

Discrete Time
Signal

د. محمد لوری

2023/10/15

Discrete time Signal

DTS
①

The discrete Time Signal Can be represented in four ways:

- ① Functional Representation.
- ② Graphical Representation.
- ③ Tabular Representation.
- ④ Sequence Representation.

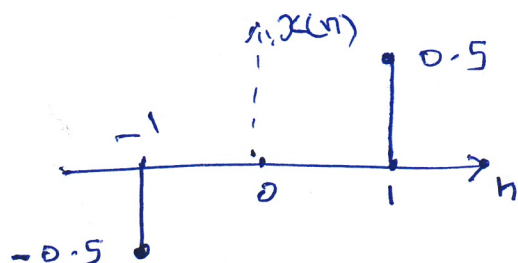
① Functional Representation

Ex $x(n) = 0.5n \quad -1 \leq n \leq 1$ & 0 elsewhere
 $x(-1) = -0.5 \quad x(0) = 0 \quad x(1) = 0.5$

② Graphical Representation

Ex $x(n) = 0.5n \quad -1 \leq n \leq 1$

$x(-1) = -0.5 \quad x(0) = 0 \quad x(1) = 0.5$



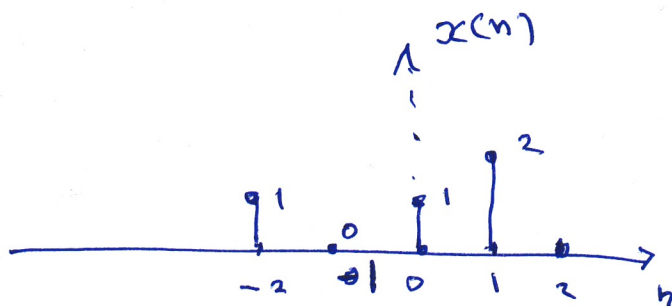
③ Tabular Representation

ex 1

n	-2	-1	0	1
$x(n)$	1	0	1	2

Find all value of $x(n)$
and Draw $x(n)$

$$x(-2)=1 \quad x(-1)=0 \quad x(0)=1 \quad x(1)=2$$



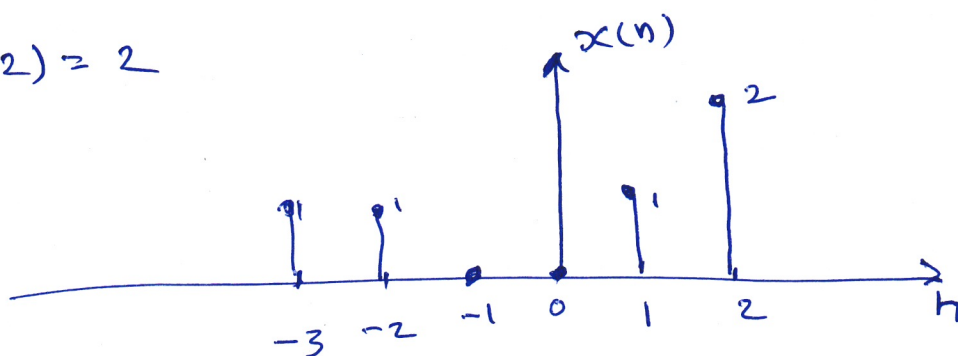
④ Sequence representation

ex

$$x(n) = \{ \overset{-3}{1}, \overset{-2}{1}, \overset{-1}{0}, \overset{0}{0}, \overset{1}{1}, \overset{2}{2} \}$$

$$x(-3)=1 \quad x(-2)=1 \quad x(-1)=0 \quad x(0)=0 \quad x(1)=1$$

$$x(2)=2$$



Basic Signal operation

DTS
③

① Basic Signal operation on Amplitude

- ① amplitude Scaling of discrete Time Signal
- ② addition of discrete Time Signal
- ③ subtraction of discrete Time Signal
- ④ multiplication of discrete Time Signal

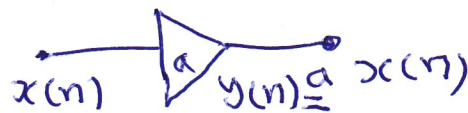
② Basic Signal operation on Time

- ① Time Scaling of signals
- ② Time shifting of discrete Time Signal
- ③ Time reversal of discrete Time Signal
- ④ combinations of operation.

operation on amplitude

DTS
4

a amplitude Scaling



ex for $x(n) = \{1, -1, 2\}$ find $y(n) = 2x(n)$

Sol.

