Classification of DSP systems

مكتبة المأمون
٠٧٨٢٧٨٦١٨٦

د. محمد فوزي محمد

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٠٧٨٢٧٨٦١٨٦

Classification of System

مسائل محلولة

Linear and No Linear [L & NL]

Static and Dynamic [S & D]

Time Variant and Time nonVariant

Causal and non Causal

Stable and unstable [BIBO]

د. محمد الامين

Linear and non linear example

ex1/ ← Time scaling

$y(n) = x(n^2)$

$x_1(n^2)$, $x_2(n^2)$

$y_1(n) = x_1(n^2)$ $\quad y_2 = x_2(n^2)$

$y_3(n) = y_1(n) + y_2(n)$

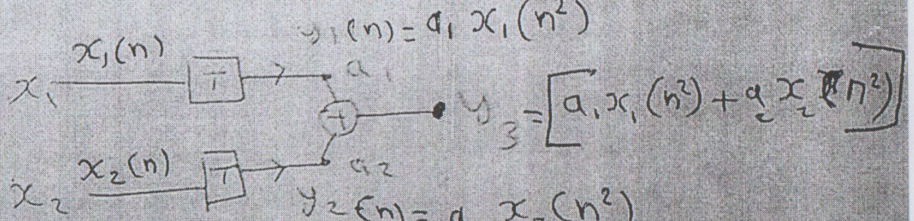
$$y_3(n) = a_1 x_1(n^2) + a_2 x_2(n^2)$$

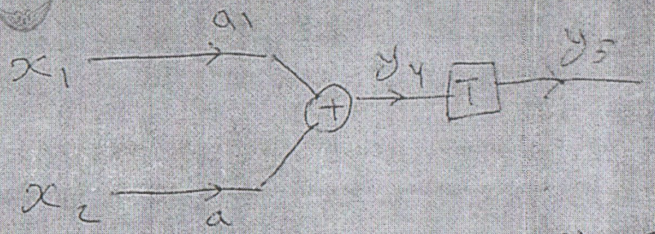
$y_4(n) = a_1 x_1(n) + a_2 x_2(n)$

$$y_5(n) = a_1 x_1(n^2) + a_2 x_2(n^2)$$

$$\therefore y_3(n) = y_5(n)$$

Linear system





الاستاذ الاول

System linearty is independent of Time scaling

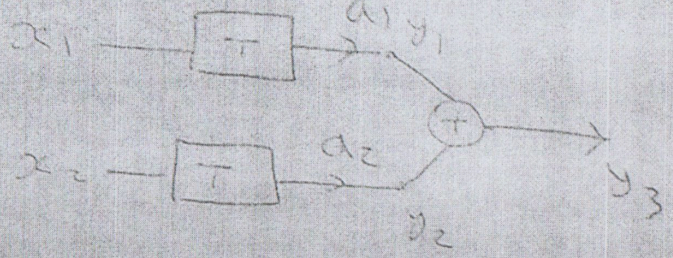
Ex 1 (dependent)

$y(n) = \sum_{\text{coeff}} x(n)$

$y_1(n) = a_1 n x_1(n)$

$y_2(n) = a_2 n x_2(n)$

$y_3(n) = a_1 n x_1(n) + a_2 n x_2(n)$



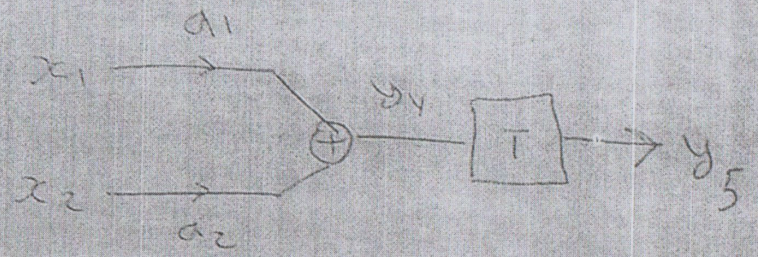
$y_4(n) = a_1 x_1(n) + a_2 x_2(n)$

$y_5(n) = T[a_1 x_1(n) + a_2 x_2(n)]$

$= a_1 n x_1(n) + a_2 n x_2(n)$

$\therefore y_5(n) = y_3(n)$

Linear system



الاستنتاج الآتي

The System linearity is independent of the Coeff used T.V or I.T.V

T.V Time variant

I.T.V independant Time variant

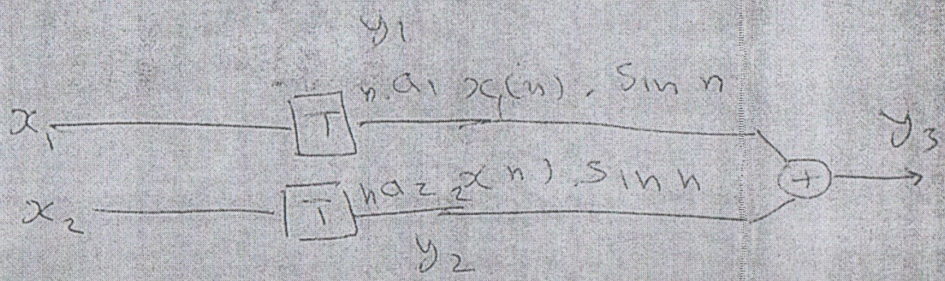
ex3 Time dependent

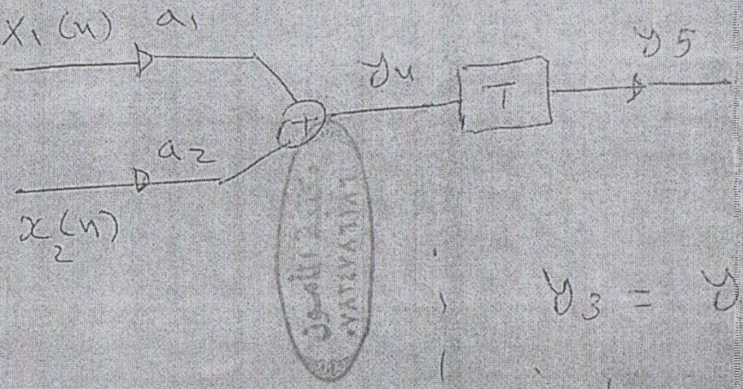
$$y(n) = \underbrace{\sin n}_{\text{coeff}} \cdot x(n)$$

$$y_1(n) = a_1 \sin n \, x_1(n)$$

$$y_2(n) = a_2 \sin n \, x_2(n)$$

$$y_3(n) = a_1 \sin n \, x_1(n) + a_2 \sin n \, x_2(n)$$





$$y_4 = [a_1 x_1(n) + a_2 x_2(n)]$$

$$y_5 = \sin n \cdot [a_1 x_1(n) + a_2 x_2(n)]$$

$$y_5 = a_1 \sin n \, x_1(n) + a_2 \sin(n) \, x_2(n)$$

$y_3 = y_5$

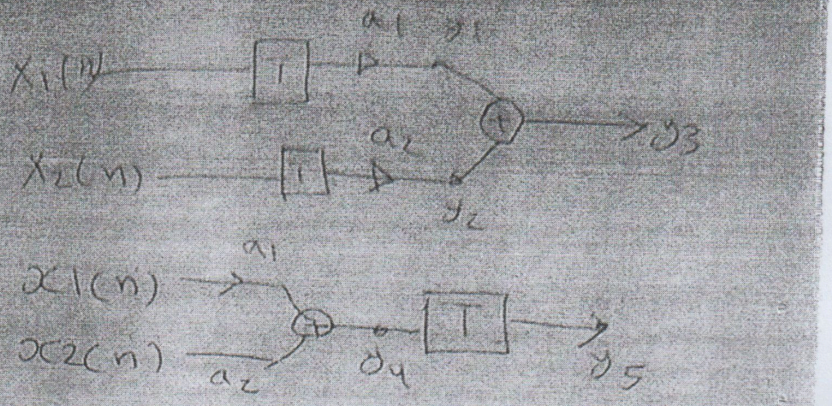
$\therefore$ Linear system

الاستنتاج الأساسي

The system linearty is independent of the coeff. used

Ex/ Find The Linear and Nonlinear of the following system

$y(n) = e^{3} x(n)$



$y_1(n) = a_1 e^{3} x_1(n)$

$y_2(n) = a_2 e^{3} x_2(n)$

$y_3(n) = a_1 e^{3} x_1(n) + a_2 e^{3} x_2(n)$

$y_4(n) = [a_1 x_1(n) + a_2 x_2(n)]$

$y_5(n) = e^{3} [a_1 x_1(n) + a_2 x_2(n)] = a_1 e^{3} x_1(n) + a_2 e^{3} x_2(n)$

$\therefore y_3 = y_5$ ∴ the system is Linear

The system linearity is independent of The Coeffie used

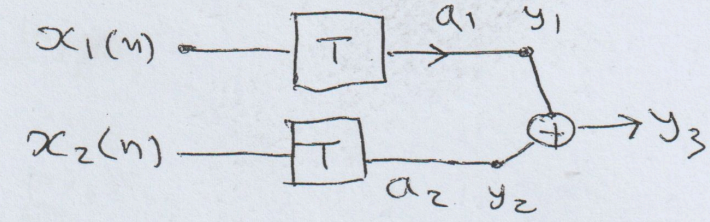
يتبع الباقي ←

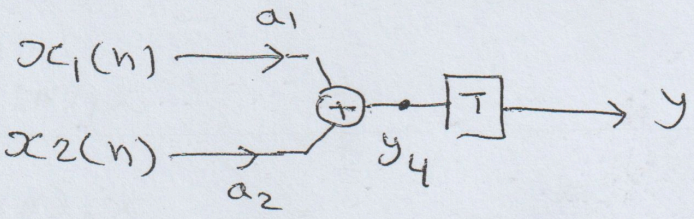
LSNL

ex 5

$$y(n) = \underbrace{2n + x(n)}_{\text{I/P}}$$

Time dependent





$$y_1(n) = [2n + x_1(n)] \cdot a_1$$

$$y_2(n) = [2n + x_2(n)] \cdot a_2$$

$$y_3(n) = 2n a_1 + a_1 x_1(n) + 2n a_2 + a_2 x_2(n)$$

$$y_4 = \underbrace{[a_1 x_1(n) + a_2 x_2(n)]}_{\text{I/P}}$$

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$$y_5 = 2n + [a_1 x_1(n) + a_2 x_2(n)]$$

