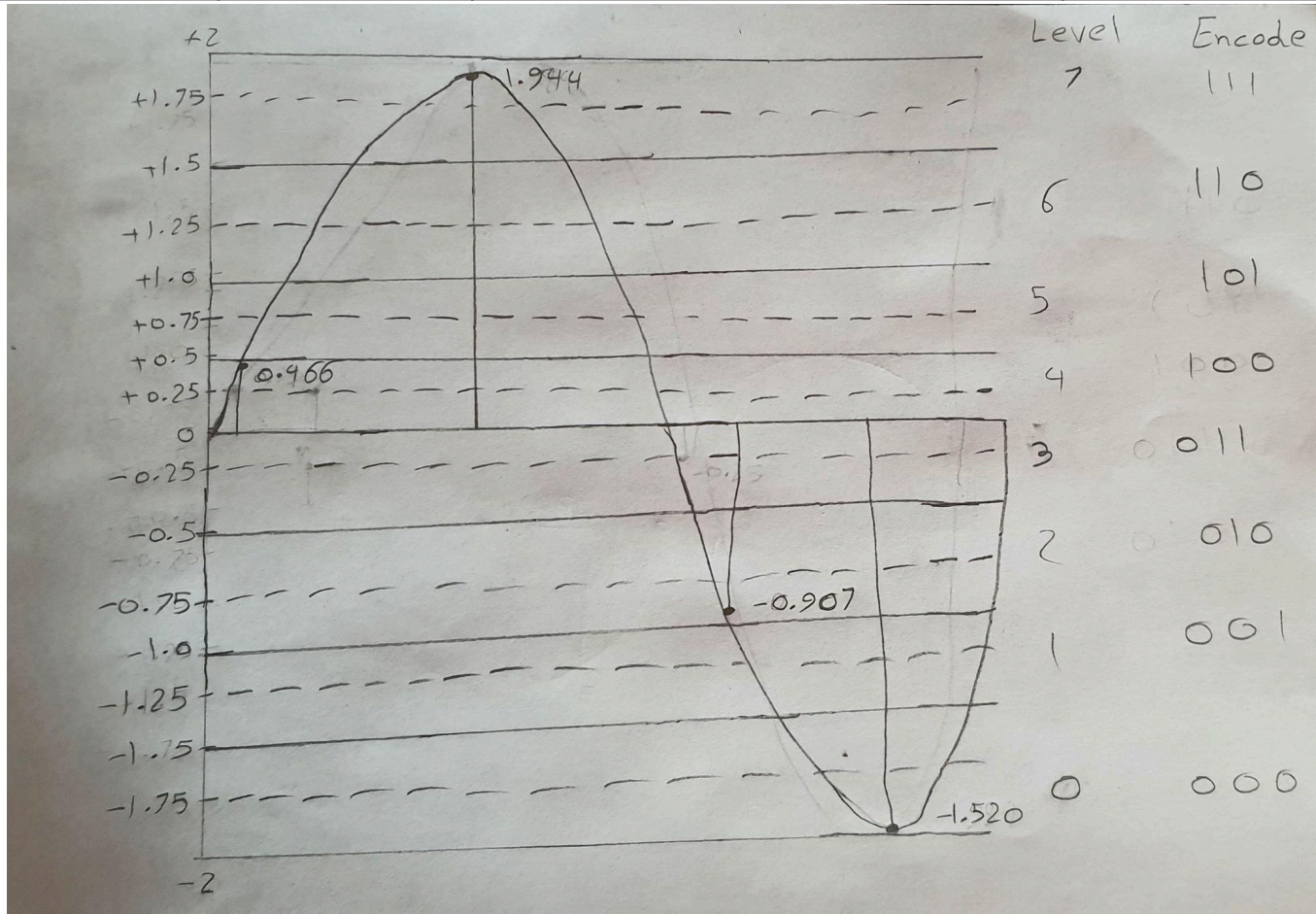
$$s_4 = 2\sin(2\pi 2500 \times 0.345 \times 10^{-3}) = -1.520$$

$$\therefore v=3, q=2^v \Rightarrow q=2^3 \Rightarrow q=8$$

$$\square = \frac{2x_{\max}}{q} \Rightarrow \square = \frac{2 \times 2}{8} = 0.5\text{V}$$



Sample value	Quantized value	Quantized error	Sample code
$S_1=0.466$	$+0.25$	$0.466-0.25=0.216$	100
$S_2=1.944$	$+1.75$	$1.944-1.75 = 0.194$	111
$S_3=-0.907$	-0.75	$-0.907+0.75=-0.157$	010
$S_4=-1.520$	-1.75	$-1.520+1.75 = 0.23$	000



Q2/an analog signal of bandwidth 3.5kHz is converted to digital form by a PCM system and then transmitted over a transmission channel of supporting a transmission rate of 55bits/sec.

If the input analog signal has peak value of 2 volts and rms of 0.2v calculate the signal to noise raio in dB

Sol/

The maximum frequency of the signal is 3.5 kHz,
 $W = 3,5\text{kHz}$

Therefore sampling frequency will be

$$\begin{aligned}f_s &\geq 2W \\&\geq 2 \times 3.5 \text{ kHz} \\&\geq 7 \text{ kHz}\end{aligned}$$

The signaling rate is given by equation,

$$r = v f_s$$

Putting values of $r = 55 \times 10^3$ bits/sec and $f_s = 7 \times 10^3$ Hz in above equation

$$55 \times 10^3 \leq v \times 7 \times 10^3$$

$$v \geq 7.857 \text{ bits}$$

$$v = 8 \text{ bits}$$

The rms value of the signal is 0.2V. Therefore the normalized signal power will be,

$$\text{Normalized signal power} = \frac{(0.2)^2}{1}$$

[$R = 1$ for normalized power]

$$P = 0.04\text{w}$$

The maximum signal to noise ratio is given by equation,

$$\frac{S}{N} = \frac{3P}{x_{max}^2} \times 2^{2v}$$

Putting the values of $p = 0.04$, $v = 8$ and $x_{\max} = 1$ in above equation,

$$\frac{S}{N} = \frac{3 \times 0.04 \times 2^{2 \times 8}}{2} = 3,932.16 = 49.8\text{dB}$$

Chapter 10: Delta Modulation

We have seen in PCM that, it transmits all the bits which are used to code the sample, Hence the sampling rate, r and transmission channel Bandwidth are large, To overcome this problem DELTA MODULATION (DM) is used.

