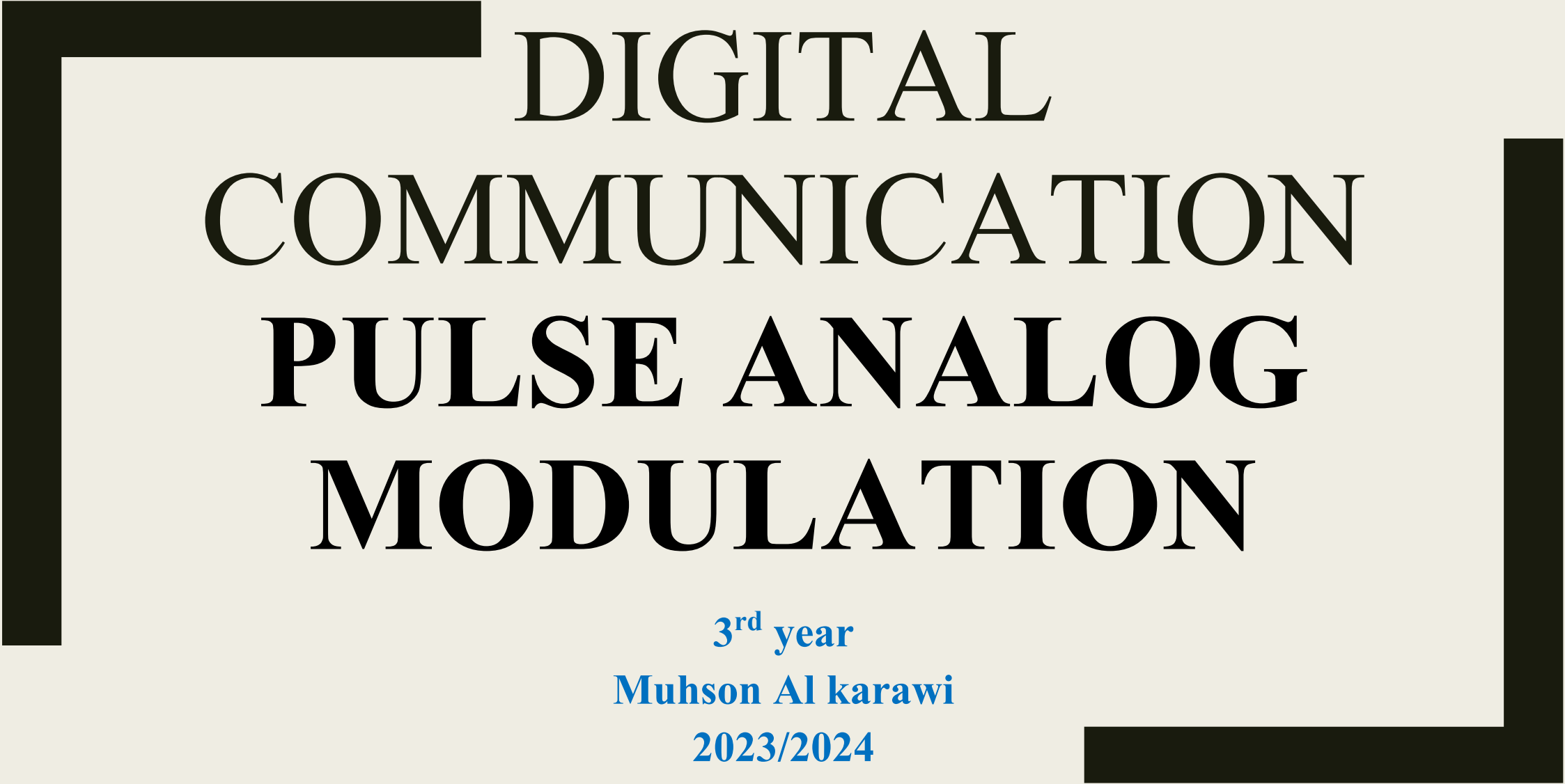


A thick black L-shaped frame is positioned on the left and bottom edges of the slide, framing the central text.