$$a = 2$$

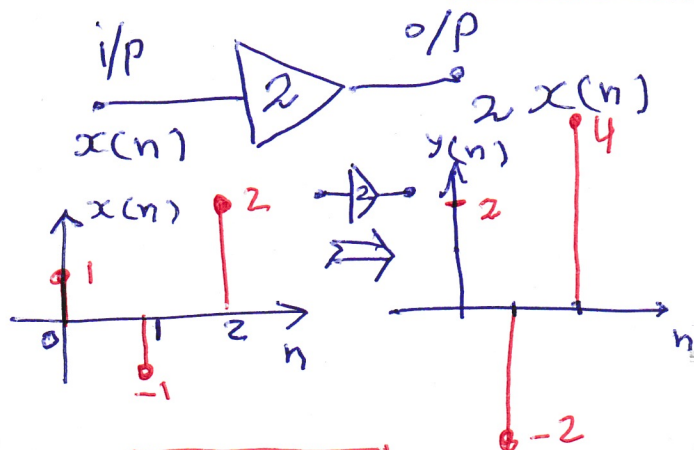
$$y(n) = 2x(n)$$

(Scaling by)
2

$$x(n) = \{1, -1, 2\}$$

$$y(n) = 2x(n)$$

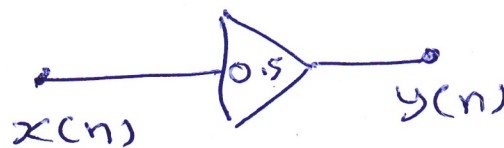
$$= \{2, -2, 4\}$$



ex if $x(n) = 0.5n$ for $-2 \leq n \leq 2$
and zero elsewhere

$$x(n) = \begin{cases} 0.5n & -2 \leq n \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

find $y(n) = 0.5x(n)$

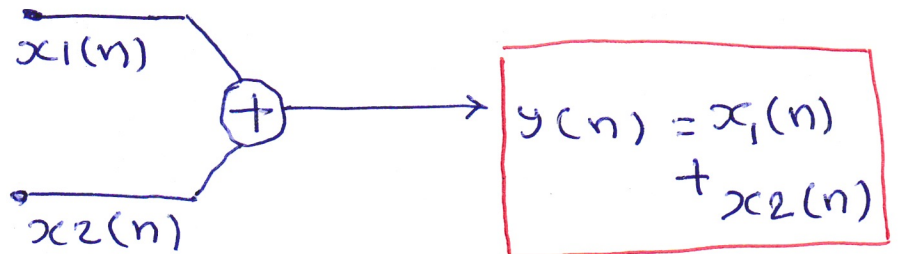


Sol.

$$x(n) = \{-1, -0.5, 0, 0.5, 1\}$$

$$y(n) = \{-0.5, -0.25, 0, 0.25, 0.5\}$$

b) addition of Discrete Time Signals

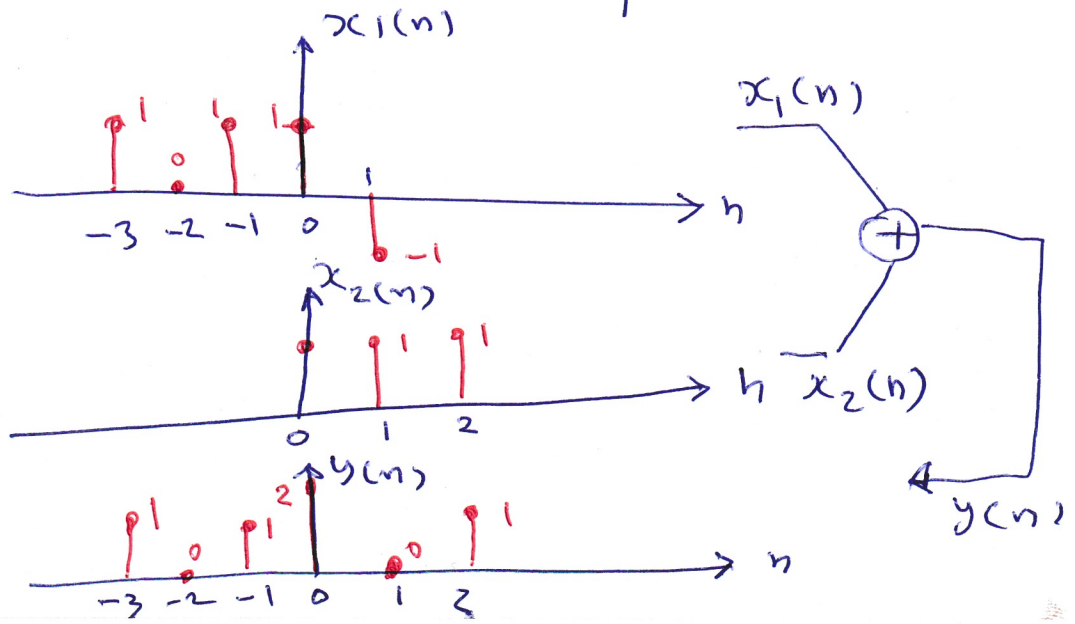


ex] if $x_1(n) = \{1, 0, 1, 1, -1\}$
 and $x_2(n) = \{1, 1, 1\}$

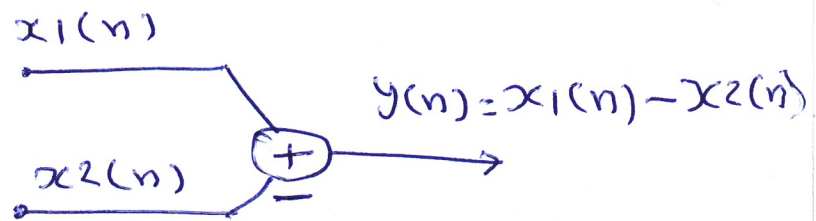
Find $y(n) = x_1(n) + x_2(n)$

$$y(n) = \begin{matrix} & -3 & -2 & -1 & 0 & 1 \\ \{ & 1, & 0, & 1, & 1, & -1 \} \\ + & & & & \uparrow & \\ \{ & 0, & 0, & 0, & 1, & 1, & 1 \} \\ & & & & \uparrow & \end{matrix}$$

$$y(n) = \{1, 0, 1, 2, 0, 1\}$$



[C] Subtraction of Discrete Time Signals



ex if

$$x_1(n) = \begin{cases} u(n) & 0 \leq n \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$x_2(n) = \begin{cases} n^2 & 0 \leq n \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

