

The inverse Laplace transforms
by using partial fraction

طريقة التجزئة بالحدود

(Ex) $\mathcal{L}^{-1} \frac{4s - 5}{s^2 - s - 2}$

Sol.

$\mathcal{L}^{-1} \frac{4s - 5}{(s - 2)(s + 1)}$

$\left[\frac{4s - 5}{(s - 2)(s + 1)} = \frac{A}{s - 2} + \frac{B}{s + 1} \right] \times \frac{(s - 2)(s + 1)}{(s - 2)(s + 1)}$

$4s - 5 = A(s + 1) + B(s - 2)$

$4s - 5 = As + A + Bs - 2B$

$4 = A + B \quad \dots \text{--- (1)}$

$-5 = -A + 2B \quad \dots \text{--- (2)}$

$9 = 3B \Rightarrow \boxed{B = 3}$

$$4 = A + B$$

$$4 = A + 3$$

$$\Rightarrow \boxed{A=1}$$

نوع B في ①

$$\mathcal{L}^{-1} \frac{4s-5}{(s-2)(s+1)} = \mathcal{L}^{-1} \frac{1}{s-2} + \mathcal{L}^{-1} \frac{3}{s+1}$$

$$= e^{2t} + 3 \mathcal{L}^{-1} \frac{1}{s-(-1)}$$

$$= e^{2t} + 3e^{-t}$$

Ex) $\int \frac{5s^2 + 8s - 1}{(s+3)(s^2+1)} ds$

Sol.

$$\frac{5s^2 + 8s - 1}{(s+3)(s^2+1)} = \frac{A}{s+3} + \frac{Bs+C}{s^2+1}$$

فرض

$$\frac{As^0}{s+3}, \quad \frac{Bs^1 + Cs^0}{s^2+1}, \quad \frac{Bs^2 + Cs^1 + Ds^0}{s^3+1}$$

$$\frac{5s^2 + 8s - 1}{(s+3)(s^2+1)} = \left[\frac{A}{s+3} + \frac{Bs+C}{s^2+1} \right] * (s+3)(s^2+1)$$

$$5s^2 + 8s - 1 = A(s^2+1) + (Bs+C)(s+3)$$

$$5s^2 + 8s - 1 = As^2 + A + Bs^2 + 3Bs + Cs + 3C$$

$$5 = A + B \quad \dots \textcircled{1}$$

$$8 = 3B + C \quad \dots \textcircled{2}$$

$$-1 = A + 3C \quad \dots \textcircled{3}$$

نأخذ معادلة ③, ①

$$-1 = A + 3C$$

$$+5 = A + B$$

$$-6 = -B + 3C \quad \text{--- (4)}$$

$$[8 = 3B - C] \times 3$$

نأخذ معادلة ②

نحتاج $3C$

$$-6 = -B + 3C$$

$$+24 = +9B + 3C$$

$$-30 = -10B \Rightarrow B = 3$$

نعوض قيمة B في ①

$$5 = A + B$$

$$B = A + 3$$

$$A = 2$$

نعوض قيمة A في ③

$$-1 = A + 3C$$

$$-1 = 2 + 3C$$

$$C = -1$$

$$\int \frac{5s^2 + 8s - 1}{(s+3)(s^2+1)} = \int \frac{2}{s+3} + \int \frac{3s-1}{s^2+1}$$

$$= 2 \int \frac{1}{s+3} + \int \frac{3s}{s^2+1} - \int \frac{1}{s^2+1}$$

$$= 2e^{-3t} + 3\cos t - \sin t$$