$$y_5 = 2n + a_1 x_1(n) + a_2 x_2(n)$$

$y_3 \neq y_5$ non-Linear System

نتيجة

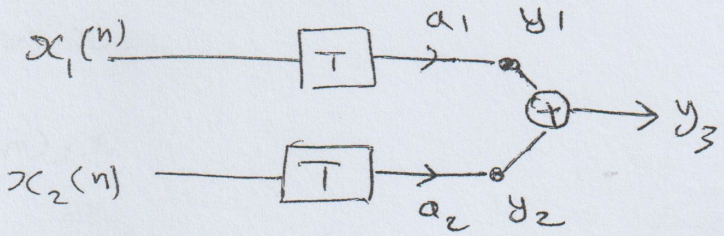
if any added / subtracted Term other Than i/p and o/p is available in The system relationship Then The system will be nonlinear

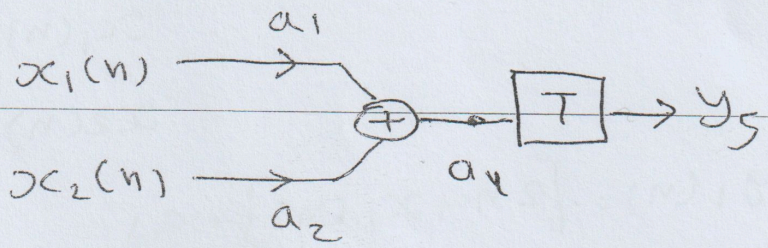
الاستنتاج

L & NL

ex 6

$$y(n) = \underset{\text{Time independent}}{2} + x(n) \leftarrow \text{I/P}$$





$$y_1(n) = [2 + x_1(n)] \cdot a_1$$

$$y_2(n) = [2 + x_2(n)] \cdot a_2$$

$$y_3(n) = 2a_1 + a_1 x_1(n) + 2a_2 + a_2 x_2(n)$$

$$y_4 = [x_1(n) \cdot a_1 + x_2(n) \cdot a_2] \leftarrow \text{I/P}$$

$$y_5 = 2 + [x_1(n) \cdot a_1 + x_2(n) \cdot a_2]$$

$$y_5 = 2 + x_1(n)\, a_1 + x_2(n)\, a_2$$

$\therefore y_3 \neq y_4$

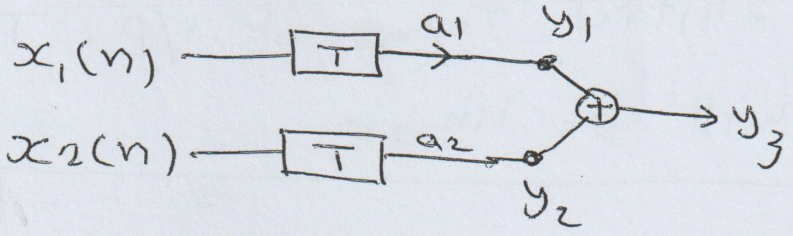
if any added/subtracted term other than i/p and o/p is available in the system relationship then the system will be nonlinear

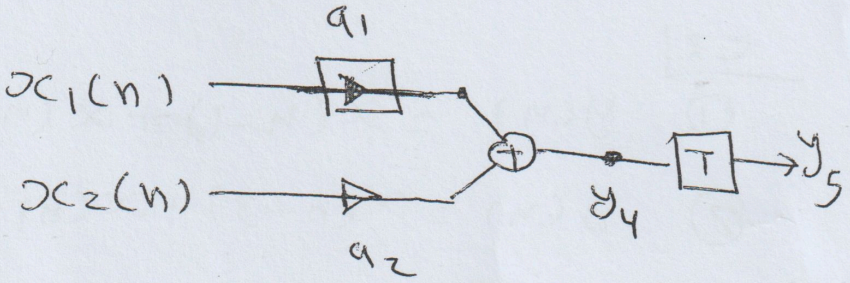
ex 7

(7)

L S NL

$$x(n) = \underbrace{x(n-2)}_{i/p\,1} + \underbrace{x(n+3)}_{i/p\,2}$$





$$y_1(n) = a_1\left[x_1(n-2) + x_1(n+3)\right]$$

$$y_2(n) = a_2\left[x_2(n-2) + x_2(n+3)\right]$$

$$y_3 = a_1 x_1(n-2) + a_1 x_1(n+3) + a_2 x_2(n-2) + a_2 x_2(n+3)$$

$$y_4(n) = \left[a_1 x_1(n) + a_2 x_2(n)\right]$$

$$y_5 = \underbrace{\left[a_1 x_1(n-2) + a_2 x_2(n-2)\right]}_{i/p\,1} + \underbrace{\left[a_1 x_1(n+3) + a_2 x_2(n+3)\right]}_{i/p\,2}$$

$$y_5 = a_1 x_1(n-2) + a_1 x_1(n+3) + a_2 x_2(n-2) + a_2 x_2(n+3)$$

$y_3 = y_5$ ∴ Linear system

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if The o/p is summation of Time shifted terms of i/p. Then The system will be linear

ex

① $y(n) = x(n-1) + x(n+1) + x(n+2)$

② $y(n) = x(n-3) + x(n) + x(n+5)$

③ $y(n) = x(n) + x(n-10)$

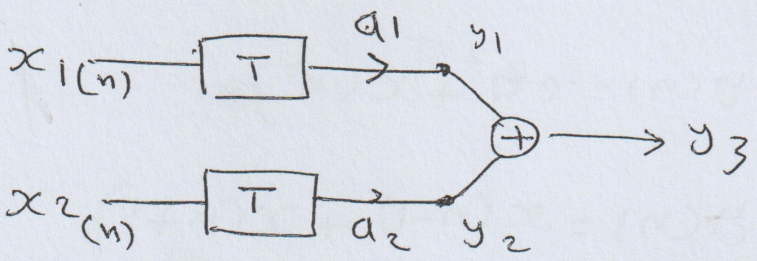
L & NL

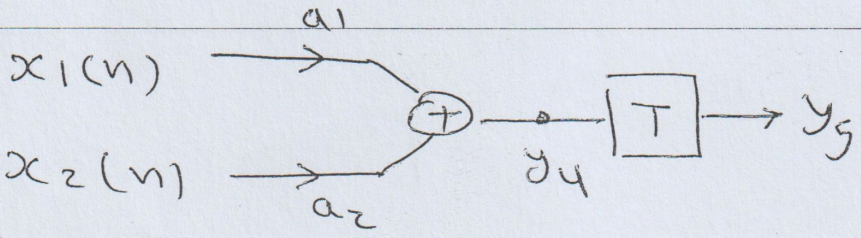
ex 8 | (9) ← SNL

$$y(n) = \int_{-\infty}^{n} x(n)\, dn$$

integral and differentiation operators are linear

ارسم تتابع الخرج





$$y_1(n) = a_1 \int_{-\infty}^{n} x_1(n)\, dn$$

$$y_2(n) = a_2 \int_{-\infty}^{n} x_2(n)\, dn$$

$$y_3(n) = a_1 \int_{-\infty}^{n} x_1(n)\, dn + a_2 \int_{-\infty}^{n} x_2(n)\, dn$$

$$y_3(n) = \int_{-\infty}^{n} a_1 x_1(n)\, dn + \int_{-\infty}^{n} a_2 x_2(n)\, dn$$

$$y_3(n) = \int_{-\infty}^{n} [a_1 x_1(n) + a_2 x_2(n)]\, dn$$

$$y_4(n) = [a_1 x_1(n) + a_2 x_2(n)]$$

$$y_5(n) = \int_{-\infty}^{n} [a_1 x_1(n) + a_2 x_2(n)]\, dn$$

H.W.

