

PCM Quantization noise and S/N

Ex.6.1.1 Derive the expression for signal to quantization noise ratio for PCM system that employs linear quantization technique. Assume that input to the PCM system is a sinusoidal signal. [May - 98, 8-Marks]

OR

A PCM system uses a uniform quantizer followed by a v bit encoder. Show that rms signal to quantization noise ratio is approximately given by $(1.8 + 6v)$ dB.

[May-96 8-Marks, Dec-97 8-Marks]

Sol. : Assume that the modulating signal be a sinusoidal voltage, having peak amplitude A_m . Let this signal cover the complete excursion of representation levels.

The power of this signal will be,

$$P = \frac{V^2}{R} \quad \text{Here } V = \text{rms value}$$
$$= \left[A_m / \sqrt{2} \right]^2 \quad \dots (6.1.33)$$

When $R = 1$, the power P is normalized, i.e.,

Normalized power : $P = \frac{A_m^2}{2}$ with $R = 1$ in above equation.

$\therefore$ Signal to quantization noise ratio is given by equation 6.1.27 as,

$$\frac{S}{N} = \frac{3P}{x_{\max}^2} \times 2^{2v}$$

Here

$$P = \frac{A_m^2}{2} \quad \text{and} \quad x_{\max} = A_m$$

Putting these values in the above equation,

$$\frac{S}{N} = \frac{3 \times \frac{A_m^2}{2}}{A_m^2} \times 2^{2v} = \frac{3}{2} \times 2^{2v} = 1.5 \times 2^{2v}$$

Expressing signal to noise power ratio in dB,

$$\begin{aligned} \left(\frac{S}{N} \right) \text{dB} &= 10 \log_{10} \left(\frac{S}{N} \right) = 10 \log_{10} (1.5 \times 2^{2v}) \\ &= 10 \log_{10} (1.5) + 10 \log_{10} 2^{2v} \\ &= 1.76 + 2v \times 10 \times 0.3 \end{aligned}$$

Thus,

$$\left(\frac{S}{N} \right) \text{dB in PCM} : \left(\frac{S}{N} \right) \text{dB} = 1.8 + 6v ; \text{ for sinusoidal signal}$$

... (6.1.34)

Ex.6.1.2 A Television signal with a bandwidth of 4.2 MHz is transmitted using binary PCM. The number of quantization levels is 512.
Calculate,

- i) Code word length ii) Transmission bandwidth
iii) Final bit rate iv) Output signal to quantization noise ratio.

[Dec-96, 8-Marks]

Sol. : The bandwidth is 4.2 MHz, means highest frequency component will have frequency of 4.2 MHz i.e.,

$$W = 4.2 \text{ MHz}$$

Quantization levels $q = 512$

i) Number of bits and quantization levels are related in binary PCM as,

$$q = 2^v$$

i.e. $512 = 2^v$

$$\log 512 = v \log 2$$

or $v = \frac{\log 512}{\log 2}$

$$= 9 \text{ bits}$$

... (Ans)

Thus the code word length is 9 bits.

ii) From equation 6.1.8 the transmission channel bandwidth is given as,

$$B_T \geq vW$$

$$\geq 9 \times 4.2 \times 10^6 \text{ Hz}$$

$$B_T \geq 37.8 \text{ MHz}$$

... (Ans)

iii) The final bit rate will equal to signaling rate. From equation 6.1.5 signaling rate is given as

$$r = v f_s$$

Sampling frequency $f_s \geq 2W$ by sampling theorem.

$$\therefore f_s \geq 2 \times 4.2 \text{ MHz} \quad \text{since } W = 4.2 \text{ MHz}$$

$$\therefore f_s \geq 8.4 \text{ MHz}$$

Putting this value of ' f_s ' in equation for signaling rate,

$$r = 9 \times 8.4 \times 10^6$$

$$= 75.6 \times 10^6 \text{ bits/sec}$$

... (Ans)

From equation 6.1.6 transmission bandwidth is also obtained as,

$$B_T \geq \frac{1}{2} r$$

$$\geq \frac{1}{2} \times 75.6 \times 10^6 \text{ bits/sec}$$

or $B_T \geq 37.8 \text{ MHz}$ which is same as the value obtained earlier.