DIGITAL COMMUNICATION PULSE ANALOG MODULATION

3rd year

Muhson Al karawi

2023/2024

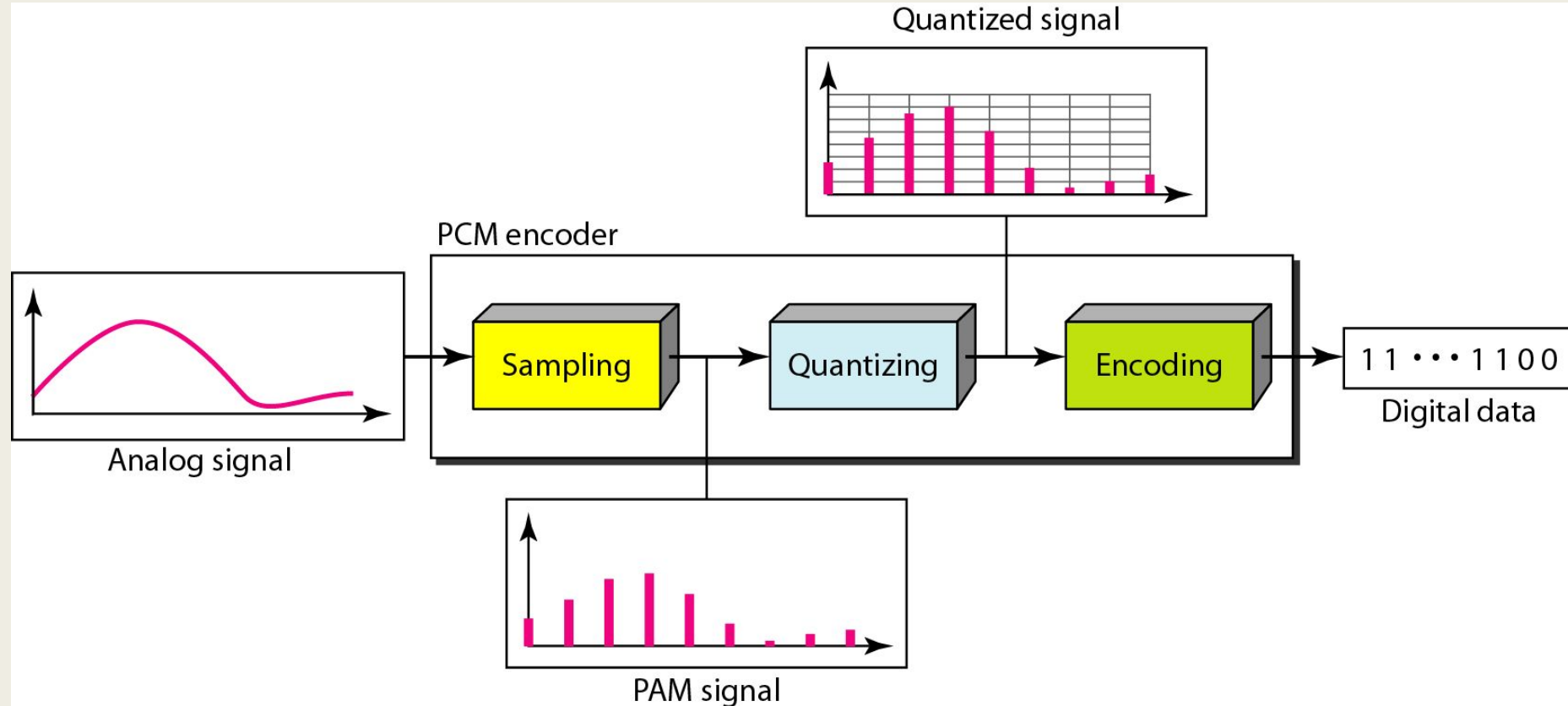
Pulse Analog Modulation

Analog to Digital Conversion

Sampling Theorem

- Almost all the natural information signal exist in **analog** form
- The modern communication systems are **digital**
- The analog signal(continues- time signal) to be transmitted by digital communication system should be converted to **Digital form** (Discrete-time signal)
- The first step in this direction, is to take **samples** from original signal (Discretization)
- Ho - many samples must be taken?, in order to good signal representation and ability of original signal reconstruction ?
- This problem is solved by a mathematical tool known as Sampling theorem
- With the help of this theorem , a continuous – time signal may be completely represented and recovered from its samples taken uniformly.