(10)

L & NL

① $y(n) = [\log(n) + 3n^2] \cdot x(n)$

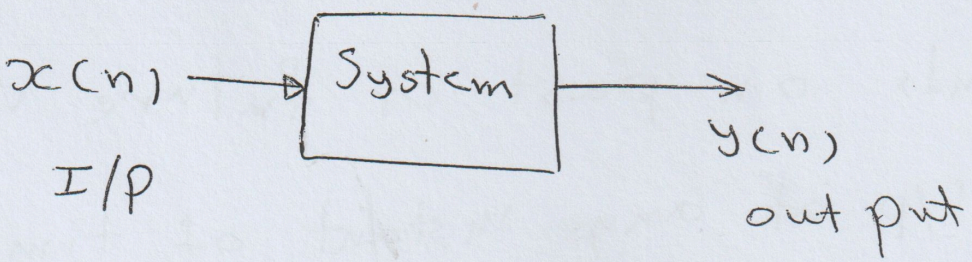
② $y(n) = n.u(n) \cdot x(n)$

③ $y(n) = 2n^3 + x(n^3)$

④ $y(n) = x(n-1) + x(n+1)$

⑤ $y(n) = \frac{\partial}{\partial n} x(n)$

Static and Dynamic System



$y(n) = x(n-1)$ past i/p

$y(n) = x(n)$ present i/p

$y(n) = x(n+1)$ future i/p

Static System [memory less system]

o/p of system depends on present values of input i/p

ex

① $y(n) = 2x(n)$

② $y(n) = 5x(n)$

③ $y(0) = 2x(0)$

Dynamic System: o/p of system depends on past or future values of i/p at any instant of time

[system with memory]

ex

① $y(n) = x(n) + x(n-1)$

present i/p — past i/p

② $y(n) = x(n+1) + x(n)$

future i/p — present i/p

③ $y(n) = x(2n)$ ← Time scalling

④ $y(n) = x(-n)$ ← Time reverse

⑤ $y(n) = \int_{-A}^{n} x(n)\, dn$

التكامل يبدأ من اقل قيمة في A- الى n

⑥ $y(n) = x[\sin(n)]$

⑦ $y(n) = x(n-1)$

⑧ $y(n) = x(n+1)$ ← Time shifting

⑨ $y(n) = x(n^2)$

⑩ $y(n) = \frac{d}{dn} x(n)$

تفاضل اي قيمة

ex| $y(n) = x(2n)$ ← Time Scalling

$n = 0$

$y(0) = x(0)$ present i/p

$n = -1$

$y(-1) = x(-2)$ past i/p

$n = 1$

$y(1) = x(2)$ future i/p

∴ the system is Dynmic

depend of past, present and futrae inpuds

The system Dynmic if The system Time scalling

ex

$y(n-2) = x(n-2)$

past o/p past i/p

$n=0 \quad \therefore y(-2) = x(-2)$

$\therefore$ — past o/p past i/p

$\therefore$ The system is static

ex $y(n+1) = x(n+1)$

future o/p future i/p

if $n=0 \quad y(1) = y(1)$

future o/p future o/p

$\therefore$ The system is static

ex $y(n) = x(-n)$ ← Reverse Time

if $n=0 \quad y(0) = x(0)$ present i/p

$n=-1 \quad y(-1) = x(1)$

past o/p future i/p

$n=1 \quad y(1) = x(-1)$

future o/p past i/p

The system is Dynmic

ex

$$y(n) = 2x^2(n)$$

$n = 0 \qquad y(0) = 2x^2(0)$

present o/p present i/p

$$y(1) = 2x^2(1)$$

∴ The system is static

ex

$$y(n) = x(n) + x(n-1)$$

$n = 1$

$$y(1) = x(1) + x(0)$$

future i/p present i/p

∴ The system is Dynamic

ex

$$y(n) = \sin(n)\, x(n)$$

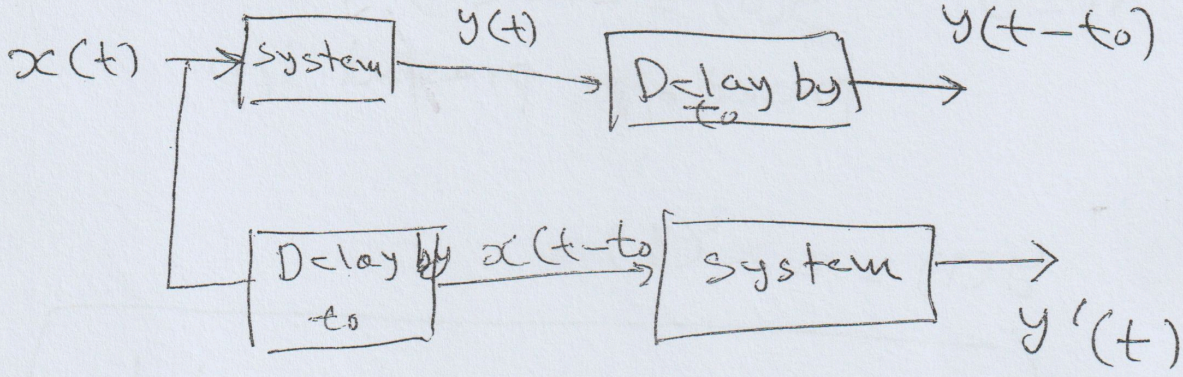
$n = 0$

$$y(0) = 0$$

$n = (1) \qquad y(1) = \sin(1)\, x(n)$

∴ The system is static

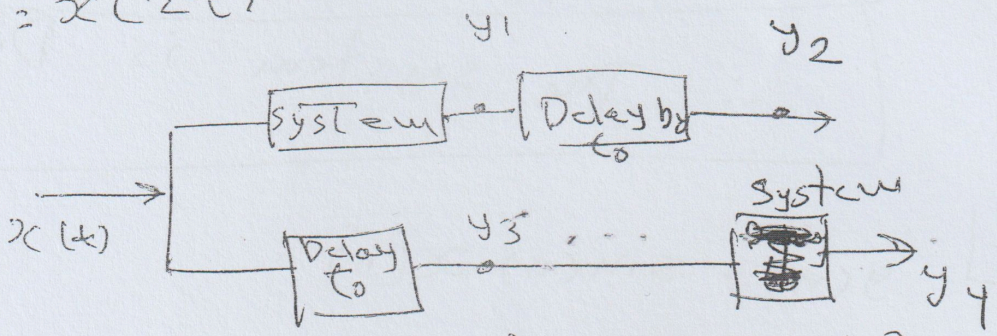
Time - Invariant & Time Variant System



If $y'(t) = y(t-t_0)$ → The system is Time invaraint

If $y'(t) \neq y(t-t_0)$ → The system Time Varaint

ex $y(t) = x(2t)$

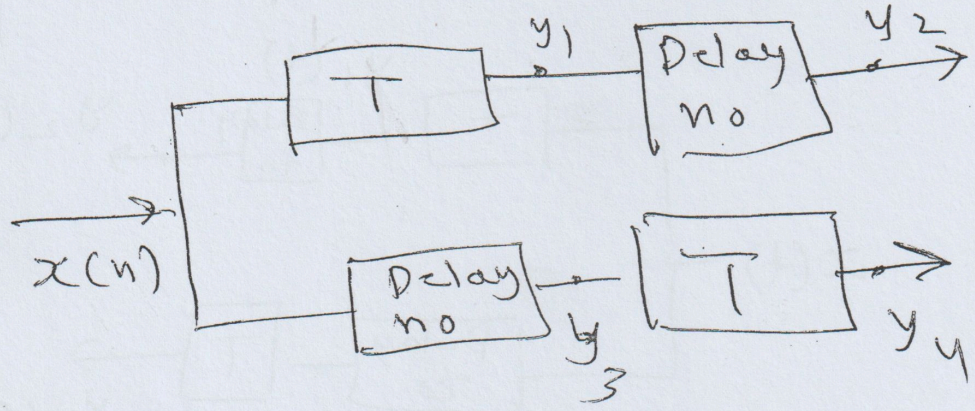


$y_1 = x(2t)$ $y_2 = x(2(t-t_0)) = x(2t-2t_0)$

$y_3 = x(2(t-t_0))$ $y_3 = x(2t-2t_0)$ $y_4 = x(2t-t_0)$

$\therefore y_2 \neq y_4$ $\therefore$ The System Time Invariant

ex 2 / $y(n) = 2 + x(n)$



$y_1(n) = 2 + x(n)$ $\qquad y_2(n) = 2 + x(n-n_0)$

$y_3(n) = x(n-n_0)$ $\qquad y_4(n) = 2 + x(n-n_0)$

$y_2(n) = y_4(n)$

$\therefore$ The system is Time invaria

H.W / check TIV and TV properties of following system ....