iv) The signal to noise ratio

$$\left(\frac{S}{N} \right) \text{dB} \leq 4.8 + 6v \text{ dB} \quad \text{by equation 6.1.32}$$

$$\leq 4.8 + 6 \times 9$$

$$\leq 58.8 \text{ dB}$$

... (Ans)

Ex.6.1.3 The bandwidth of signal input to the PCM is restricted to 4 kHz. The input varies from -3.8 V to $+3.8\text{ V}$ and has the average power of 30 mW . The required signal to noise ratio is 20 dB . The modulator produces binary output. Assume uniform quantization.

i) Calculate the number of bits required per sample.

ii) Outputs of 30 such PCM coders are time multiplexed. What is the minimum required transmission bandwidth for the multiplexed signal ? **[Dec-94, 8-Marks]**

Sol. : The given value of signal to noise ratio is 20 dB .

i.e.
$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \left(\frac{S}{N}\right) = 20\text{ dB}$$

$$\therefore \frac{S}{N} = 100$$

i) From equation 6.1.27, the signal to quantization noise ratio is given as,

$$\frac{S}{N} = \frac{3P \cdot 2^{2v}}{x_{\max}^2}$$

Here
$$x_{\max} = 3.8\text{ V}, P = 30\text{ mW} \text{ and } \frac{S}{N} = 100$$

$$\therefore 100 = \frac{3 \times 30 \times 10^{-3} 2^{2v}}{(3.8)^2}$$

$$\therefore v = 6.98 \text{ bits}$$

$$= 7 \text{ bits}$$

... (Ans)

ii) The maximum frequency is,

$$W = 4 \text{ kHz}$$

The transmission bandwidth is given by equation 6.1.8 as,

$$B_T \geq vW$$

Since there are 30 PCM coders which are time multiplexed, the transmission bandwidth will be,

$$B_T \geq 30 \times v \cdot W$$

$$\geq 30 \times 7 \times 4 \text{ kHz}$$

$$\geq 840 \text{ kHz}$$

... (Ans)

Signaling rate is two times the transmission bandwidth as given by equation 6.1.1.
i.e.,

$$\text{Signaling rate } r = 840 \times 2 \text{ bits/sec} = 1680 \text{ bits/sec}$$

Ex.6.1.4 The information in an analog signal voltage waveform is to be transmitted over PCM system with an accuracy of $\pm 0.1\%$ (full scale). The analog voltage waveform has a bandwidth of 100 Hz and an amplitude range of -10 to $+10$ volts.

- Determine the maximum sampling rate required.
- Determine the number of bits in each PCM word.
- Determine minimum bit rate required in the PCM signal.
- Determine the minimum absolute channel bandwidth required for the transmission of the PCM signal.

[Dec-91, 8-Marks]

Sol. : Here an accuracy is given as $\pm 0.1\%$. That is quantization error should be $\pm 0.1\%$.

or the maximum quantization error should be $\pm 0.1\%$

or

$$\epsilon_{\max} = \pm 0.1\% = \pm 0.001$$

From equation 6.1.2 the maximum quantization error for an uniform quantizer is given as,

$$\epsilon_{\max} = \left| \frac{\delta}{2} \right|$$

or

$$\left| \frac{\delta}{2} \right| = 0.001$$

That is

$$\text{Step size } \delta = 2 \times 0.001 = 0.002$$

From equation 6.1.10, the step size, number of levels and maximum value of the signal are related as

$$\delta = \frac{2x_{\max}}{q} \quad \text{Here } |x_{\max}| = 10 \text{ volts}$$

$\therefore$ Putting values of δ and $x_{\max}$,

$$0.002 = \frac{2 \times 10}{q}$$

or

$$q = \frac{20}{0.002} \\ = 10,000$$

That is the number of levels are 10,000.