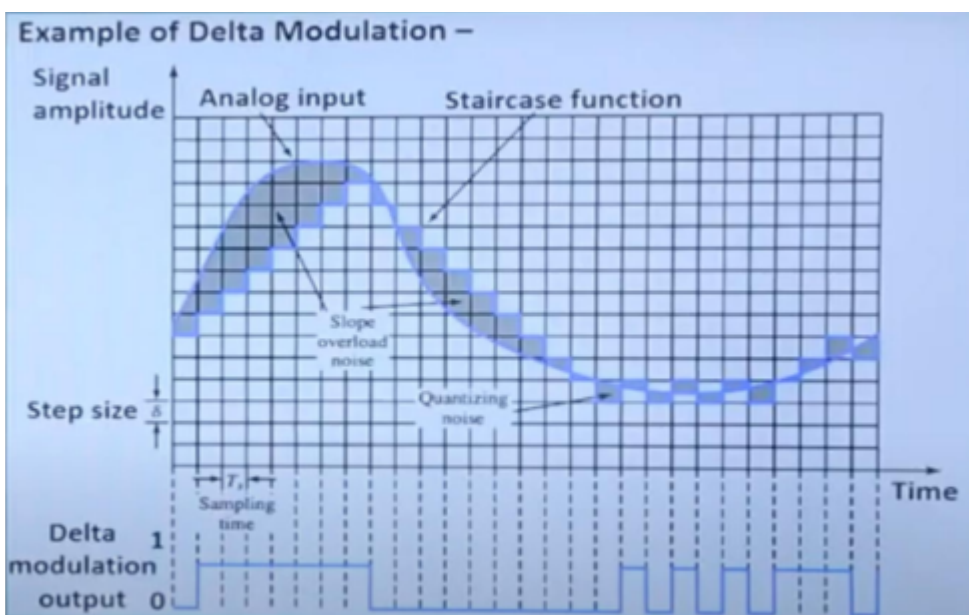
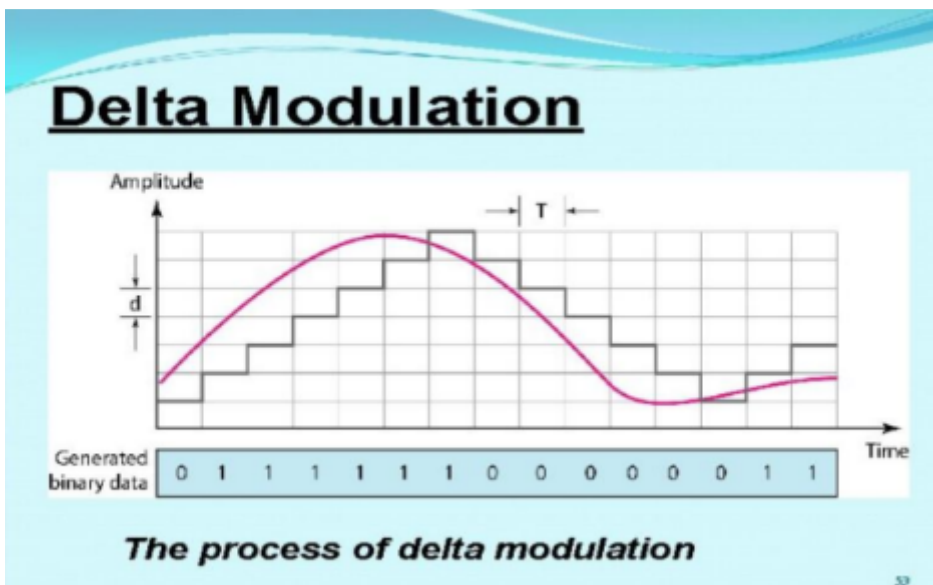
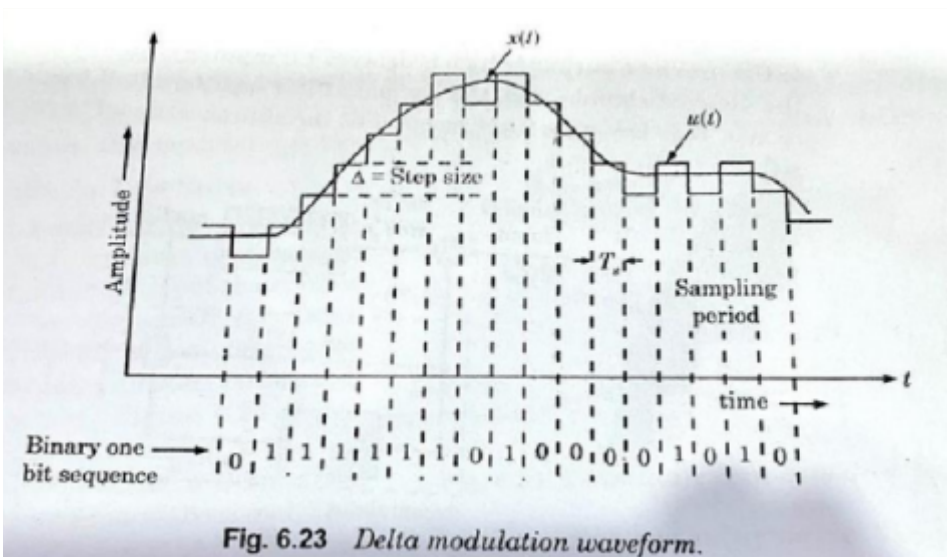
How it works:

- Delta Modulation(DM) transmits only one bit per sample.
- Present sample value is compared with the previous sample value, and the indication, whether the amplitude is increased or decreased is detected and sent.
- Input signal $x(t)$ is approximated to step signal $u(t)$ by the Delta Modulator.
- This step size is fixed

The difference between the input signal $x(t)$ and staircase approximated signal $u(t)$ confined to two levels, i.e. $+\delta$ and $-\delta$, If the difference is positive, then the approximated signal is increased by one step size δ and '1' is transmitted or If the difference is negative, then approximated signal $U(t)$ is reduced by δ and '0' is transmitted, Thus for each sample, only one binary bit is transmitted.

The fig. in the next slide shows:

- The analog input signal $x(t)$ to Delta Modulator
- The staircase approximated signal ($u(t)$)
- a Delta Modulator binary output



- The principle of Delta Modulation can be explained by the following set of equations
- The error between the sampled value of $x(t)$ and last approximated sample is given:

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

Where

$e(nT_s)$ = error at present sample

$X(nT_s)$ = sampled signal value

$\hat{x}(nT_s)$ = last sample approximation of staircase waveform

If we assume $u(nT_s)$ as the present sample approximation of staircase output,

Then

$$u(n-1)T_s = \hat{x}(nT_s)$$

last sample approximation of staircase waveform

Let us define a quantity $b(nT_s) = \delta \cdot \text{Sign}[e(nT_s)]$

This means that depending on the sign of error $e(nT_s)$, the sign of step size δ is decided

In other words, we can write:

$$\begin{aligned} b(nT_s) &= +\delta \quad \text{if } x(nT_s) \geq \hat{x}(nT_s) \\ &= -\delta \quad \text{if } x(nT_s) < \hat{x}(nT_s) \end{aligned}$$