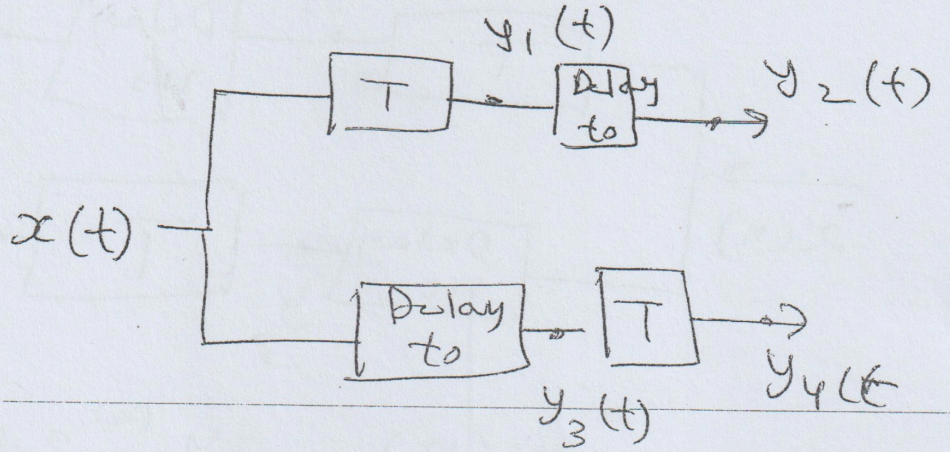
① $y(n) = x(\sin n)$

② $y(n) = x(n+2)$

③ $y(t) = \cos t + x(t)$

ex]

$y(t) = x(\cos t)$



$y_1(t) = x(\cos t)$ $\quad$ $y_2(t) = x(\cos(t-t_0))$

$y_3(t) = x(t-t_0)$ $\quad$ $y_4(t) = x(\cos(t)-t_0)$

$y_2(t) \neq y_4(t)$

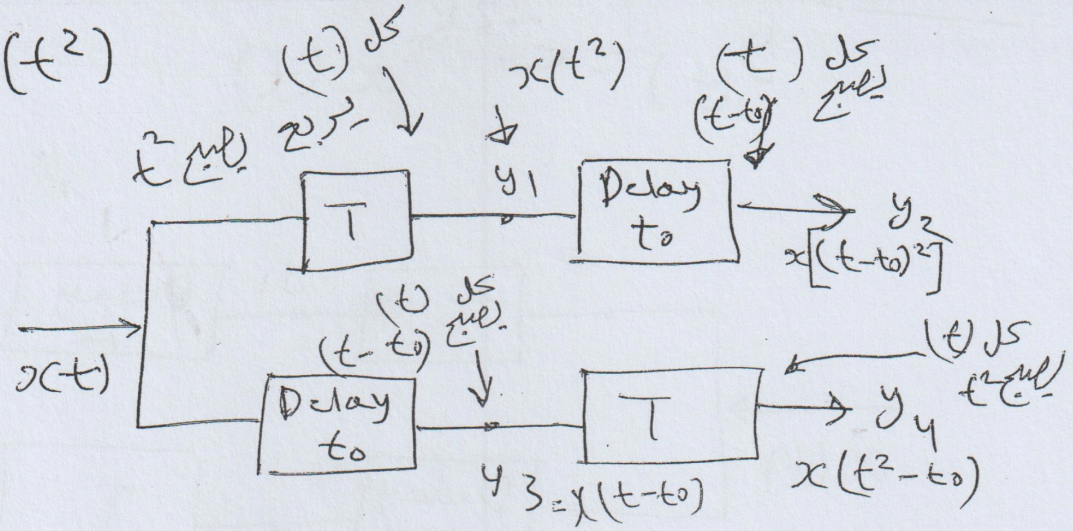
$\therefore$ The system is Time variant [TV]

ex] $y(t) = x(\text{Tan}t)$ as above

The system is Time Variant
TV

ex

$y(t) = x(t^2)$



$y_1(t) = x(t^2)$ $\quad y_2(t) = x((t-t_0)^2)$

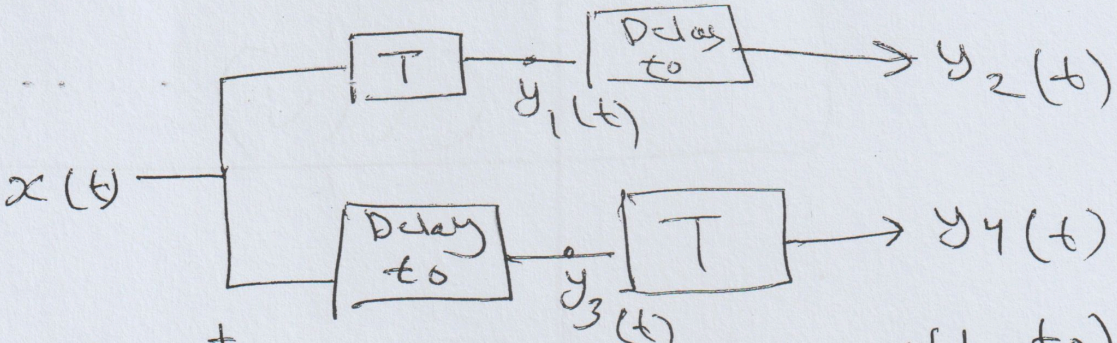
$y_3(t) = x(t-t_0)$ $\quad y_4(t) = x(t^2-t_0)$

$\therefore y_2 \neq y_4$

The system is T.V.

ex

$y(t) = x(t)\, e^{-t}$



$y_1(t) = x(t)\, e^{-t}$

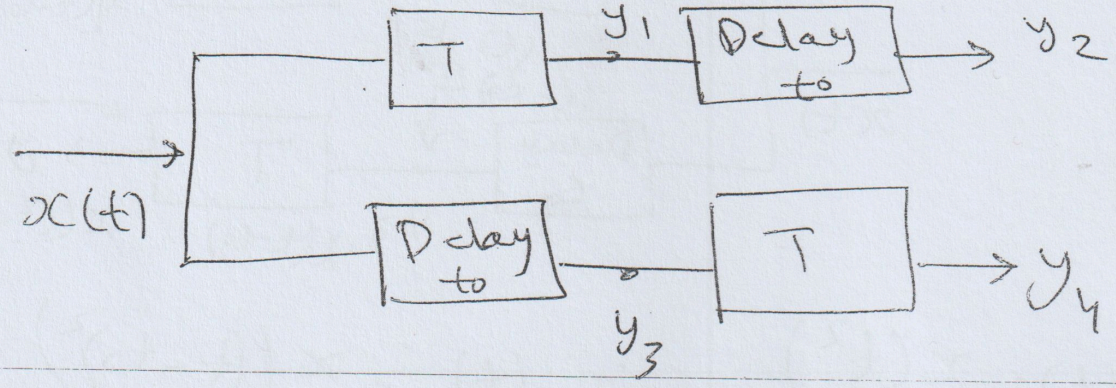
$y_2(t) = x(t-t_0)\, e^{-(t-t_0)}$

$y_3(t) = x(t-t_0)$

$y_4(t) = x(t-t_0)\, e^{-t}$

ex

$$y(t) = e^{-2k}\, x(t)$$



$$y_1(t) = e^{-2k}\, x(t) \qquad y_2(t) = e^{-2k}\, x(t-t_0)$$

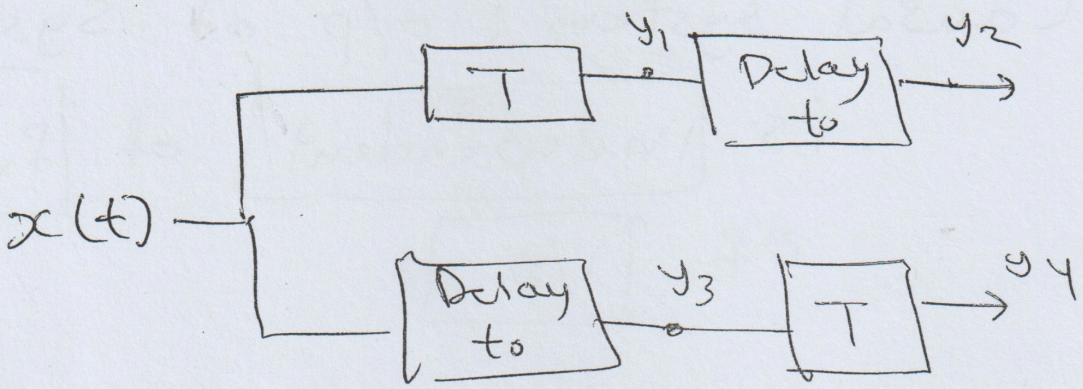
$$y_3(t) = x(t-t_0) \qquad y_4(t) = e^{-2k}\, x(t-t_0)$$

$$y_2 = y_4$$

∴ The system is Time invariant (TIV)

ex

$$y(t) = \tan(t+1)\, x(t)$$



$y_1(t) = \tan(t+1)\, x(t)$ · $y_2(t) = \tan(t-t_0+1)\, x(t-t_0)$

$y_3(t) = x(t-t_0)$ $\quad y_4 = \tan(t+1)\, x(t-t_0)$

$\therefore y_2 \neq y_4$

The system is Time Variant

T.V.

Causal and non Causal system

Causal system : o/p of system is **independent** of **future value** of **i/p**

Non - Causal system o/p of system **depends** on **future values** of **i/p** at any instant of time

ex $y(n) = x(n) + \frac{1}{x(n-1)}$

for $n = 1$

$$y(1) = x(1) + \frac{1}{x(0)}$$

↑ present o/p — ↑ present i/p — ↑ past i/p

the system is Causal ∴

ex

$$y(t) = 2x(t) + \frac{1}{x^2(t)}$$

if $n = 0$

$$\therefore y(0) = 2x(0) + \frac{1}{x^2(0)}$$

present o/p ↑ present i/p · ↑ present i/p

The system is causal

ex

$$y(n) = x(n) + 2x(n+1)$$

for $n = 0$

$$y(0) = x(0) + 2x(1)$$

↑ present i/p ↑ future i/p

the system is non causal

ex

$$y(t) = x(t) + x(t-1) + x(t+1)$$

if $t = 0$

$$y(0) = x(0) + x(-1) + x(1)$$

$\therefore$ non Causal system

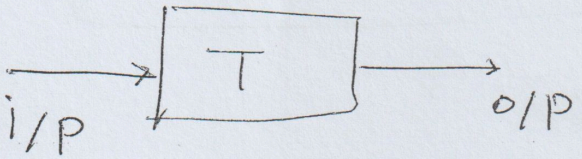
Stable and unstable System

Bounded i/p Bounded o/p criteria

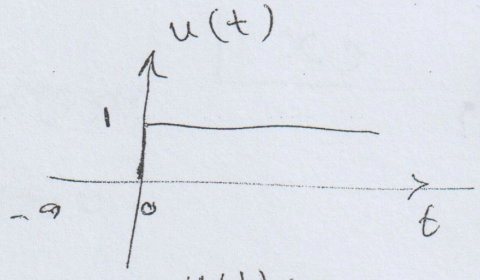
BIBO: for a stable system o/p should be bounded for bounded i/p at each and every instant of Time