Find $y(n) = x_1(n) - x_2(n)$

Sol.

$$x_1(n) = \{ \overset{0}{1}, \overset{1}{1}, \overset{2}{1}, \overset{3}{1} \}$$

$$x_2(n) = \{ \overset{0}{0}, \overset{1}{1}, \overset{2}{4}, \overset{3}{9} \}$$

$y(n)$

$$y(n) = \{ \underset{\uparrow}{1}, 0, -3, -8 \}$$

1-d multiplication of Discrete Time Signals

DT S
Z

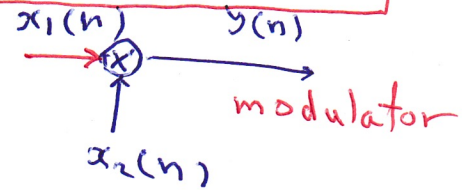
ex 1/ if $x_1(n) = [1 \ 1 \ 2]$

and $x_2(n) = [2 \ 0 \ 2]$

Find

$$y(n) = x_1(n) \cdot x_2(n)$$

Signal multiplier
or modulator



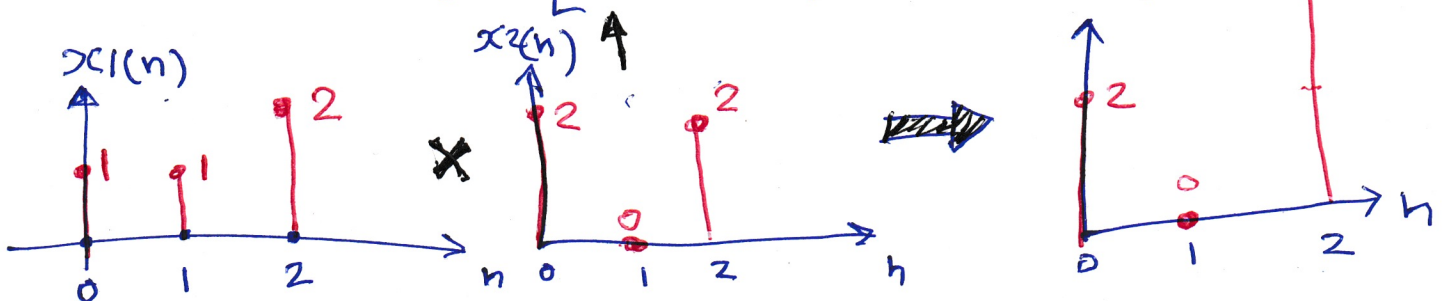
$$y(n) = x_1(n) \cdot x_2(n)$$

Sol. /

$$y(n) = x_1(n) \cdot x_2(n)$$

$$y(n) = [1 \ 1 \ 2] \cdot [2 \ 0 \ 2]$$

$$y(n) = [2 \ 0 \ 4]$$



ex 2/ if $x_1(n) = [1 \ 1 \ 2]$

and $x_2(n) = [2 \ 0 \ 2]$

Find $y(n) = x_1(n) \cdot x_2(n)$

Sol.

$$y(n) = x_1(n) \cdot x_2(n) = [1 \ 1 \ 2] \cdot [2 \ 0 \ 2]$$

$$= [0 \ 0 \ 4 \ 0 \ 0]$$

$n=-2 \quad n=-1 \quad n=0 \quad n=1 \quad n=2$

H.w graph representation

Time Scaling of Discrete Time Signals

a) compressed signals

The discrete Time Sequence $x(n)$ is

Compressed in Time by multiplying The index n by an integer K

$$x[Kn]$$

$K-1 = \text{digit, place, ...}$

ex let $x(n) = \{3, 3, 2, 1, 3, 1, 1, 2, 1\}$

$\uparrow$

find $y(n) = x(2n)$

Sol. $K = 2$ compressed ratio $= K-1 = 2-1 = 1$

$$x(n) = \left\{ \overset{-4}{\checkmark} 3, \overset{-2}{\checkmark} \boxed{3}, 2, \overset{0}{\checkmark} \boxed{1}, 3, \overset{2}{\checkmark} \boxed{1}, 1, \overset{2}{\checkmark} \boxed{2}, 1 \right\}$$

$\uparrow$

$$x(2n) = \left\{ \overset{-5}{\checkmark} 3, \overset{-3}{\checkmark} 1, \overset{-1}{\checkmark} 1, \overset{1}{\checkmark} 2 \right\}$$

$\uparrow$

- $n = 0 \quad y(n) = x(2n) \Rightarrow y(0) = x(0) = 1$
- $n = 1 \quad y(n) = x(2n) \Rightarrow y(1) = x(2) = 2$
- $n = 2 \quad y(n) = x(2n) \Rightarrow y(2) = x(4) = 0$
- $n = -1 \quad y(n) = x(2n) \Rightarrow y(-1) = x(-2) = 1$