Also, If $b(nT_s) = +\delta$ binary “1” is transmitted

And If $b(nT_s) = -\delta$ binary “0” is transmitted

DM – transmitter

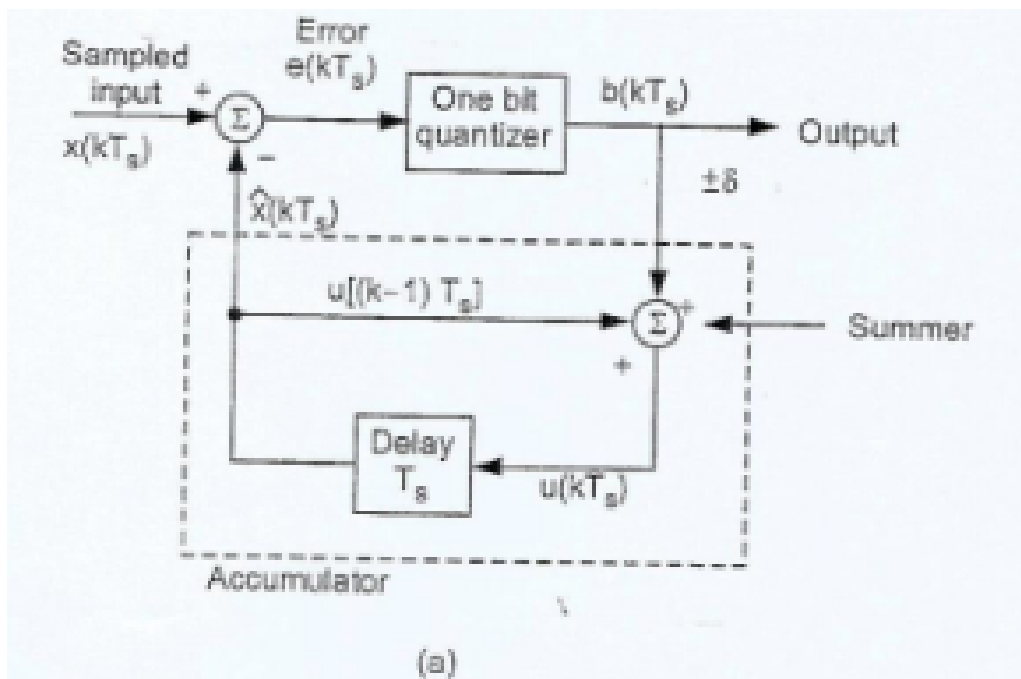
The summer in the accumulator adds quantizer output ($\pm\delta$) With the previous sample approximation. This gives present sample approximation. I.e.

$$u(n T_s) = u(nT_s - T_s) + [\pm\delta]$$

Or

$$= u[(n-1) T_s] + b(n T_s)$$

fig. below shows the transmitter of DM



The previous sample approximation $u [(n-1)T_s]$ is restored by delaying one

Sample period T_s . The sampled input signal $x(nT_s)$ and staircase approximated signal $x(nT_s)$ are subtracted to get error signal $e(NT_s)$.

Depending on the sign of $e(NT_s)$ one bit quantizer produces an output step $+\delta$ or $-\delta$. If it is '+', binary '1' is transmitted, and if it '-' then '0' is transmitted.

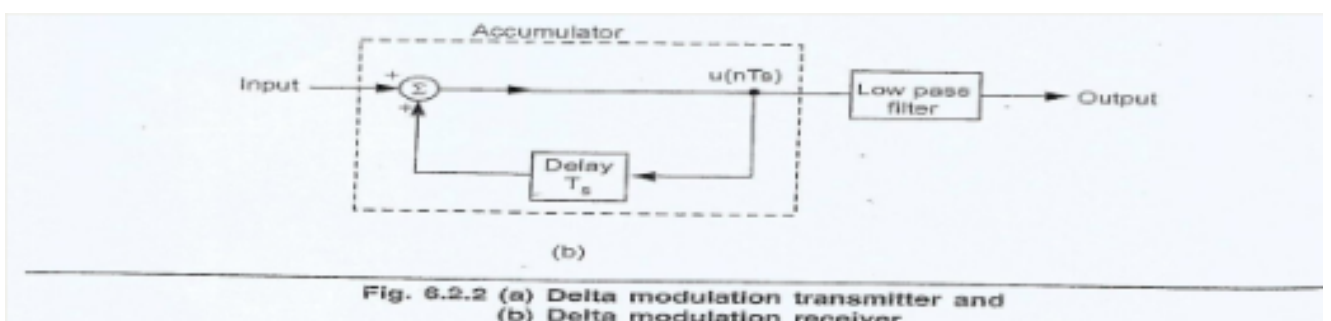


Fig. 6.2.2 (a) Delta modulation transmitter and (b) Delta modulation receiver

At the receiver shown in fig. 6.2.2 (b), the accumulator and low-pass filter are used. The accumulator generates the staircase approximated signal output and is delayed by one sampling period T_s . it is then added to the input signal. If input is binary '1' then it adds $+\delta$ step to the previous output (which is delayed). If input is binary '0' then one step ' δ ' is subtracted from the delayed signal. The low-pass filter has the cutoff frequency equal to highest frequency in $x(t)$. This filter smoothen the staircase signal to reconstruct $x(t)$.

Advantages Of Delta Modulation(DM)

The Delta Modulation has certain advantages over PCM as under:

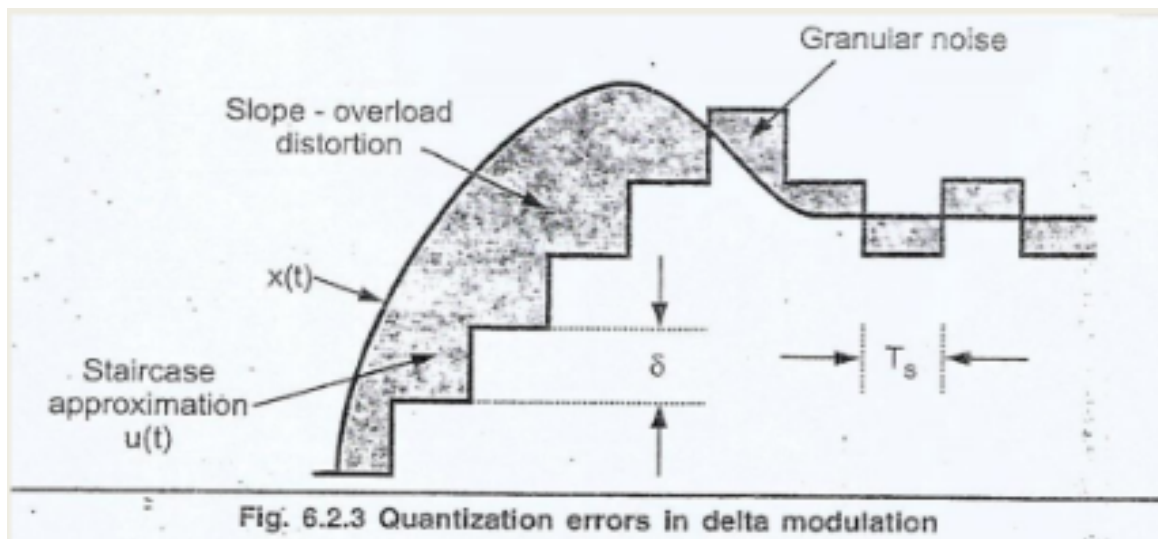
- since, the Delta modulation transmits only one bit per sample, therefore the signaling rate and transmission channel bandwidth is quite small for delta Modulation compared to PCM
- The transmitter and receiver implementation is very much simple for delta Modulation.
- There is no analog to digital conversion required in DM