Figure 4.21 *Components of PCM encoder*



Statement of sampling theorem

- The sufficient number of samples of the signal must be taken so that the original signal is represented in its samples completely. Also, it should be possible to recover or reconstruct the original signal completely.
- The statement of sampling theorem can be given in two parts as:
 - I. A band limited signal of finite energy, which has no frequency component higher than f_m Hz, is completely described by its sample values at uniform intervals(T_s) less than or equal to $1/(2f_m)$ second apart.
 - II. A band limited signal of finite energy, which has no frequency components higher than f_m Hz may be completely recovered from the knowledge of its samples taken at the rate of $(2 f_m)$ samples per second.

Combining the two parts, the sampling theorem may be stated as under:

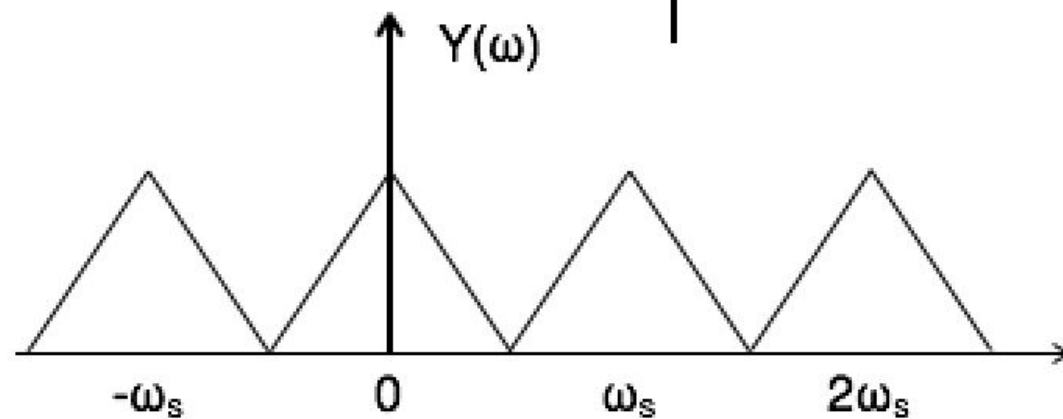
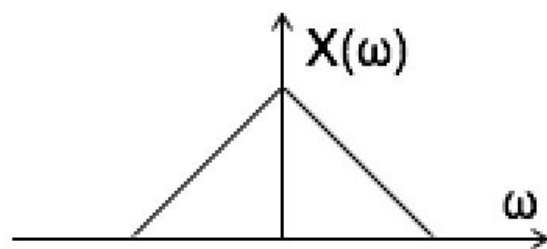
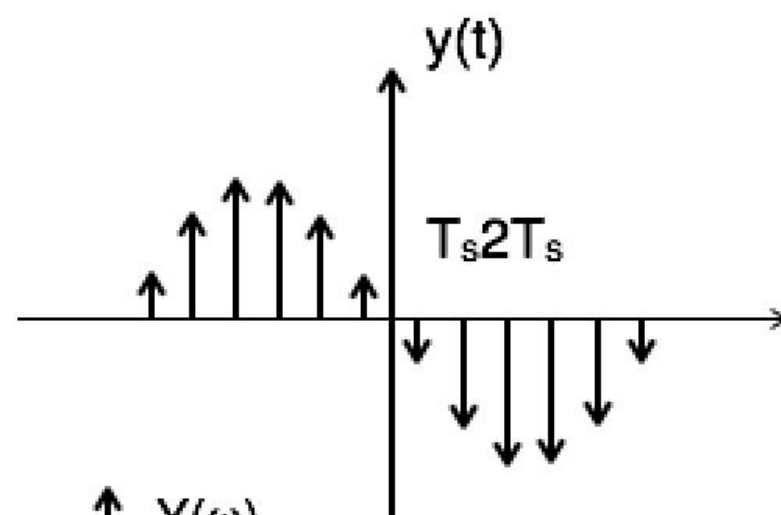