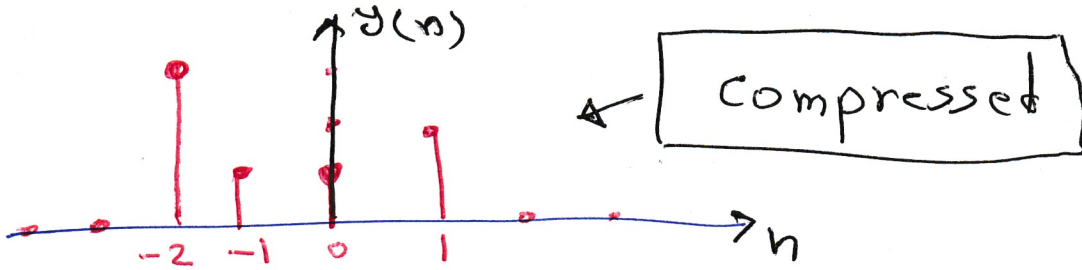
$$n = -2$$

$$y(-2) = x(-4) = 3$$

$$y(-3) = x(-6) = 0$$

$$n = -3$$

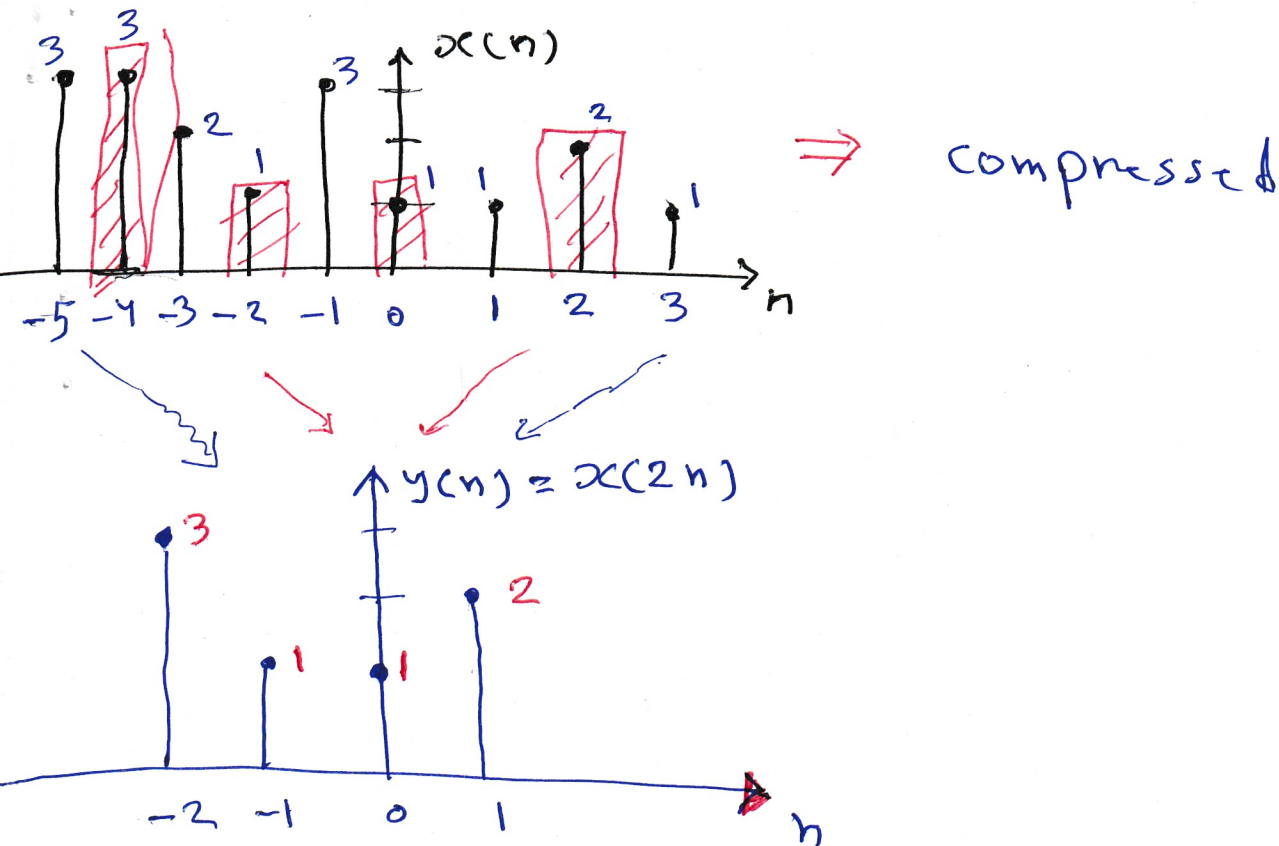
$$\therefore y[n] = \begin{bmatrix} 0 & 3 & 1 & 1 & 2 & 0 \\ -3 & -2 & -1 & 0 & 1 & 2 \end{bmatrix}$$



called downSampling

or

decimation



b expanded signals

DTS
10

The discrete Time Sequence $x[n]$ is
expanded in time by dividing the index n
by an integer m . To produce The Time Scaled
Sequence $x[n/m]$