Drawbacks (disadvantages) of Delta - Modulation

The delta – modulation has two major drawbacks as under:

1. Slope overload distortion

2. Granular or idle noise



1. Slope overload distortion(Startup error)

- This distortion arises because of large dynamic range of the input signal
- As can be observed from Fig. 6.2.3 in slide 14, the rate of rise of input signal $x(t)$ is so high that the staircase signal cannot approximate it, the step size δ becomes too small for staircase signal $u(t)$ to follow the step segment of $x(t)$.
- Hence, there is a large error between the staircase approximated signal and the original input signal $x(t)$.
- This error or noise is known as slope overload distortion.
- To reduce this error, the step size must be increased when slope of signal $x(t)$ is high.
- Since the step size of Delta Modulator remains fixed, its maximum or minimum slopes occur along straight lines.
- Therefore, this modulator is also known as Linear delta Modulator(LDM)

2. Granular or idle noise

- Granular or idle noise occurs when the step size is too large compared to small variations in the input signal.
- This means that for very small variations in the input signal, the staircase signal is changed by large amount δ because of large step size.
- Fig. 6.2.3 shows that when the input signal is almost flat, the staircase $u(t)$ keeps on oscillating by $\pm\delta$ around the signal
- The error between the input and approximated signal is called granular noise
- The solution to this problem is to make step size small
- The large step size is required to accommodate wide dynamic range of the input signal(to reduce slope overload distortion) And small steps are required to reduce granular noise.
- Adaptive Delta Modulation (ADM) is the modification

Example – Delta Modulation (DM)

Ex.6.3.11: The input to the DM is $m(t)$ $0.01 t$. The DM operates at a sampling frequency of 20 Hz and has a step size of 2 mV. Sketch the delta modulator output $\Delta(t)$ and $\hat{m}(t)$.

Sol.: Fig. 6.3.7 shows the DM output. The signal $m(t)$ is sketched for $t = 0$ to 1 sec. The signal $m(t)$ rises linearly with Time. The DM changes its output in steps of 2 mV when reconstructed signal $\hat{m}(t)$ is less than $m(t)$ output is increased by δ . When $\hat{m}(t)$ is greater than $m(t)$, output is reduced by δ . The change in $\hat{m}(t)$ takes place at sampling instants. When $\hat{m}(t)$ is increased by ' δ ', modulator output is $\Delta(t) = 1$. When $\hat{m}(t)$ is reduced by ' δ ', modulator output is $\Delta(t) = 0$. The sampling frequency is $f_s = 20\text{Hz}$. Hence sampling duration will be $T_s = \frac{1}{f_s} = \frac{1}{20} = 0.05$ seconds. Fig. 6.3.7 shows $m(t)$, $\hat{m}(t)$ and $\Delta(t)$ for 0 to 1 sec with sampling durations of 0.05 sec

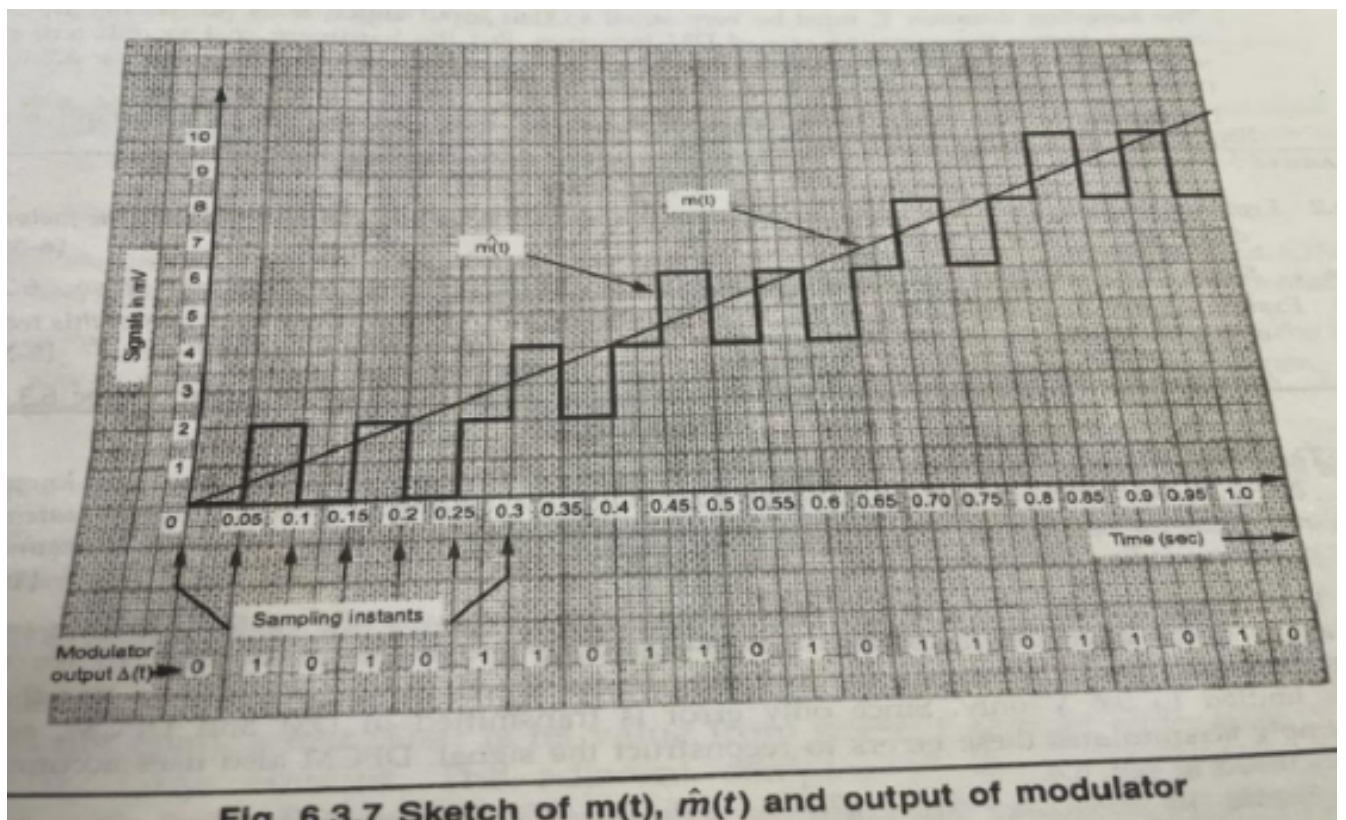


Fig. 6.3.7 Sketch of $m(t)$, $\hat{m}(t)$ and output of modulator

Chapter 11: ADAPTIVE DELTA MODULATION (ADM)

To overcome the quantization errors due to slope overload and granular noise, The step size δ is made adaptive to variations in the input signal $x(t)$.