Bounded signal dc, sint, cost, u(t)

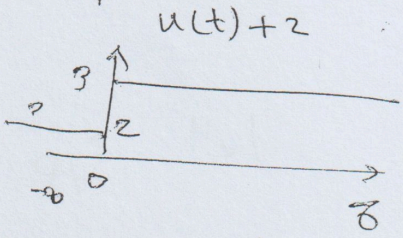
ex:1 $y(t) = x(t) + 2$



bounded i/p as u(t)



$\therefore$ o/p $y(t) = u(t) + 2$



$\therefore$ The System is stable o/p Bounded

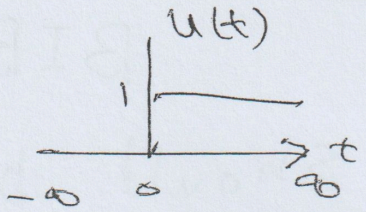
BIBO

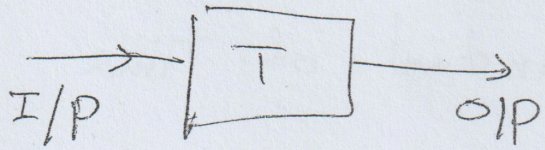
ex 2

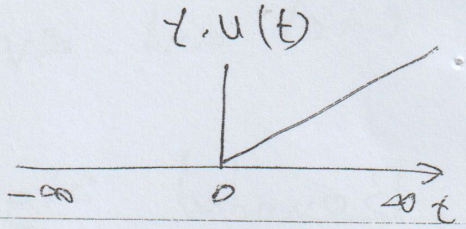
$y(t) = t\,x(t)$

let the i/p is $u(t)$

bounded i/p







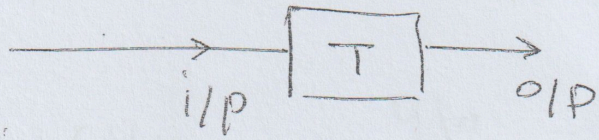
$y(t) = t \cdot u(t)$

$\therefore$ o/p unbounded

The system is unstable

ex 3

$y(t) = \sin[x(t)]$



let i/p bounded

$x(t) = u(t)$

$\therefore$ $y(t) = \sin[u(t)]$ $\quad -1 \le \sin[\,] \le 1$

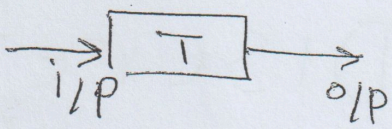
$\therefore$ bounded

BIBO

stable

ex 4|

$$y(t) = \int_{-\infty}^{t} x(t)\, dt$$

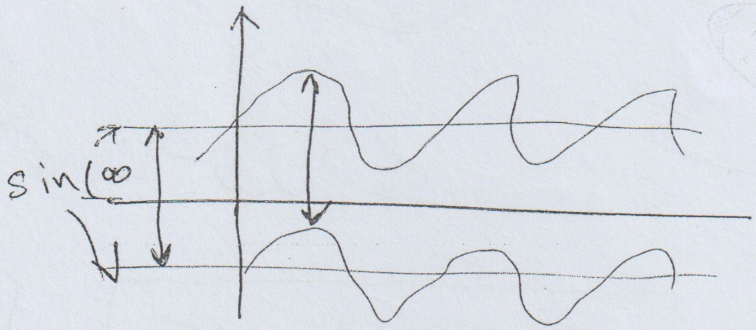


let $x(t) = \cos t$

$$y(t) = \int_{-\infty}^{t} \cos t\, dt = \sin(t)\Big]_{-\infty}^{t}$$

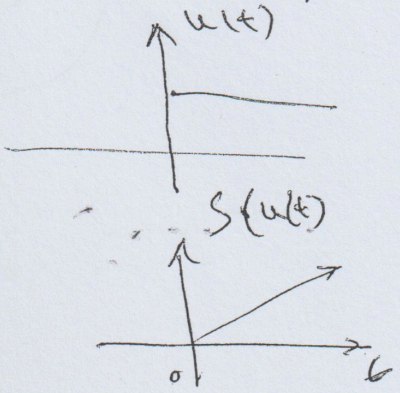
$$y(t) = \sin(t) - \sin(-\infty)$$

$$y(t) = \sin(t) + \underbrace{\sin\infty}_{-1 \le \sin\infty \le 1}$$



for this bounded o/p

if $x(t) = u(t)$



$$y(t) = \int_{-\infty}^{t} u(t)\, dt = \int_{0}^{t} d(t)$$

$$y(t) = t\Big]_{0}^{t} = t$$

$\therefore$ o/p unbounded

The System is unstable.

Discrete Time Signal

د. محمد نوری محسنی

2023/10/15

Discrete time Signal

The discrete Time Signal Can be represented in four ways :.

① Functional Representation.

② Graphical Representation.

③ Tabular Representation.

④ Sequence Representation.

① Functional Representation

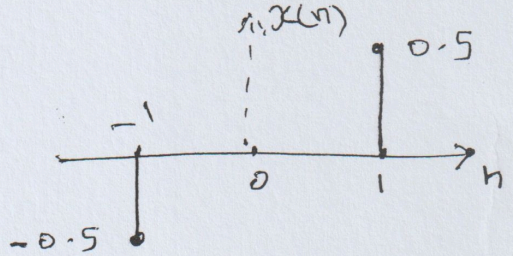
Ex $x(n) = 0.5n \quad -1 \le n \le 1$ & 0 elsewere

$x(-1) = -0.5 \quad x(0) = 0 \quad x(1) = 0.5$

② Graphical Representation

Ex $x(n) = 0.5n \quad -1 \le n \le 1$

$x(-1) = -0.5 \quad x(0) = 0 \quad x(1) = 0.5$



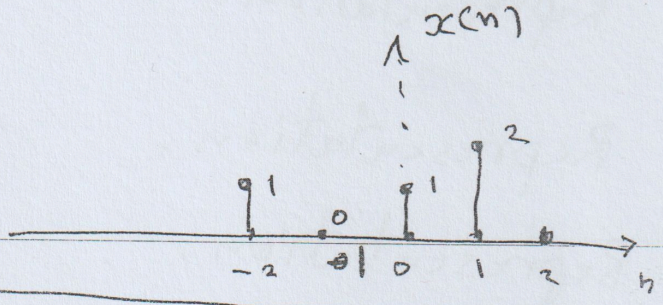
③ Tabular Representation

ex1

n	-2	-1	0	1
x(n)	1	0	1	2

Find all value of x(n) and Draw x(n)

$x(-2)=1 \quad x(-1)=0 \quad x(0)=1 \quad x(1)=2$



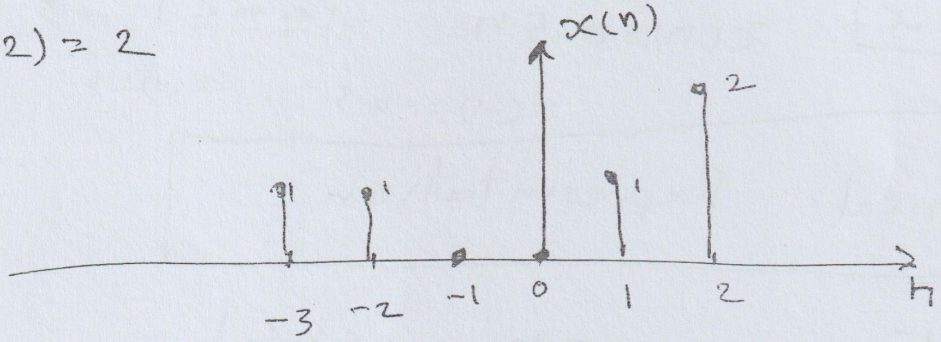
④ Sequence representation

ex

$$x(n) = \{\underset{-3}{1}, \underset{-2}{1}, \underset{-1}{0}, \underset{0}{0}, \underset{1}{1}, \underset{2}{2}\}$$

$x(-3)=1 \quad x(-2)=1 \quad x(-1)=0 \quad x(0)=0 \quad x(1)=1$

$x(2)=2$



Basic Signal operation

DTS
3

(1) Basic Signal operation on Amplitude

(a) amplitude Scaling of discrete Time Sig

(b) addition of discrete Time Signal

(c) subtraction of discrete Time Signal

(d) multiplication of discrete Time Signal

(2) Basic Signal operation on Time

(a) Time Scaling of signals

(b) Time shifting of discrete Time Signal

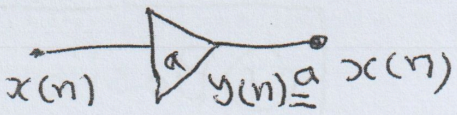
(c) Time reversal of discrete Time Signal

(d) Combinations of operation.