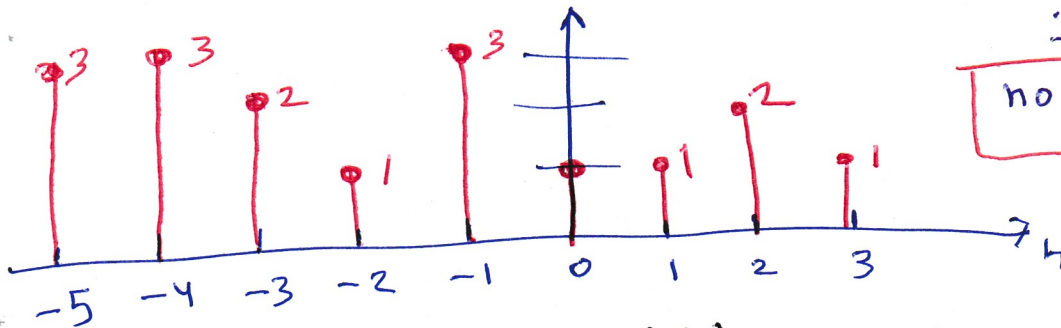
no. of Zero added = $m-1$

ex

let

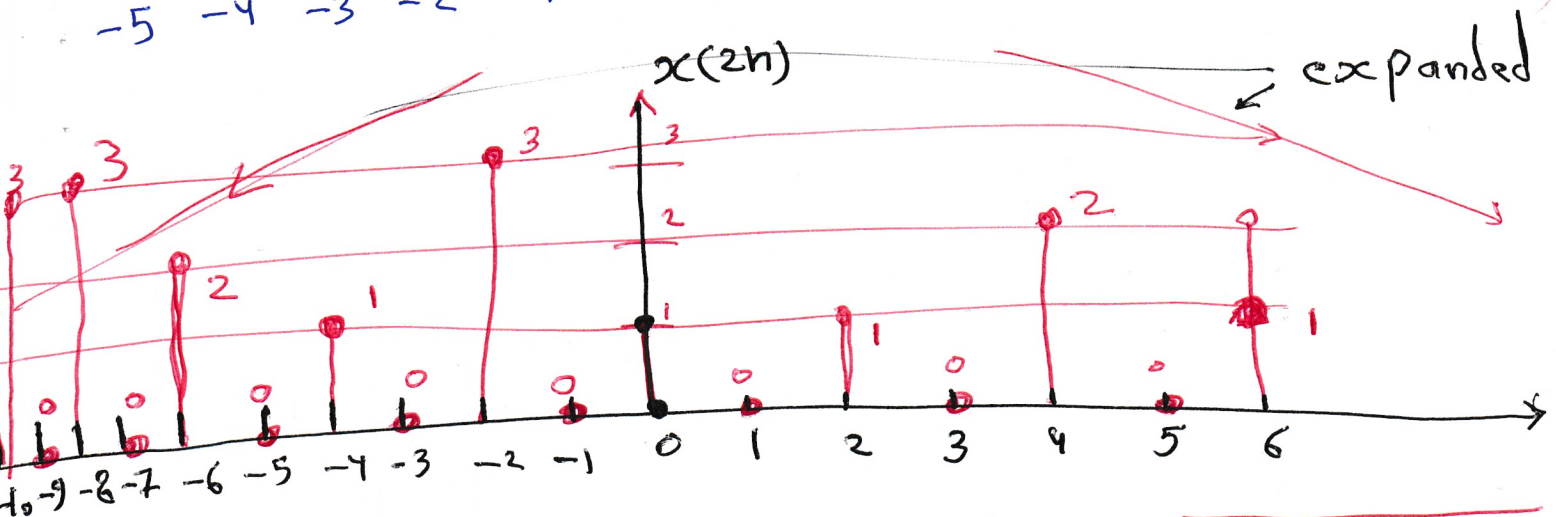
find $x(\frac{2n}{2})$

$x(n)$



$\therefore m=2$

no. of zeros added = $\frac{2-1}{2-1} = 1$



Called upsampling or interpolation

$$n = 0$$

$$y(n) = x(n/2) = x(0) = 1$$

$$n = -1$$

$$y(-1) = x(-1/2) = 0$$

$$n = -2$$

$$y(-2) = x(-1) = 3$$

$$n = -3$$

$$y(-3) = x(-1.5) = 0$$

$$n = -4$$

$$y(-4) = x(-2) = 1$$

$$n = -5$$

$$y(-5) = x(-2.5) = 0$$

$$n = -6$$

$$y(-6) = x(-3) = 2$$

$$n = -7$$

$$y(-7) = x(-3.5) = 0$$

$$n = -8$$

$$y(-8) = x(-4) = 3$$

$$n = -9$$

$$y(-9) = x(-4.5) = 0$$

$$n = -10$$

$$y(-10) = x(-5) = 3$$

DTS
⑪

$$n = 1$$

$$y(1) = x(1/2) = 0$$

$$n = 2$$

$$y(2) = x(1) = 1$$

$$n = 3$$

$$y(3) = x(1.5) = 0$$

$$n = 4$$

$$y(4) = x(2) = 2$$

$$n = 5$$

$$y(5) = x(2.5) = 0$$

$$n = 6$$

$$y(6) = x(3) = 1$$

$$y(n) = \begin{matrix} n & -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ & 3 & 0 & 3 & 0 & 2 & 0 & 1 & 0 & 3 & 0 & 1 & 0 & 1 & 0 & 2 & 0 & 1 \end{matrix}$$