a) The maximum frequency in the signal is 100 Hz i.e.,

$$W = 100 \text{ Hz}$$

By sampling theorem, minimum sampling frequency should be,

$$f_s \geq 2W \\ \geq 2 \times 100 \geq 200 \text{ Hz}$$

... (Ans)

b) We know that minimum 10,000 levels should be used to quantize the signal. If binary PCM is used, then number of bits for each samples can be calculated as,

$$q = 2^v$$

Here,

q = number of levels

v = bits in PCM,

$$\therefore 10,000 = 2^v$$

$$\log_{10} 10,000 = v \log_{10} 2$$

$$\text{or } v = \frac{\log 10,000}{\log 2} = 13.288$$

$$\text{or } v = 14 \text{ bits}$$

... (Ans)

c) From equation 6.1.5 the bit rate or signaling rate is given as,

$$r = v f_s$$

$$= 14 \times 200$$

$$\geq 2800 \text{ bits per second.}$$

d) The transmission channel for PCM is given by equation 6.1.6 as,

$$B_T \geq \frac{1}{2} r$$

$$\geq \frac{1}{2} \times 2800$$

$$\geq 1400 \text{ Hz}$$

... (Ans)

Ex.6.1.5 A PCM system uses a uniform quantizer followed by a 7-bit binary encoder. The bit rate of the system is equal to 50×10^6 bits/sec.

a) What is the maximum message bandwidth for which the system operates satisfactorily?

b) Determine the output signal to quantization noise ratio when a full load sinusoidal modulating wave of frequency 1 MHz is applied to the input.

Sol. : a) Let us assume that the message bandwidth be W Hz. Therefore sampling frequency should be,

$$f_s \geq 2W$$

The number of bits $v = 7$ bits

From equation 6.1.5 the signaling rate is given as,

$$r = v \cdot f_s$$

$$\therefore r \geq 7 \times 2W$$

$$\therefore 50 \times 10^6 \geq 14W \quad (\text{putting value of } r)$$

$$\therefore W \leq 3.57 \text{ MHz} \quad \dots (\text{Ans})$$

Thus the maximum message bandwidth is 3.57 MHz.

b) The modulating wave is sinusoidal. For such signal, the signal to quantization noise ratio is given by equation 6.1.34 as,

$$\left(\frac{S}{N} \right)_{dB} = 1.8 + 6v$$

$$= 1.8 + 6 \times 7 \quad (\text{putting for } v = 7)$$

$$= 43.8 \text{ dB}$$

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... (Ans)

Ex.6.1.6 The information in an analog waveform with maximum frequency $f_m = 3$ kHz is to be transmitted over an M -level PCM system where the number of pulse levels is $M=16$. The quantization distortion is specified not to exceed 1% of peak to peak analog signal.

- i) What is the maximum number of bits per sample that should be used in this PCM system ?
- ii) What is the minimum sampling rate and what is the resulting bit transmission rate ?

[Dec-94, 8-Marks]

Sol. : i) Since the number of levels given here are $M=16$,

$$q = M = 16$$

Then bits and levels in binary PCM are related as,

$$q = 2^v \quad v = \text{bits}$$

$$16 = 2^v$$

or

$$v = 4$$

ii) Since $f_m = W = 3$ kHz

$$\text{minimum } f_s \geq 2W \quad \text{by sampling theorem}$$

$$\therefore f_s \geq 2 \times 3 \text{ kHz}$$

or

$$\geq 6 \text{ kHz}$$

... (Ans)

Bit transmission rate or signaling rate is given by equation 6.1.5 as,

$$r = v f_s$$

$$\geq 4 \times 6 \times 10^3$$

$$\geq 24 \times 10^3 \quad \text{bits per second}$$

... (Ans)

Ex.6.1.7 A Signal of bandwidth 3.5 kHz is sampled quantized and coded by a PCM system. The coded signal is then transmitted over a transmission channel of supporting a transmission rate of 50 k bits/sec. Calculate the maximum signal to noise ratio that can be obtained by this system.

The input signal has peak to peak value of 4 volts and rms value of 0.2 V.