operation on amplitude

DTS 4

a amplitude Scaling



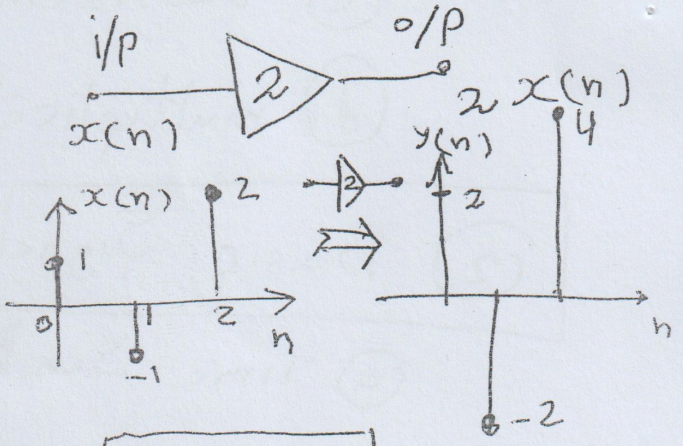
ex| for $x(n) = \{1, -1, \underset{\uparrow}{2}\}$ find $y(n) = 2x(n)$

Sol.| $a = 2$ $y(n) = 2x(n)$ (scaling by 2)

$x(n) = \{1, -1, \underset{\uparrow}{2}\}$

$y(n) = 2x(n)$

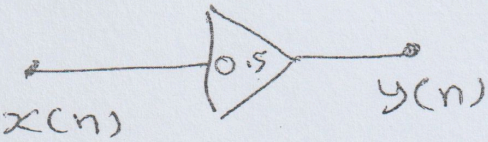
$= \{2, -2, \underset{\uparrow}{4}\}$



ex| if $x(n) = 0.5n$ for $-2 \le n \le 2$ and zero elsewhere

$$x(n) = \begin{cases} 0.5n & -2 \le n \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

find $y(n) = 0.5x(n)$

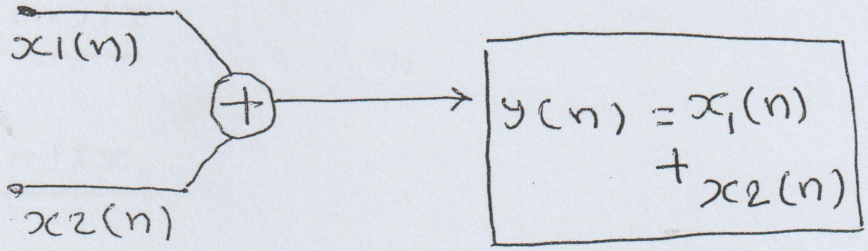


Sol.|

$x(n) = \{-1, -0.5, \underset{\uparrow}{0}, 0.5, 1\}$

$y(n) = \{-0.5, -0.25, 0, 0.25, 0.5\}$

(b) addition of Discrete Time Signals



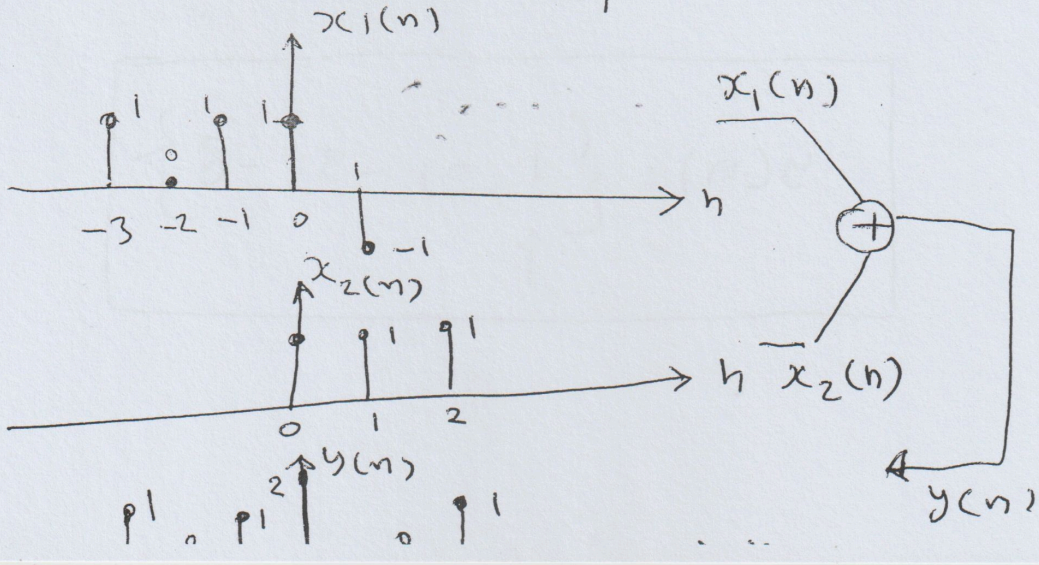
ex if $x_1(n) = \{1, 0, 1, \underset{\uparrow}{1}, -1\}$

and $x_2(n) = \{\underset{\uparrow}{1}, 1, 1\}$

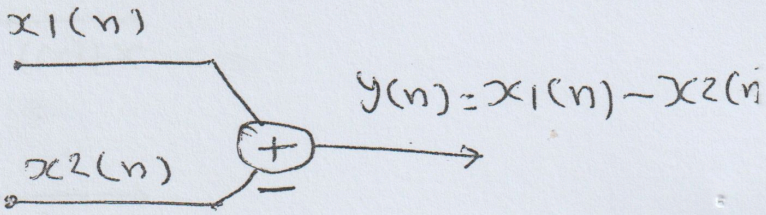
Find $y(n) = x_1(n) + x_2(n)$

$$y(n) = \{\overset{-3}{1}, \overset{-2}{0}, \overset{-1}{1}, \underset{\uparrow}{\overset{0}{1}}, \overset{1}{-1}\} + \{0, 0, 0, \underset{\uparrow}{1}, 1, 1\}$$

$$y(n) = \{1, 0, 1, \underset{\uparrow}{2}, 0, 1\}$$



C Subtraction of Discrete Time Signals



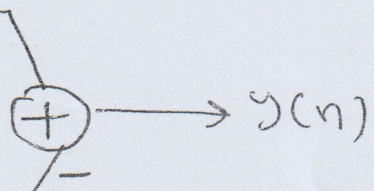
ex] if $x_1(n) = \begin{cases} u(n) & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$

$x_2(n) = \begin{cases} n^2 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$

Find $y(n) = x_1(n) - x_2(n)$

sol.] $x_1(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$

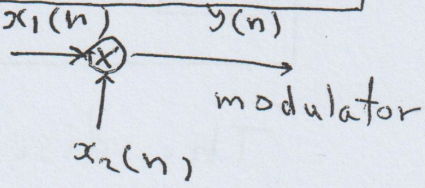
$x_2(n) = \{\underset{\uparrow}{0}, 1, 4, 9\}$



$$y(n) = \{\underset{\uparrow}{1}, 0, -3, -8\}$$

1-d multiplication of Discrete Time Signals

Signal multiplier or modulator



ex1 if $x_1(n) = [1 \;\; 1 \;\; \underset{\uparrow}{2}]$

and $x_2(n) = [\underset{\uparrow}{2} \;\; 0 \;\; 2]$

Find $y(n) = x1(n) \cdot x2(n)$

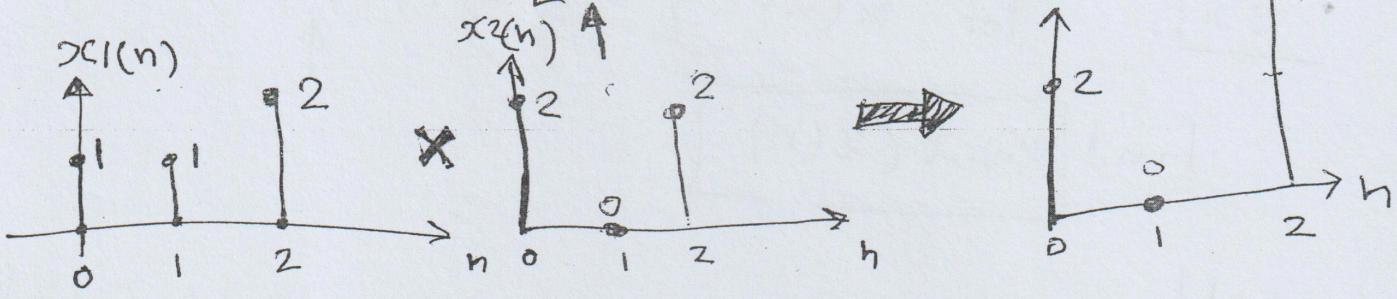
$y(n) = x_1(n) \cdot x_2(n$

Sol.

$$y(n) = x1(n) \cdot x2(n)$$

$$y(n) = [\underset{\uparrow}{1} \;\; 1 \;\; 2] \cdot [\underset{\uparrow}{2} \;\; 0 \;\; 2]$$

$$y(n) = [2 \;\; \underset{\uparrow}{0} \;\; 4]$$



ex2 if $x_1(n) = [1 \;\; 1 \;\; \underset{\uparrow}{2}]$

and $x2(n) = [\underset{\uparrow}{2} \;\; 0 \;\; 2]$

Find $y(n) = x1(n) \cdot x2(n)$

Sol.