- Practically in the steep segment of the signal $x(t)$, the step size is increased.
- While, when the input signal is varying slowly, the step size is reduced.
- Then the method is called Adaptive Delta Modulation (ADM)
- The adaptive delta modulators can take continuous changes in step size or discrete changes in step size.

ADM TRANSMITTER

- The logic for step size control(in the transmitter) is added and it used to increases or decreases step size according to certain rule depending on one bit quantizer output.
- For example if one bit quantizer output is high(1), then step size may be doubled for next sample. If one bit quantizer output is low, then step size may be reduced by one step
- Fig. in next slide, shows waveforms of adaptive delta modulator and sequence of bits transmitted.

ADAPTIVE DELTA MODULATION - TRANSMITTER

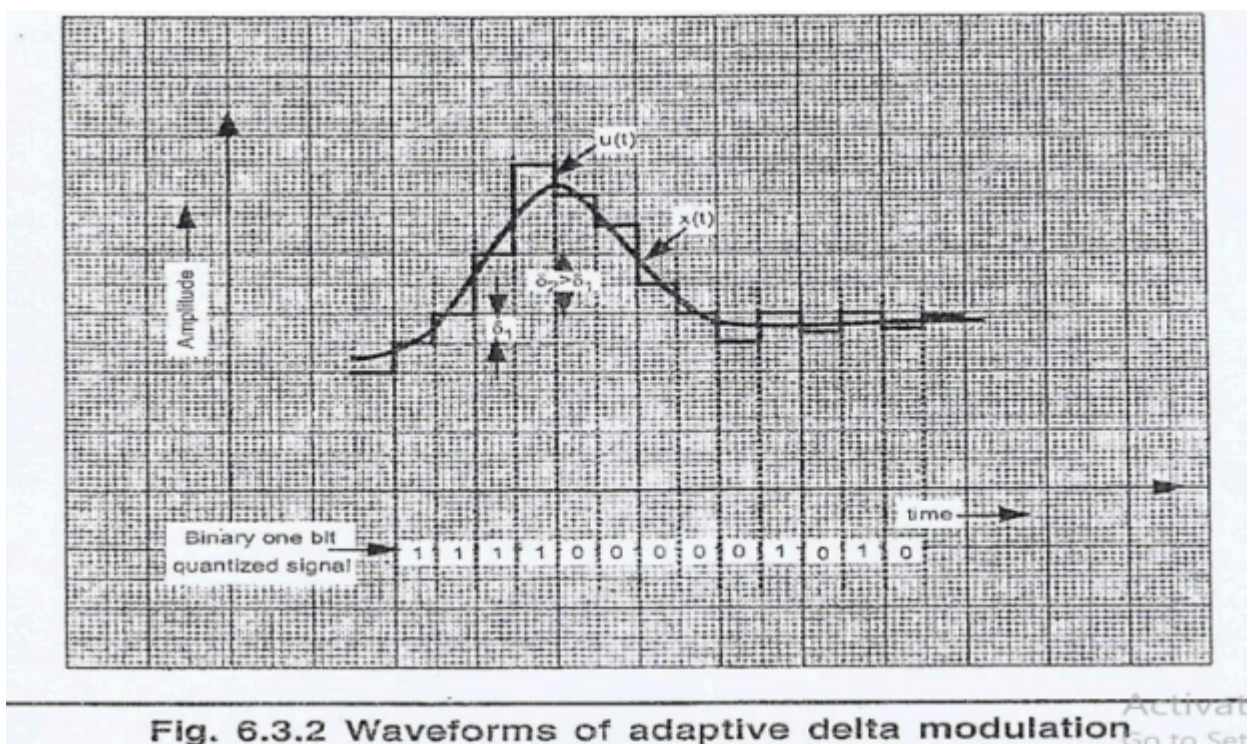
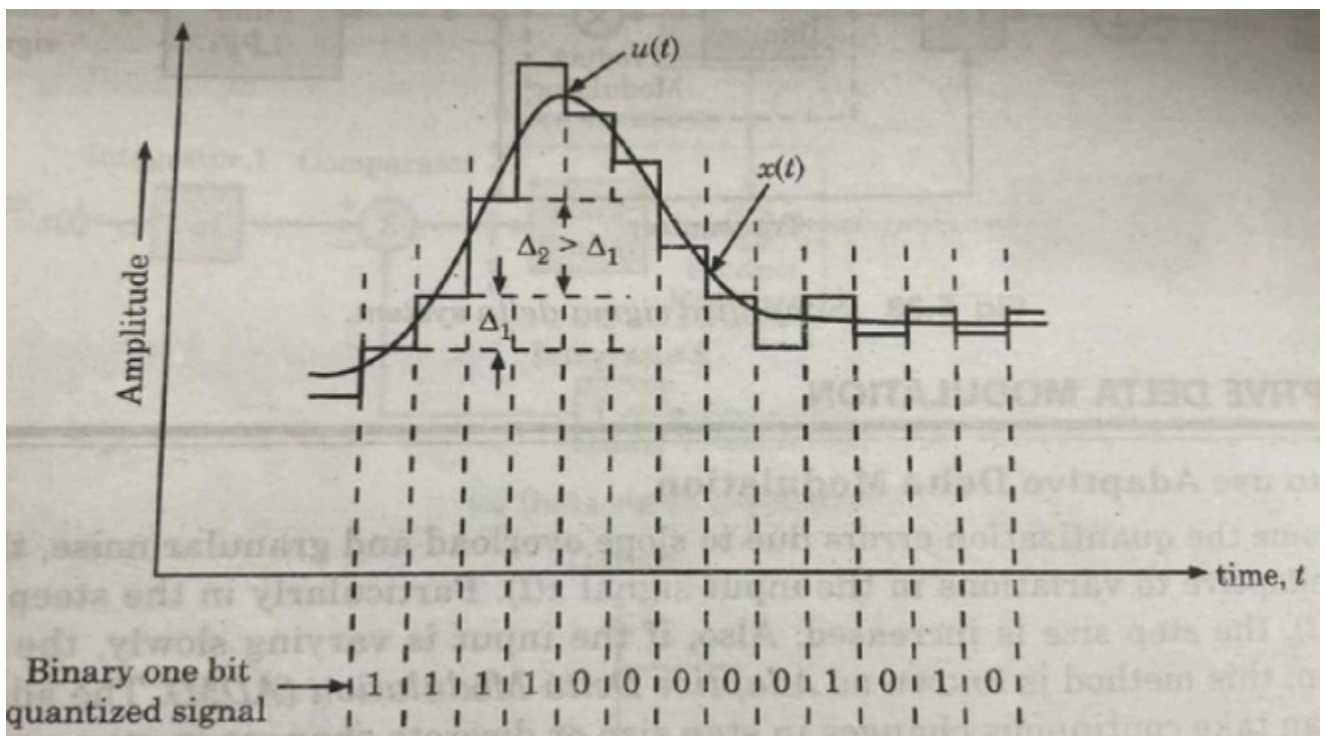
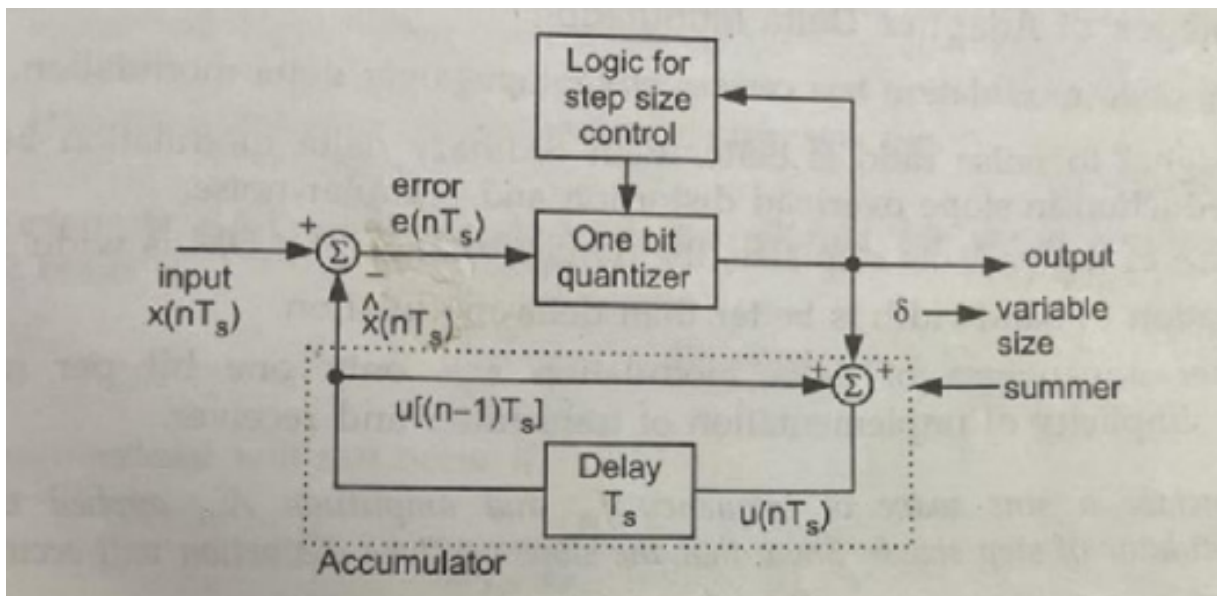