↑

Time Reversal of Discrete Time Signals

discrete Time reverse

replace $[n]$ with $[-n]$

reversed discrete Time signal of $x(n)$

is $x[-n]$

ex | if $x(n) = [0 \ 1 \ 2 \ 3]$

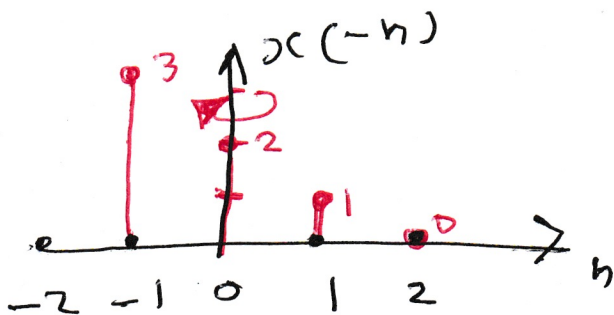
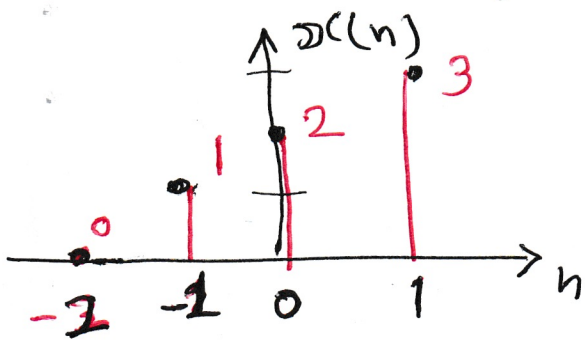
Find

$$y(n) = x(-n)$$

Sol.

if $x(n) = [0 \ 1 \ 2 \ 3]$

$$\therefore y(n) = [3 \ 2 \ 1 \ 0] = x(-n)$$

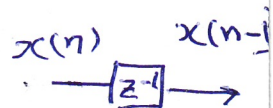


Time shift of Discrete Time signals

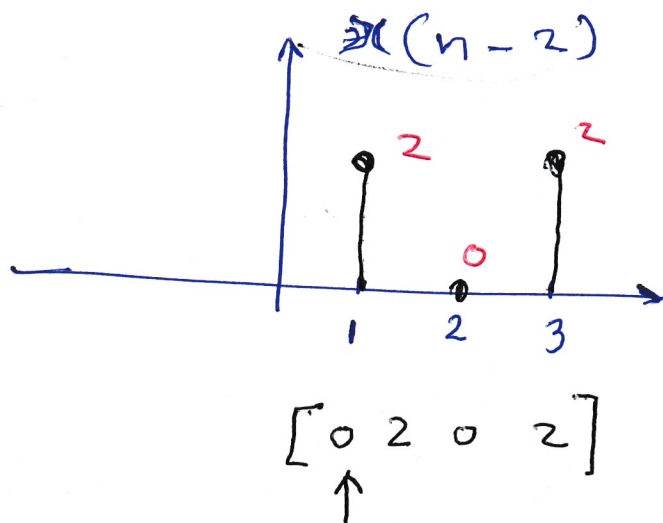
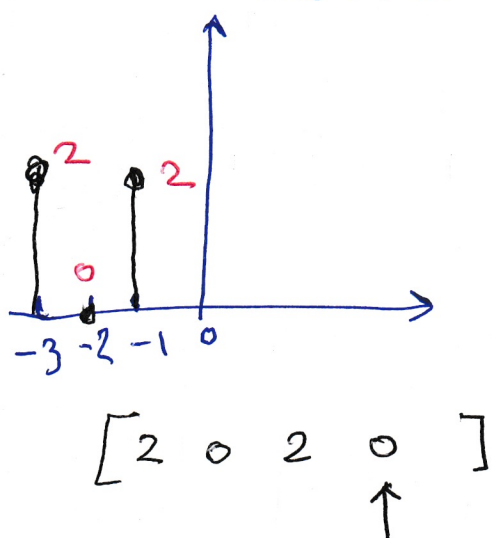
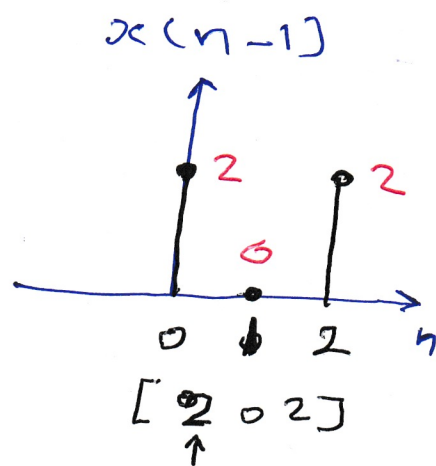
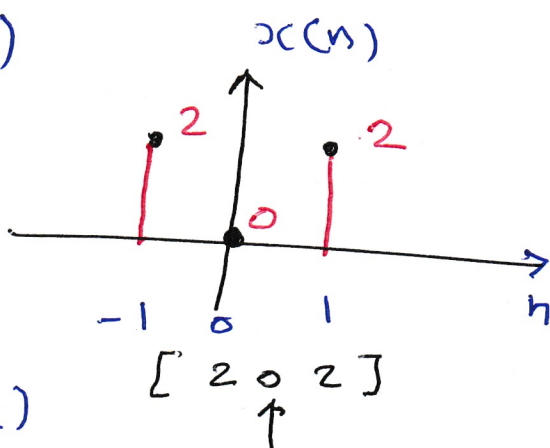
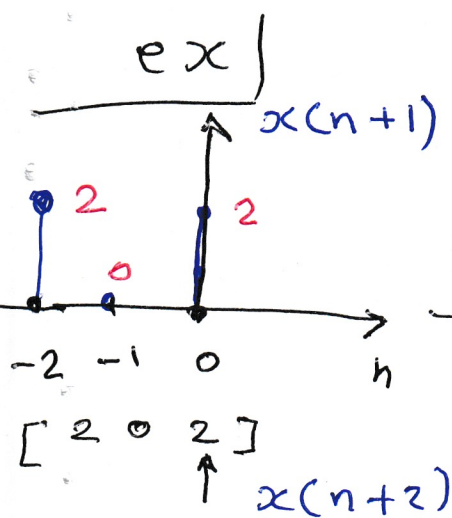
DTS
13