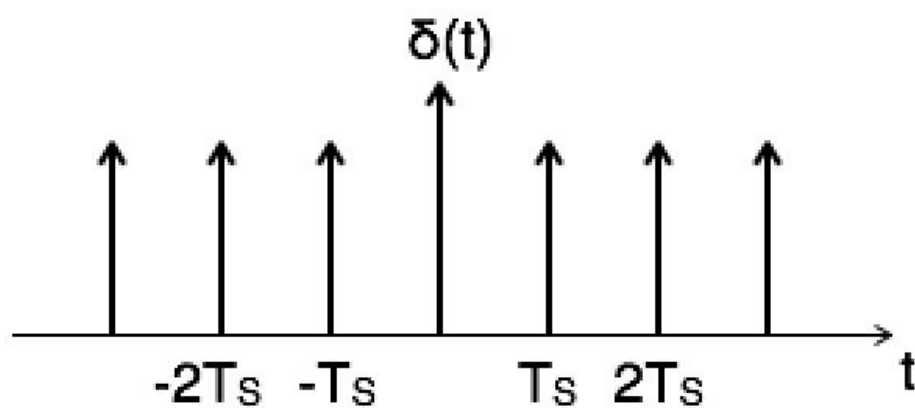
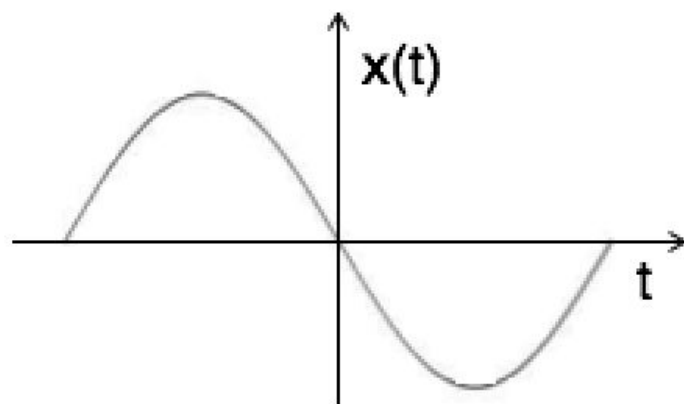
- *‘A continuous – time signal may be completely represented in its samples and recovered back , If the sampling frequency is $F_s \geq 2 F_m$. Here F_s is the sampling frequency and F_m is the maximum frequency present in the signal.’*

Signal sampling theorem

Statement: A continuous time signal can be represented in its samples and can be recovered back when sampling frequency f_s is greater than or equal to the twice the highest frequency component of message signal

Proof: Consider a continuous time signal x . The spectrum of $x(t)$ is a band limited to f_m Hz i.e. the spectrum of x is zero for $|\omega| > \omega_m$.

Sampling of input signal $x(t)$ can be obtained by multiplying $x(t)$ with an impulse train $\delta(t)$ of period T_s . The output of multiplier is a discrete signal called sampled signal which is represented with $y(t)$ in the following diagrams.