Fig. 6.3.2 Waveforms of adaptive delta modulation

ADM RECEIVER

- In the receiver of Adaptive Delta Modulator, the first part generates the step size from each incoming bit.
- Exactly the same process is followed as that in transmitter. The previous input and present input decides the step size.
- It is then given to an accumulator which builds up staircase waveform.
- The low-pass filter then smoothens output staircase waveform to reconstruct the smooth signal.

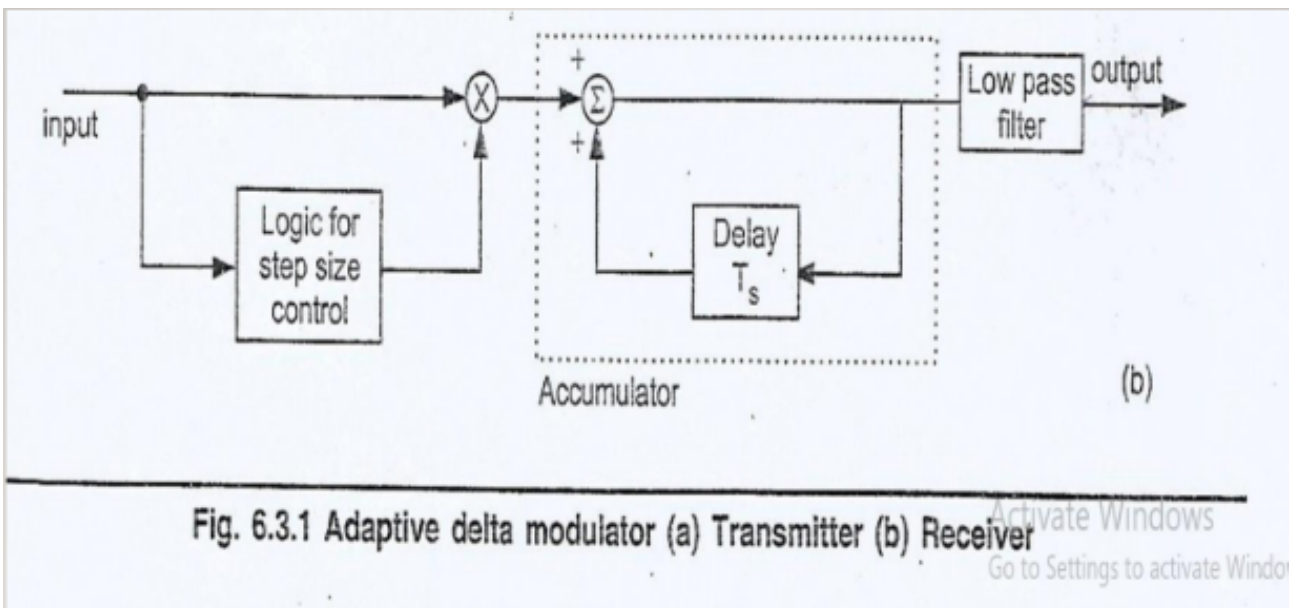


Fig. 6.3.1 Adaptive delta modulator (a) Transmitter (b) Receiver

ADVANTAGES OF ADAPTIVE DELTA MODULATOR

- The signal to noise ratio is better than ordinary Delta Modulator because of the reduction in Slope overload distortion and Granular noise.
- Because of the variable step size, the dynamic range of ADM is wide
- Utilization of bandwidth is better than Delta Modulation
- Only one bit per sample is required (same as in DM)
- Simplicity of implementation of transmitter and receiver (same as in DM)

Ex.6.3.1: Consider a sine wave of frequency f_m and amplitude to a delta modulator of step size δ . Show that the slope overload distortion will occur if

$$A_m > \frac{\delta}{2\pi f_m T_s}$$

Where T_s is the sampling period.