$$y(n) = x_1(n) \cdot x2(n) = [1 \;\; 1 \;\; \underset{\uparrow}{2}] \cdot [\underset{\uparrow}{2} \;\; 0 \;\; 2]$$

$$= \overset{\substack{n=-2 \\ h=-1}}{[} \overset{n=0}{0} \;\; 0 \;\; \underset{\uparrow}{4} \;\; 0 \;\; \overset{n=2}{0}]$$

H.w graph representation

Time Scaling of Discrete Time Signals

a| compressed signals

The discrete Time Squence $x(n)$ is **Compressed** in Time by multiplying The index n by an integer K

$$x[Kn]$$

$K-1$ = عدد الارقام المحذوفة

ex| let $x(n) = \{3, 3, 2, 1, 3, \underset{\uparrow}{1}, 1, 2, 1\}$

Find $y(n) = x(2(n))$

Sol.| $K = 2$ compressed ratio $= K-1 = 2-1 = 1$

$$x(n) = \{\underset{-4}{3}, \boxed{3}, \underset{-2}{2}, \boxed{1}, \underset{0}{3}, \boxed{1}, \underset{2}{1}, \boxed{2}, 1\}$$

$$x(2(n)) = \{\underset{-2}{3}, \underset{-1}{1}, \underset{0}{1}, \underset{1}{2}\}$$

$n=0$ $y(n) = x(2n) \Rightarrow y(0) = x(0) = 1$

$n=1$ $y(n) = x(2n) \Rightarrow y(1) = x(2) = 2$

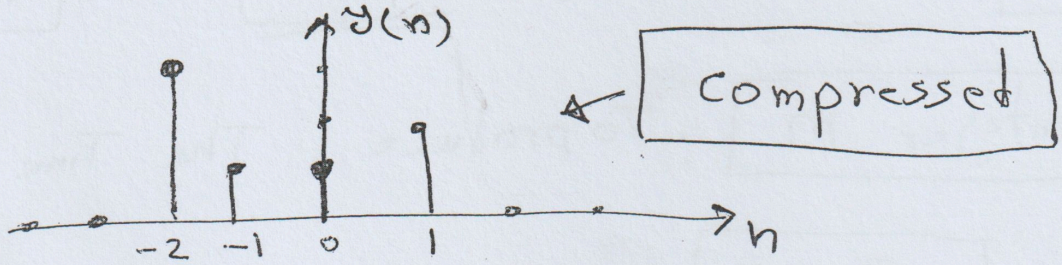
$n=2$ $y(n) = x(2n) \Rightarrow y(2) = x(4) = 0$

$n=-2$

$y(-2) = x(-4) = 3$

$n=-3$ $\quad y(-3) = x(-6) = 0$

$\therefore \quad y[n] = \underset{-3}{0}\ \underset{-2}{3}\ \underset{-1}{1}\ \underset{0}{\uparrow 1}\ \underset{1}{2}\ \underset{2}{0}$

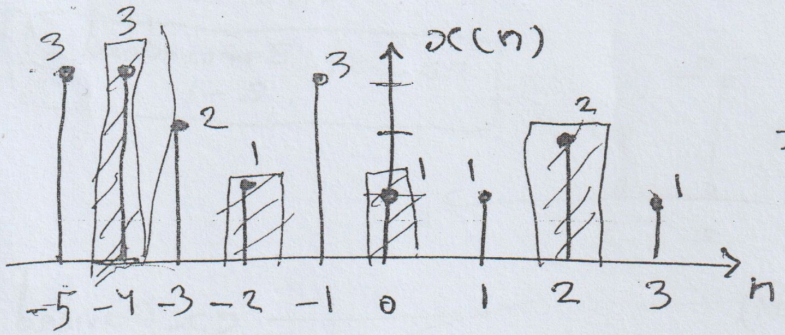


Compressed

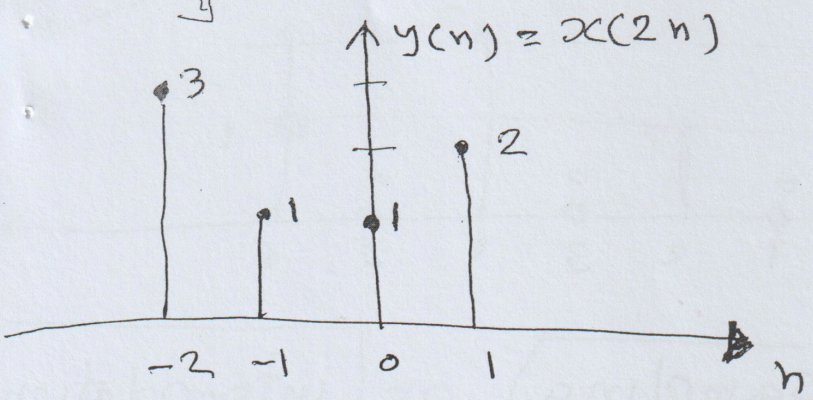
Called downSampling

or

decimation



$\Rightarrow$ compressed



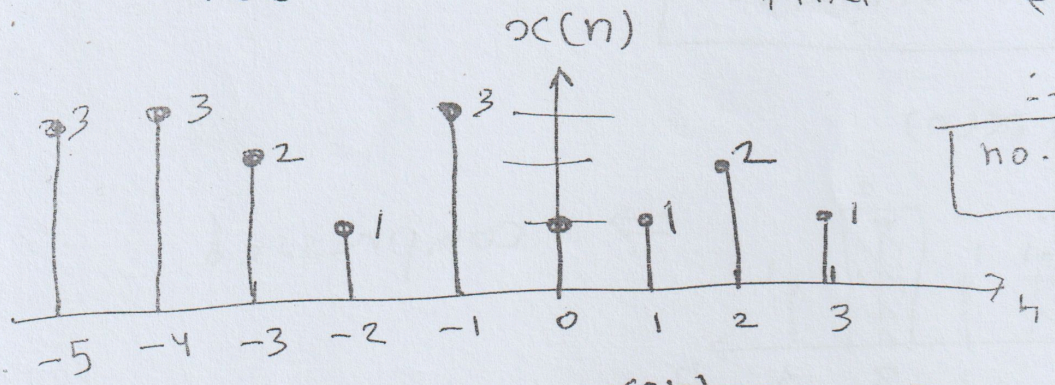
b expanded signals

The discrete Time Sequence $x[n]$ is **expanded** in time by **dividing** the **index n** by an **integer m**. To produce The Time Scale Sequence $x[n/m]$

no. of zero added = m − 1

ex

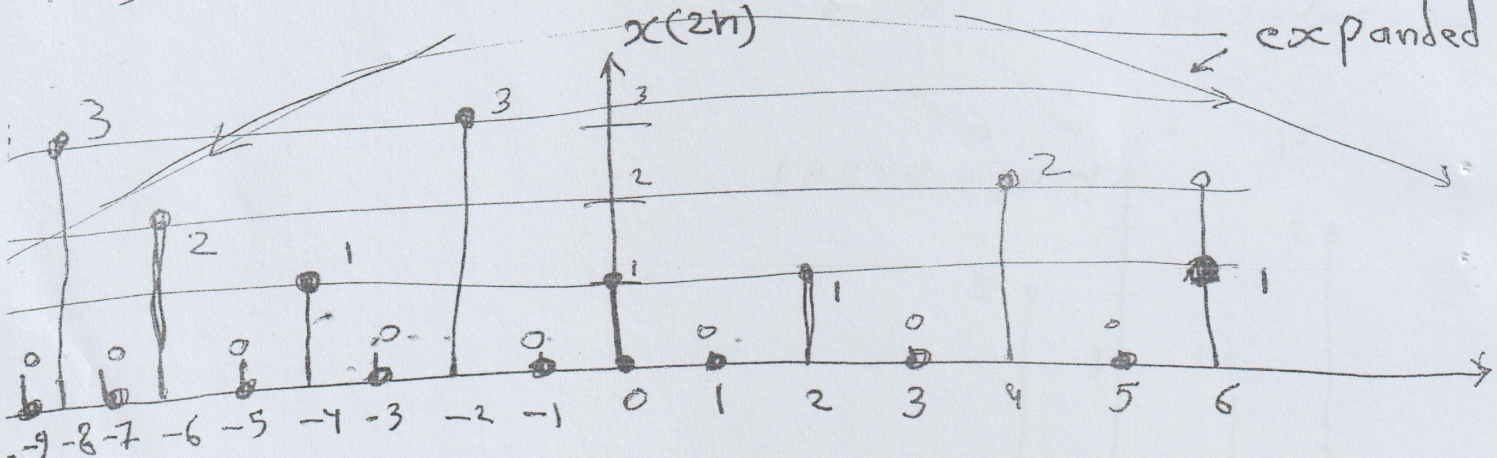
let



find $x(\frac{n}{2})$

$\therefore m = 2$

no. of zeros added = 1
2 − 1



Called **upsampling** or **interpolation**

$n = 0$

$y(n) = x(n/2) = x(0) = 1$

$n = -1$

$y(-1) = x(-1/2) = 0$

$n = -2$

$y(-2) = x(-1) = 3$

$n = -3$

$y(-3) = x(-1.5) = 0$

$n = -4$

$y(-4) = x(-2) = 1$

$n = -5$

$y(-5) = x(-2.5) = 0$

$n = -6$

$y(-6) = x(-3) = 2$

$n = -7$

$y(-7) = x(-3.5) = 0$

$n = -8$

$y(-8) = x(-4) = 3$

$n = -9$

$y(-9) = x(-4.5) = 0$

$n = -10$

$y(-10) = x(-5) = 3$

$n = 1$

$y(1) = x(1/2) = 0$

$n = 2$

$y(2) = x(1) = 1$

$n = 3$

$y(3) = x(1.5) = 0$

$n = 4$

$y(4) = x(2) = 2$

$n = 5$

$y(5) = x(2.5) = 0$

$n = 6$

$y(6) = x(3) = 1$

n	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y(n)	3	0	3	0	2	0	1	0	3	0	1	0	1	0	2	0	1