Here, you can observe that the sampled signal takes the period of impulse. The process of sampling can be explained by the following mathematical expression:

$$\text{Sampled signal } y(t) = x(t) \cdot \delta(t) \dots \dots (1)$$

$s(t) = 0$ except at $t = 0$ 

The trigonometric Fourier series representation of δt is given by

$$\delta(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_s t + b_n \sin n\omega_s t) \dots \dots (2)$$

Where $a_0 = \frac{1}{T_s} \int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t) dt = \frac{1}{T_s} \delta(0) = \frac{1}{T_s}$

The area under $s(t)$
 $\dots \int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} s(t) dt = 1$

$$a_n = \frac{2}{T_s} \int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t) \cos n\omega_s t dt = \frac{2}{T_s} \delta(0) \cos n\omega_s 0 = \frac{2}{T}$$

$$b_n = \frac{2}{T_s} \int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t) \sin n\omega_s t dt = \frac{2}{T_s} \delta(0) \sin n\omega_s 0 = 0$$

Substitute above values in equation 2.

$$\therefore \delta(t) = \frac{1}{T_s} + \sum_{n=1}^{\infty} \left(\frac{2}{T_s} \cos n\omega_s t + 0 \right)$$

Substitute δt in equation 1.

$$\rightarrow y(t) = x(t) \cdot \delta(t)$$

$$= x(t) \left[\frac{1}{T_s} + \sum_{n=1}^{\infty} \left(\frac{2}{T_s} \cos n\omega_s t \right) \right]$$

$$= \frac{1}{T_s} [x(t) + 2 \sum_{n=1}^{\infty} (\cos n\omega_s t) x(t)]$$

$$\because \cos(\omega_s t) = \frac{1}{2} \delta(\omega - \omega_s) + \frac{1}{2} \delta(\omega + \omega_s)$$

$$y(t) = \frac{1}{T_s} [x(t) + 2 \cos \omega_s t \cdot x(t) + 2 \cos 2\omega_s t \cdot x(t) + 2 \cos 3\omega_s t \cdot x(t) \dots \dots]$$

Take Fourier transform on both sides.

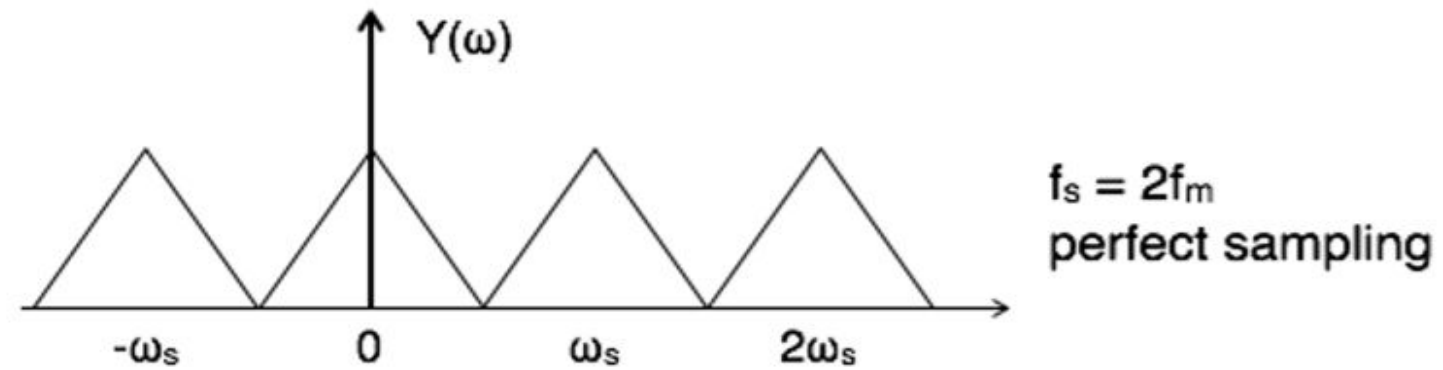
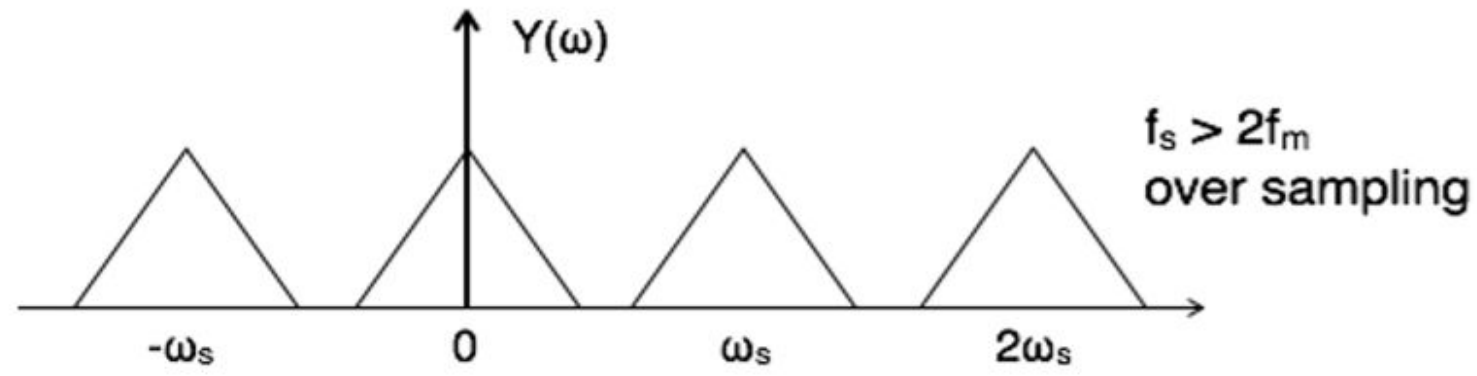
$$\because \omega_s = 2\pi f_s$$