Sol: the input signal $x(t)$ to the modulator is sinusoidal signal, i.e. $x(t) = A_m \sin(2\pi F_m t)$

The slope of any signal is its first derivative with respect to time, and the slope is maximum when the 1st derivative is maximum.

$$\ast \text{ slope of input signal } x(t) = \frac{d}{dt}x(t)$$

$$= \frac{d}{dt} | A_m \sin(2\pi F_m t) | = A_m \cdot 2\pi F_m \cdot \cos(\pi F_m t)$$

And this slope is maximum when $(A_m \cdot 2\pi F_m \cdot \cos(\pi F_m t))$ is maximum, this happens when the term $(\cos(2\pi F_m t))$ equal to its maximum value i.e. = 1

$$\ast \text{ slope maximum of } x(t) = A_m 2\pi F_m$$

$$\text{The slope maximum of delta modulator} = \frac{\text{Step size } \delta}{\text{sampling period}}$$

$$= \frac{\delta}{T_s}$$

Slope overload distortion will take place if slope of input signal is greater than slope of delta modulator, i.e.

$$A_m \cdot 2\pi F_m > \frac{\delta}{T_s} \quad \text{or} \quad A_m > \frac{\delta}{2\pi F_m T_s}$$

Ex.6.3.2: A delta modulator system is designed to operate at five times the Nyquist rate for a signal with 3 kHz bandwidth. Determine the maximum amplitude of a 2 kHz input sinusoid for which the delta modulator does not have slope over load. Quantizing step size is 250mV. Derive the formula that you use.

sol: In example 6.3.1 we have derived the relation for slope overload distortion which will occur if,

$$A_m > \frac{\delta}{2\pi F_m T_s} \text{ by equation 6.3.1}$$

∴ Slope overload will not occur if,

$$A_m < \frac{\delta}{2\pi F_m T_s}$$

The maximum frequency in the signal is,

$$W = 3 \text{ kHz}$$

$$\text{Nyquist rate} = 2W = 2 \times 3 \text{ kHz} = 6 \text{ kHz}$$

$$\text{Sampling frequency} = 5 \text{ times nyquist rate}$$

$$F_s = 5 \times 6 \text{ kHz} = 30 \text{ kHz}$$

$$\text{Sampling interval } T_s = \frac{1}{f_s} = \frac{1}{30 \times 10^3}$$

$$\begin{aligned} \text{Step size } \delta &= 250 \text{ mV} = 250 \times 10^{-3} \\ &= 0.25 \text{ V} \end{aligned}$$

$$\text{Given that } f_m = 2 \text{ kHz} = 2 \times 10^3 \text{ Hz}$$

∴ putting these values in equation 6.3.4

$$A_m < \frac{0.25}{2\pi \times 2 \times 10^3 \times \frac{1}{30 \times 10^3}} \Rightarrow A_m < 0.6 \text{ volts}$$

Ex.6.3.4: Find the signal amplitude for minimum quantization error in a delta modulation system if step size is 1 volts having repetition period 1 ms. The information signal operates at 100Hz.

sol.: The quantization error is minimum when slope overload distortion is absent. Then the quantization error is due to granular noise only. We know that the slope overload distortion is absent if,

$$A_m > \frac{\delta}{2\pi F_m T_s}$$

Here A_m is the signal amplitude

δ is the step size. Given 1v

F_m is signal frequency. given 100Hz

T_s is sampling duration. Given 1ms

Hence signal amplitude becomes,

$$A_m < \frac{1}{2\pi \times 100 \times 1 \times 10^{-3}}$$

$$A_m < 1.6V$$

That is, the signal amplitude should be less than 1.6 volts to have minimum quantization error

Ex.6.3.5: Derive an expression for signal to quantization noise power ratio for delta modulation. Assume that no slope overload distortion exists.

Sol.:(i) To obtain signal power:

In example 6.3.1 we have derived that slope overload distortion will not occur if $A_m > \frac{\delta}{2\pi F_m T_s}$

Here A_m is peak amplitude of sinusoidal signal

δ is the step size

F_m is the signal frequency

T_s is the sampling period.

From above equation, the maximum signal amplitude

will be, $A_m > \frac{\delta}{2\pi F_m T_s}$

Signal power is given as, $p = \frac{V^2}{R}$

Here V is the rms value of the signal. Here $V = \frac{A_m}{\sqrt{2}}$.

hence above equation becomes,

$$P = \frac{A_m^2}{2} / R$$

Normalized signal power is obtained by taking $r = 1$.

$$\text{Hence, } P = \frac{A_m^2}{2}$$

Putting for A_m from equation 6.3.6,

$$P = \frac{\delta^2}{8\pi^2 F_m^2 T_s^2}$$

Ex.6.3.7: A DM system is tested with a 10 kHz sinusoidal signal with 1V peak to peak at the input. It is sampled at 10 times the nyquist rate.

(i) what is the step size required to prevent slope overload?

(ii) what is the corresponding SNR?

sol.: Given data

Signal frequency, $f_m = 10\text{kHz}$

Signal amplitude, $A_m = \frac{1V}{2} = 0.5V$

Nyquist rate $= 2 \times f_m = 2 \times 10\text{kHz} = 20\text{kHz}$

Sampling frequency, $f_s = 10 \times \text{Nyquist rate}$
 $= 10 \times 20\text{ kHz} = 200\text{kHz}$

$$T_s = \frac{1}{f_s} = \frac{1}{200 \times 10^3} \text{sec}$$

(i) to obtain step size

From equation 6.3.4 we have,

$$A_m < \frac{\delta}{2\pi f_m T_s}$$

Under this condition slope overload will not occur, From above equation step size will be,

$$\delta > 2\pi f_m T_s A_m$$

Putting values in above equation,

$$\delta > 2\pi \times 10,000 \times \frac{1}{200 \times 10^3} \times 0.5$$

$$> 0.157V$$

Thus the step size greater than 157mV will prevent the slope overload.