Sol. : The maximum frequency of the signal is 3.5 kHz,

i.e. $W = 3.5 \text{ kHz}$

Therefore sampling frequency will be

$$\begin{aligned} f_s &\geq 2W \\ &\geq 2 \times 3.5 \text{ kHz} \\ &\geq 7 \text{ kHz} \end{aligned}$$

The signaling rate is given by equation 6.1.5 as,

$$r = v f_s$$

Putting values of $r = 50 \times 10^3$ bits/sec and $f_s \geq 7 \times 10^3$ Hz in above equation.

$$50 \times 10^3 \leq v \cdot 7 \times 10^3$$

$$\therefore v \geq 7.142 \text{ bits}$$

$$\therefore v \approx 8 \text{ bits}$$

The rms value of the signal is 0.2 V. Therefore the normalized signal power will be,

$$\text{Normalized signal power} = \frac{(0.2)^2}{1} \quad [R = 1 \text{ for normalized power}]$$

$$\text{i.e., } P = 0.04 \text{ W}$$

The maximum signal to noise ratio is given by equation 6.1.27 as,

$$\frac{S}{N} = \frac{3P \cdot 2^{2v}}{x_{\max}^2}$$

Putting the values of $P = 0.04$, $v = 8$ and $x_{\max} = 2$ in above equation,

$$\begin{aligned} S/N &= \frac{3 \times 0.04 \times 2^{2 \times 8}}{4} \\ &= 1966.08 \approx 33 \text{ dB} \end{aligned}$$

Ex.6.1.10 A 7 bit PCM system employing uniform quantization has an overall signaling rate of 56 k bits per second. Calculate the signal to quantization noise ratio that would result when its input is a sine wave with peak to peak amplitude equal to 5. Calculate the dynamic range for the sine wave inputs in order that the signal to quantization noise ratio may be less than 30 dBs. What is the theoretical maximum frequency that this system can handle ?
[May-96, 8-Marks]

Sol. : The number of bits in the PCM system are

$$v = 7 \text{ bits}$$

Assume that 5 V peak to peak voltage utilizes complete range of quantizer. Then from equation 6.1.34 we can find the signal to quantization noise ratio as,

$$\left(\frac{S}{N}\right)_{dB} = 1.8 + 6v \text{ dB}$$

$$= 1.8 + 6 \times 7$$

$$= 43.8 \text{ dB}$$

By equation 6.1.5 signaling rate is given as,

$$r = v f_s$$

Putting $r = 56 \times 10^3$ bits/second and $v = 7$ bits in above equation we get,

$$56 \times 10^3 = 7 \cdot f_s$$

$$\therefore \text{Sampling frequency, } f_s = 8 \times 10^3 \text{ Hz}$$

$$\text{By sampling theorem, } f_s \geq 2W$$

$\therefore$ Maximum frequency that can be handled is given as,

$$\begin{aligned} W &\leq \frac{f_s}{2} \\ &\leq \frac{8000}{2} \end{aligned}$$

$$\therefore W \leq 4000 \text{ Hz}$$

(Ans)

Ex..6.1.11 The bandwidth of TV video plus audio signal is 4.5 MHz. If the signal is converted to PCM bit stream with 1024 quantization levels, determine the number of bits/sec generated by the PCM system. Assume that the signal is sampled at the rate of 20% above nyquist rate. If above linear PCM system is converted to companded PCM, will the output bit rate change? Justify.

[Dec-99 8-Marks, Dec-2001 8-Marks]

Sol.: The given data is,

$$W = 4.5 \text{ MHz}$$

$$q = 1024 \text{ levels}$$

The Nyquist rate is,

$$\text{Nyquist rate} = 2W = 2 \times 4.5 = 9 \text{ MHz}$$

The sampling rate is 20% above the nyquist rate. i.e.

$$\text{Sampling rate, } f_s = 1.2 \times 9 = 10.8 \text{ MHz}$$

We know that quantization levels q and number of bits v are related as,

$$q = 2^v$$

$$\therefore 1024 = 2^v$$

$$\therefore v = 10 \text{ bits}$$

The number of bits/sec generated by PCM system is called bit rate or signaling rate. i.e.,

$$\begin{aligned} \text{Signaling rate, } r &= v f_s \\ &= 10 \times 10.8 \times 10^6 \text{ bits/sec} \\ &= 108 \times 10^6 \text{ bits / sec} \end{aligned}$$

The output bit rate doesnot change if linear PCM is converted into companded PCM. Companded PCM is used to improve the signal to noise ratio.