$$Y(\omega) = \frac{1}{T_s} [X(\omega) + X(\omega - \omega_s) + X(\omega + \omega_s) + X(\omega - 2\omega_s) + X(\omega + 2\omega_s) + \dots]$$

$$\therefore Y(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s) \quad \text{where } n = 0, \pm 1, \pm 2, \dots$$

To reconstruct xt , you must recover input signal spectrum $X\omega$ from sampled signal spectrum which is possible when there is no overlapping between the cycles of $Y\omega$.

Possibility of sampled frequency spectrum with different conditions is given by the following diagrams:



Aliasing effect

- The overlapped region in case of under sampling represents aliasing effect which can be removed by:
 - Considering $F_s \geq 2F_m$
 - By using anti aliasing filters

Few points about Sampling Theorem

- The spectrum of sampled signal, as the sampling frequency F_s is greater twice than maximum signal frequency, the spectrum will repeat periodically without overlapping.
- The spectrum of sampled signal extends up to infinity and the ideal bandwidth of sampled signal is infinity.
- The original desired spectrum is centered at $\omega=0$ and having bandwidth or maximum frequency equal to ω_m . The desired spectrum may be recovered by passing the sampled signal through a low pass filter with cut-off frequency ω_m .
- It may be observed, that for the case $F_s > 2F_m$, the successive cycles of sampled signal are not overlapping each other.
- For the case $F_s = 2F_m$, although the successive cycles are not overlapping each other, but they are touching each other.
- For the case $F_s < 2F_m$, the successive cycles will overlap each other and hence in this case, the original spectrum cannot be extracted out of the spectrum.
- Hence for reconstruction without distortion, we must have, $F_s \geq 2F_m$

Nyquist rate and Nyquist Interval

- ❑ Sampling rate is the number of samples per second
- ❑ Sampling interval is the distance in time between successive samples.
- ❑ Nyquist rate is the sampling frequency when $F_s = 2 F_m$ and is also called **minimum sampling rate**.
- ❑ Nyquist Interval is the interval of Nyquist sampling frequency and is also called **maximum sampling Interval**.
- ❑ $T_s = 1 / (2F_m)$