For a discrete Time Signal $x[n]$ and an integer $n_1 > 0$

- $x[n - n_1]$ is a delayed signal



- $x[n + n_1]$ is an advanced signal



Combinations of Operations

Time scaling, shifting and reversal can all be combined.

ex/ $y(n) = x(2n/3)$

$$x(n) = [2, 1, 2, 4, 5, 0, 6]$$

Sol/

$$x(2n) = [2, 2, 5, 6]$$

compression by 2
 $k=2$

$$m=3$$

$$\text{Zeros no.} = m-1 = 2$$

$$x(2n/3) = [2, 0, 0, 2, 0, 0, 5, 0, 0, 6]$$

expansion
by 3

$$m=3$$

or

$$m=3 \quad \text{no. of Zeros} = m-1 = 3-1 = 2$$

$$x(n/3) = [2, 0, 0, 1, 0, 0, 2, 0, 0, 4, 0, 0, 5, 0, 0, 0, 0, 6]$$

expansion

$$x[2 \frac{n}{3}] = [2, 0, 0, 2, 0, 0, 5, 0, 0, 6]$$

ex

$$x(n) = [1, 2, 3, 4, 5]$$

Find $y(n) = x(n \cdot 2) = x(-2n)$

Sol.

$$x(n) = [1, 2, 3, 4, 5]$$

reverse

$$x(-n) = [5, 4, 3, 2, 1]$$

compression by 2 $k=2$

$$x(-2n) = [5, 3, 1]$$

$$\therefore y(n) = [5, 3, 1]$$

or

$$x(n) = [1, 2, 3, 4, 5]$$

$$x(2n) = [1, 3, 5]$$

reverse

compression
by 2
 $k=2$

$$x(-2n) = [5, 3, 1]$$

Q. 1

ex/

$$x(n) = \{7, 1, 2, 6, 3\}$$

↑

Find

$$y(n) = x(-n-1)$$

Sol.

$$x(n) = \{7, 1, 2, 6, 3\}$$

$$x(n-1) = \{7, 1, 2, 6, 3\}$$

↑

reverse of Time

$$x(-n-1) = \{3, 6, 2, 1, 7\}$$

↑

or

$$x(n) = \{7, 1, 2, 6, 3\}$$

↑

reverse of time

$$x(-n) = \{3, 6, 2, 1, 7\}$$

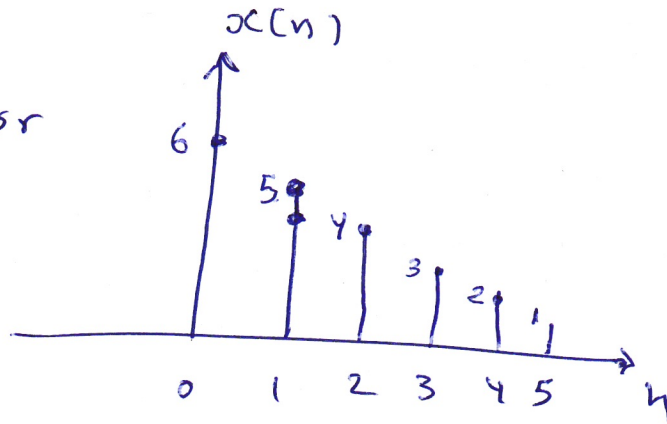
↑

$$x(-n-1) = x(-[n+1]) = \{3, 6, 2, 1, 7\}$$

↑

فصلی
الواجب

ex/ for



Find $y(n) = x(3n+4)$

sol.