(ii) to obtain signal to noise ratio signal to noise ratio of delta modulation system is given by equation 6.3.12 as,

$$\frac{S}{N} = \frac{\delta^2}{8\pi^2 w F_m^2 T_s^2}$$

This is post filtered signal to noise ratio. In this example value of ‘w’ is not given. Hence we will calculate signal to noise ratio from equation 6.3.7 and equation 6.3.10 as,

$$\frac{S}{N} = \frac{\frac{\delta^2}{8\pi^2 w F_m^2 T_s^2}}{\frac{\delta^2}{3}}$$

$$= \frac{\delta^2}{8\pi^2 w F_m^2 T_s^2}$$

Putting values in above equation.

$$\frac{S}{N} = \frac{3}{8\pi^2 \times (10,000)^2 \times \frac{1}{(200 \times 10^3)^2}}$$

$$= 15.2 = 11.8 \text{ dB.}$$

Ex.6.3.10:in a single integration DM scheme, the voice signal is sampled at a rate of 64kHz. The maximum signal amplitude is 1 volt, voice signal bandwidth is 3.5kHz.

i)Determine the minimum value of step size to avoid slope overload.

ii)determine granular noise N_o

iii)Assuming signal to be sinusoidal,calculate signal power and signal to noise ratio.

iv)Assuming that noice signal amplitude is uniformly distributed in the range (-1,1) determine the signal power and signal to noise ratio.

sol.:The given data is,

$$F_s = 65\text{kHz}, A_m = 1\text{V}, W=3.5\text{K=kHz}.$$

i)to obtain step size

Slope overload will not occur if,

$$A_m < \frac{\delta}{2\pi F_m T_s}$$

Putting values in above equation with

$$T_s = \frac{1}{64 \times 10^3} \text{ and } f_m = 3.5\text{kHz}$$

$$1 < \frac{\delta}{2\pi \times 3500 \times \frac{1}{64 \times 10^3}}$$

$$\delta > 0.3436 \text{ volts}$$

ii) to obtain granular noise power

Noise power is given by equation 6.3.11 as,

$$\begin{aligned} N_c &= \frac{WT_s \delta^2}{3} = \frac{W\delta^2}{3f_c} \\ &= \frac{3.5 \times 10^3 \times (0.3436)^2}{3 \times 64 \times 10^3} \\ &= 2.15 \text{ mW} \end{aligned}$$

iii) to obtain S/N ratio

Amplitude of the signal is 1 V. For sinusoidal signal, the signal power will be

$$P = \frac{A_m^2}{2} = \frac{1}{2} \text{ W}$$

Hence signal to noise ratio will be,

$$\begin{aligned} \frac{S}{N} &= \frac{\text{Signal power}}{\text{Noise power}} \\ &= \frac{1/2}{2.15 \times 10^{-3}} = 232.3 \quad \text{or} \quad \left(\frac{S}{N}\right)_{\text{dB}} = 10 \log_{10} \frac{S}{N} = 23.66 \text{ dB} \end{aligned}$$

COMPARISON BETWEEN PCM, DM, AND ADM

No	Parameter of comparison	PCM	DM	ADM
1	Number of bits	It can use 4,8,16 bit per sample	It uses only one bit for one sample	It uses only one bit for one sample
2	Level and	The number of levels(q) depend on number of bits, level size is kept fixed	Step size is kept fixed and can not be varied	According to the signal variation, step size varies(i.e. adapted)
3	Quantization error and distortion	Quantization error depends on number of levels used	Slope overload distortion and granular noise are present.	Quantization noise is present but other errors are absent.
4	Transmission bandwidth	Highest bandwidth is required since number of bits are high	Lowest bandwidth is required	Lowest bandwidth is are absent.
5	feedback	There is no feedback in transmitter or receiver	Feedback exists in transmitter	Feedback exists
6	Complexity of implementation	System complex	simple	simple

Chapter 12: DIGITAL PULSE MODULATION

TECHNIQUES Review

General notes

- They are convert analog signal to digital (binary, 0.1) and make it suitable for modern digital communication systems.
- Because of this conversion, some noise (error) is permanently introduced in the result signal.
- This is because the analog to digital conversion is not ideal and perfect(there is some approximations)
- The main problems associated with Digital pulse Modulation techniques is that they need large channel Bandwidth.
- To reduce the Bandwidth, the number of bits required to represent samples of signal must be reduced.
- But if this happens, noise increase.
- A compromising between Bandwidth and noise is required in real applications.

1. Pulse Code Modulation(PCM)

The first type of Digital Pulse modulation techniques is
Pulse Code Modulation PCM

In this type(PCM):

- The signal is sampled at F_s ($F_s \geq 2F_m$),
- Quantized by comparison with (q) predefined levels
- Encoded to v bits per each q level
- These process leads to a noise or error called quantization error (ϵ)
- it has maximum value ($\epsilon_{\max}$) $= \frac{\delta}{2}$
- It has bit rate, $r = v.F_s$ bit per second
- It has bandwidth, $BT = 1/2(V.F_s)$
- In PCM, the error or noise is related to signal by Signal power/Noise power
- The general equation for this ratio is, maximum signal to quantization noise ratio:

$$\frac{S}{N} = \frac{3P}{x_{\max}^2} \cdot 2^{2v}$$

- Signal to quantization noise ratio for normalized values of power and amplitude of input signal
- $$\left(\frac{S}{N}\right)_{\text{dB}} \leq (4.8) + 6v \text{ dB}$$
- Signal to quantization noise for sinusoidal input signal is,

$$\left(\frac{S}{N}\right)_{\text{dB}} \leq (1.8) + 6v \text{ dB}$$

2. Delta Modulation(DM)

- We have seen in PCM that, it transmits all the bits which are used to code the sample.
- Hence signaling rate and transmission channel Bandwidth are large
- To overcome this problem Delta Modulation is used.
- Delta modulation transmits only one bit per sample.
- In DM, the present sample value is compared with previous sample value and according to this comparison, “1” is transmits if the difference is positive(present sample is grate the previous) and “0” is transmits if the difference is negative (present sample is less than the previous)
- That means only one bit is transmit for each sample.

Advantages of DM(compare with PCM)

1. Delta modulation transmits only one bit for each sample. thus signaling rate and transmission bandwidth is quite small.
2. The transmitter and receiver implementation

Disadvantages of DM

1. Slope overload distortion(startup error): this arises because of large dynamic range of the input signal (the input signal is more steep) i.e. has high slope. To reduce this error, the step size should be increased when slope of input signal is high.
2. Granular Noise (Hunting) : it occurs when the input signal varying slowly (low input signal slope) To reduce this error, the step size should be reduced when slope of input signal is low.
3. Thus large step size is required to accommodate wide dynamic range of the input signal (to reduce slope overload distortion) and small steps. are required to reduce granular noise
4. Adaptive Delta Modulation is the modulation to overcome these errors.

3. Adaptive Delta Modulation (ADM)

- To overcome the quantization errors due to slope overload and granular noise, the step size (δ) is made adaptive to variations in the input signal.
- Practically, in the steep segment of the input signal, the step is increased. When the input is varying slowly, the step size is reduced.
- Then the method is called Adaptive Delta Modulation ADM