Example -1

- An analog signal is expressed by,

$$x(t) = 3 \cos 50\pi t + 10 \sin 300 \pi t - \cos 100 \pi t$$

Calculate the Nyquist rate and Nyquist Interval

solution: the three angular frequencies in the signal are:

$$w_1 = 50\pi, \quad w_2 = 300\pi, \quad w_3 = 100\pi \quad \text{with } w = 2\pi f, \quad f = w/2\pi$$

$$f_1 = 50\pi / 2\pi \quad f_2 = 300\pi / 2\pi \quad f_3 = 100\pi / 2\pi$$

$$f_1 = 25 \text{ Hz} \quad f_2 = 150 \text{ Hz} \quad f_3 = 50 \text{ Hz}$$

the maximum frequency, F_m is 150 Hz

$$\text{Nyquist rate } F_s = 2 F_m = 2 \times 150 = 300 \text{ Hz}$$

$$\text{Nyquist interval } T_s = 1/F_s = 1/300 = 3.333 \text{ msec.}$$

Example -2

- Find the Nyquist rate and Nyquist Interval for the following signal
 $x(t) = \frac{1}{2} \pi \cos(4000 \pi t) \cdot \cos(1000 \pi t)$

solution: it is better to convert product to adding
with $2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$

Example:

Suppose that an analog signal is given as

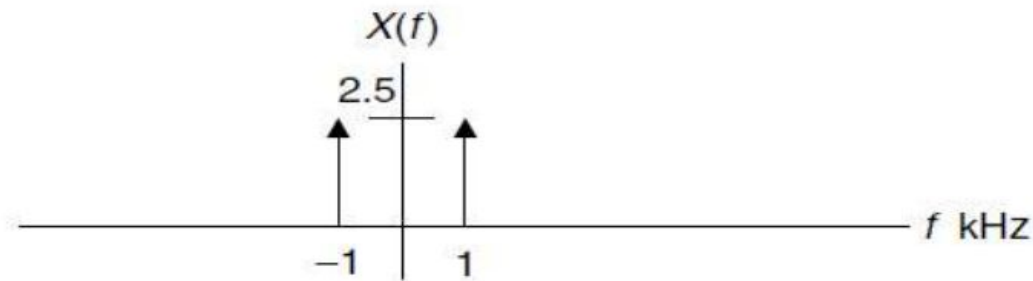
$x(t) = 5 \cos(2\pi \cdot 1000t)$, for $t > 0$, and is sampled at the rate of 8,000 Hz.

- Sketch the spectrum for the original signal.
- Sketch the spectrum for the sampled signal from 0 to 20 kHz.

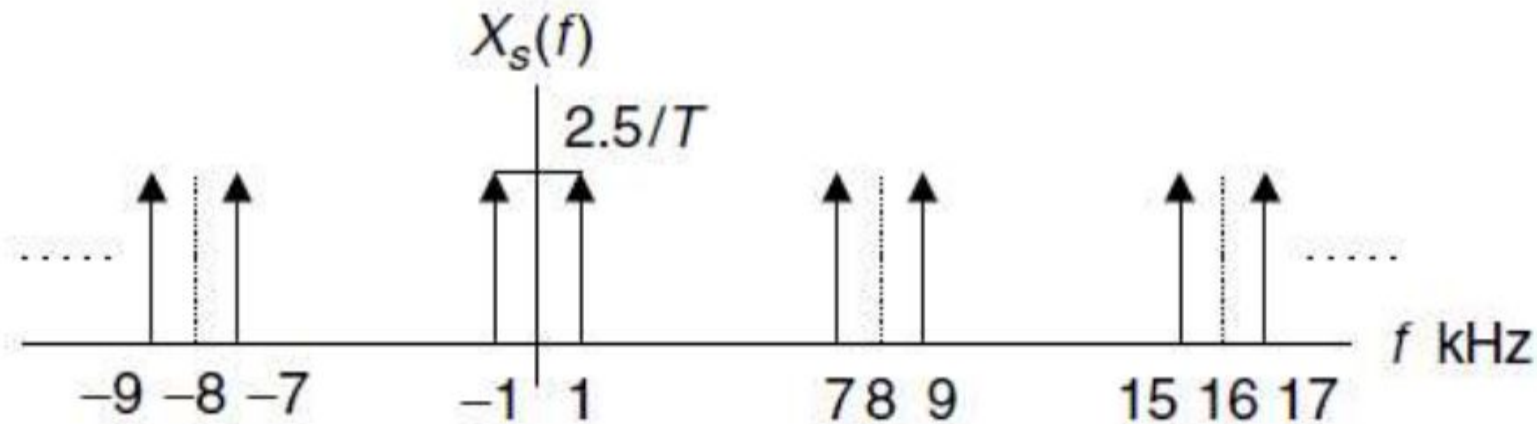
Sol.