$$x(n) = \{ \overset{0}{\underset{\uparrow}{6}}, \overset{1}{5}, \overset{2}{4}, \overset{3}{3}, \overset{4}{2}, \overset{5}{1}, 0 \}$$

$$x(n+4) = \{ 6, \boxed{5}, 4, 3, \boxed{2}, 1, \boxed{0} \}$$

$b=3$ ~~width~~ $\text{index} = 3-1=2$ $\uparrow$

$$y(n) = x(3n+4) = \{ \text{red } 5, \text{red } 2 \}$$

قائمة

$$x(3n+4)$$

for $n = -2$

$$y(-2) = x(-2) = 0$$

$$\{ \overset{-6}{3 \cdot (-2)} + \overset{+4}{4} \} = -2$$

for $n = -1$

$$y(-1) = x(+1) = 5$$

$$\{ -3 + 4 \} = +1$$

for $n = 0$

$$y(0) = x(4) = 2$$

$$\{ 3 \cdot 0 + 4 \} = 4$$

for $n = 1$

$$y(1) = x(7) = 0$$

$$\{ 3 \cdot 1 + 4 \} = 7$$

$$\therefore y(n) = \left[\underset{\uparrow}{5}, 2 \right]$$

excl if $x(n) = \{6, 5, 4, 3, 2, 1\}$

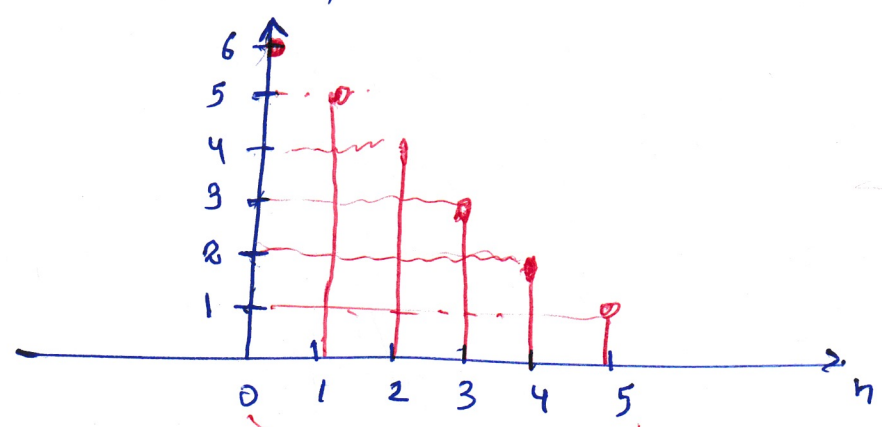
$\uparrow$ $\uparrow$ $\uparrow$
 $n=1$ $n=2$ $n=4$

Find

- ① $y(n) = x(3n+4)$
- ② $y(n) = x\left(\frac{n}{2} + 3\right)$
- ③ $y(n) = x(-n-2)$

$x(-n-2)$
 $= \{1, 2, 3, 4, 5, 6\}$

Sol. ① $x(n) = \{6, 5, 4, 3, 2, 1\}$

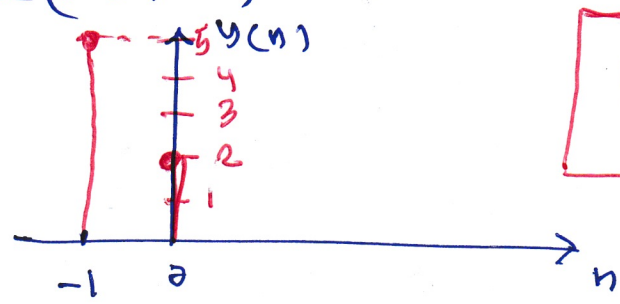


$0 \leq -n-2 \leq 5$
 $2 \leq -n \leq 7$
 $-2 \leq n \leq -7$
 $-2 \geq n \geq -7$

$x(-n-2)$
 $y(0) = -2 \quad 0$

$0 \leq 3n+4 \leq 5$
 $-4 \leq 3n \leq 1$
 $-\frac{4}{3} \leq n \leq \frac{1}{3}$
 $n = -1 \quad n = 0$

$y(0) = x(3 \cdot 0 + 4) = x(4) = 2$
 $y(-1) = x(3 \cdot (-1) + 4) = x(1) = 5$



$y(n) = \{5, 2\}$
 $\uparrow$

or

$$x(n) = \{6, 5, 4, 3, 2, 1\}$$

↑
shift by four [4]

$$x(n+4) = \{6, 5, 4, 3, 2, 1\}$$

compression by 3

$$\text{no. of zeros} = 3 - 1 = 2$$

$$y(n) = x(3n+4) = \{5, 2\}$$

نتائج

Sol of ② $y(n) = x\left(\frac{n}{2} + 3\right)$

$$x(n) = \{6, 5, 4, 3, 2, 1\}$$

↑ shift by ③

$$x(n+3) = \{6, 5, 4, 3, 2, 1\}$$

interpolation

$$\text{no. of zeros} = 2 - 1 = 1$$

interpolation

$$x\left(\frac{n}{2} + 3\right) = \{6, 0, 5, 0, 4, 0, 3, 0, 2, 0, 1\}$$

$$\textcircled{3} \quad y(n) = x(-n-2)$$

DTS
20

Sol. $y(n) = x(-(n+2))$

$$x(n) = \{ \underset{\uparrow}{6}, 5, 4, 3, 2, 1 \}$$

$$x(n+2) = \{ \underset{\uparrow}{6}, 5, 4, 3, 2, 1 \}$$

$$y(n) = x(-(n+2)) = \{ 1, 2, 3, \underset{\uparrow}{4}, 5, 6 \}$$

$H(\omega) =$ Find $\textcircled{1} \quad y(n) = u(-n+4)$

$$\textcircled{2} \quad y(n) = x\left(\frac{n}{2} - 2\right)$$

$$\text{if } x(n) = \{ 1, \underset{\uparrow}{2}, 3 \}$$