- Since the analog signal is sinusoid with a peak value of 5 and frequency of 1,000 Hz, we can write the sine wave using Euler's identity:

$$5 \cos(2\pi \times 1000t) = 5 \cdot \left(\frac{e^{j2\pi \times 1000t} + e^{-j2\pi \times 1000t}}{2} \right) = 2.5e^{j2\pi \times 1000t} + 2.5e^{-j2\pi \times 1000t},$$



- After the analog signal is sampled at the rate of 8,000 Hz, the sampled signal spectrum and its replicas centered at the frequencies $\pm n f_s$, each with the scaled amplitude being $2.5/T$, are as shown in Figure below.



Notice that the spectrum of the sampled signal contains the images of the original spectrum; that the images repeat at multiples of the sampling frequency f_s (for our example, 8 kHz, 16 kHz, 24 kHz, . . .); and that all images must be removed, since they convey no additional information.

Reconstruction Filter (Low Pass Filter)

- The original signal can be recover from its samples by using a low pass Filter
- The low pass filter is that type of filter which passes only low frequencies up to a specified cut-off frequency and rejects all other frequencies above cut-off frequency.
- The ideal LPF has sharp – change in response at cut-off frequency
- The practical LPF has amplitude response decrease slowly to zero.
- It has transition band.

Effect of Under Sampling : ALIASING

- When a continuous - time band – limited signal is sampled at a rate **lower than Nyquist rate $F_s < 2F_m$** , then the successive cycles of the spectrum of the sampled signal **overlap** with each other.
- Hence the signal is under sampled process.
- Some amount of aliasing is produced.
- Aliasing is the phenomenon in which a high frequency component in the frequency spectrum of the signal takes identity of a lower frequency component in the spectrum of the sampled signal.
- It is clear that because of the overlap due to aliasing phenomenon, it is not possible to recover original signal from sampled signal by low pass filtering since the spectral components in the overlap regions add and hence the signal is distorted.
- Since any information signal contains a large number of frequencies, so
- To decide a sampling frequency is problem.?

....Continue

- Therefore , a signal is first passed through a low pass filter which blocks all the frequencies which are above F_m Hz. This process is known as band limiting of the original signal.
- This low pass filter is called pre alias filter because it is used to prevent aliasing effect.
- After band- limiting, it becomes easy to decide sampling frequency since the maximum frequency is fixed at F_m Hz.

■ To avoid aliasing:

- - Prealias filter must be used to limit band of frequencies of frequencies of the signal to F_m Hz.
- - Sampling frequency “ F_s ” must be selected such that

- $F_s > 2 F_m$

Sampling of Bandpass Signals

- The sampling theorem for bandpass signal $X(t)$ whose maximum bandwidth is $2W$ can be completely represented into and recovered from its samples if it is sampled at minimum rate of twice the bandwidth.

Sampling techniques

They are three types

- I. Ideal sampling or instantaneous sampling or impulse sampling which is shown in study of sampling theorem. The width of samples (Δ) approaches zero. Hence the transmitted power is negligible and the method is impracticable.**
- II. Natural sampling or chopper sampling, in this type of sampling the pulse has a finite width Δ**
- III. Flat Top Sampling or Rectangular pulse sampling**
this is also a practically possible sampling method. it is very easy to get flat top samples
the top of the samples remains constant and equal to instantaneous value of baseband signal $x(t)$ at the start of sampling. The duration of each sample is Δ and sampling rate is equal to $F_s = 1/T_s$