Time Reversal of Discrete Time Signals

discrete Time reverse

replace $[n]$ with $[-n]$

reversed discretete Time signal of $x(n)$ is $x[-n]$

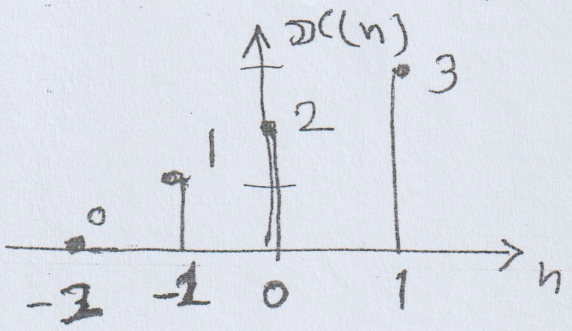
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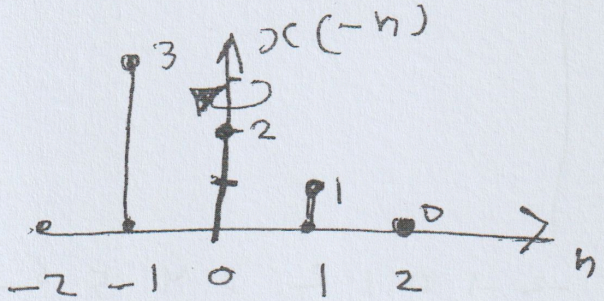
ex] if $x(n) = [0 \quad 1 \quad \underset{\uparrow}{2} \quad 3]$

Find $y(n) = x(-n)$

Sol.] if $x(n) = [0 \quad 1 \quad \underset{\uparrow}{2} \quad 3]$

$\therefore y(n) = [3 \quad \underset{\uparrow}{2} \quad 1 \quad 0] = x(-n)$





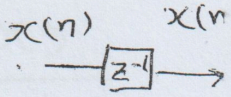
Time shift of Discrete Time signals

DTS

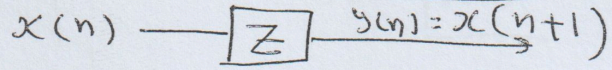
13

For a discrete Time Signal $x[n]$ and an Integer $n_1 > 0$

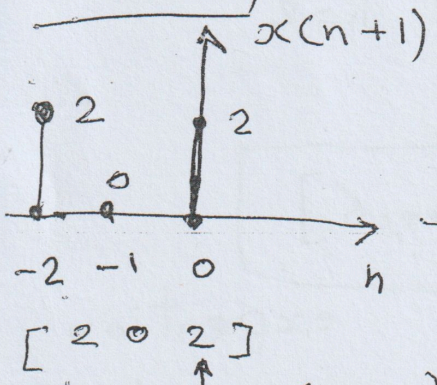
- $x[n - n_1]$ is a delayed signal



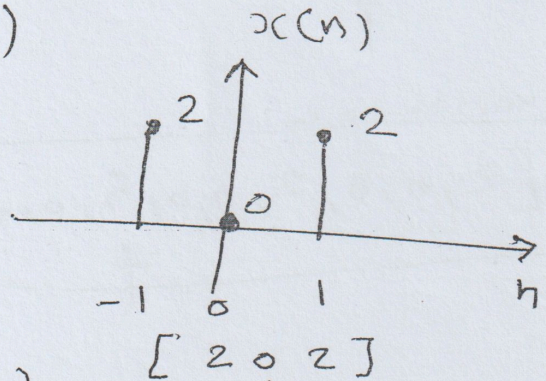
- $x[n + n_1]$ is an advanced signal



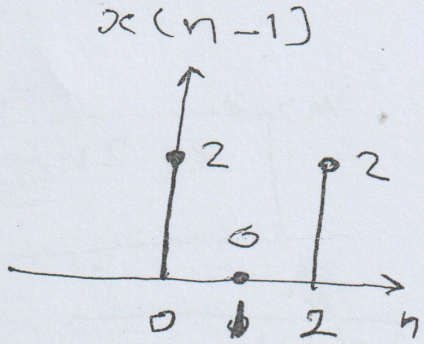
ex



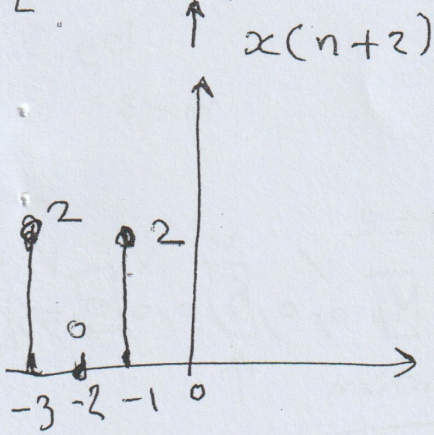
[2 0 2]
↑



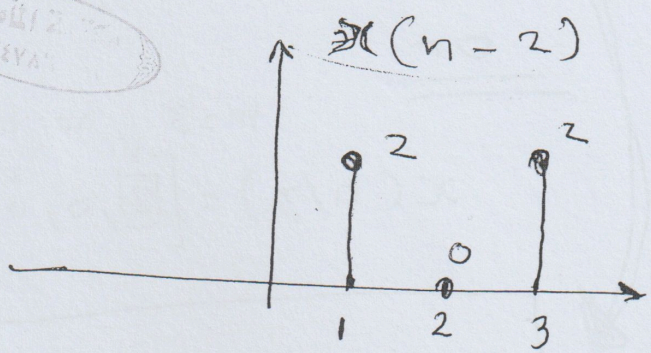
[2 0 2]
↑



[2 0 2]
↑



[2 0 2 0]
↑



[0 2 0 2]
↑

Combinations of Operations

Time scaling, shifting and reversal can all be combined.

ex/ $y(n) = x(2n/3)$

$x(n) = [\underset{\uparrow}{2}, 1, 2, 4, 5, 0, 6]$

Sol/

$x(2n) = [2, 2, 5, 6]$ comperation by 2, $K=2$

$m=3$ Zeros no. $= m-1 = 2$

$x(2n/3) = [2,0,0,2,0,0,5,0,0,6]$

expention by 3, $m=3$

أفضل

or

$m=3$ no. of zeros $= m-1 = 3-1 = 2$

$x(n/3) = [2,0,0,1,0,0,2,0,0,4,0,0,5,0,0,0,0,6]$

expention

$x[2\frac{n}{3}] = [2,0,0,2,0,0,5,0,0,6]$

ex

$x(n) = [1, 2, \underset{\uparrow}{3}, 4, 5]$

Find $y(n) = x(n \cdot 2) = x(-2n)$

Sol.

reverse

$x(n) = [1, 2, 3, 4, 5]$

$x(-n) = [5, 4, 3, 2, 1]$

comperation by 2 $k = 2$

$x(-2n) = [5, 3, 1]$

$\therefore \; y(n) = [5, 3, 1]$

or

$x(n) = [1, 2, 3, 4, 5]$

comperation by 2 $k = 2$

$x(2n) = [1, 3, 5]$ reverse

$x(-2n) = [5, 3, 1]$

ex/

$x(n) = \{7, 1, \underset{\uparrow}{2}, 6, 3\}$

Find

$y(n) = x(-n-1)$

Sol./

$x(n) = \{7, 1, \underset{\uparrow}{2}, 6, 3\}$

$x(n-1) = \{7, \underset{\uparrow}{1}, 2, 6, 3\}$

reverse of Time

$x(-n-1) = x(-(n+1)) = \{3, 6, 2, \underset{\uparrow}{1}, 7\}$

or/

$x(n) = \{7, 1, \underset{\uparrow}{2}, 6, 3\}$

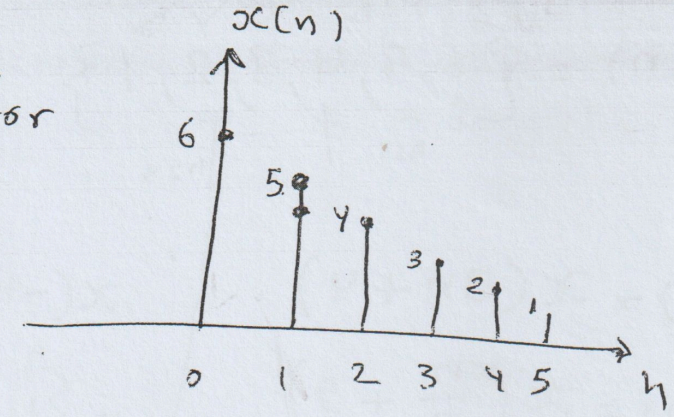
reverse of Time

$x(-n) = \{3, 6, \underset{\uparrow}{2}, 1, 7\}$

نفس الناتج

$x(-n-1) = x(-[n+1]) = \{3, 6, 2, \underset{\uparrow}{1}, 7\}$

ex/ for



Find $y(n) = x(3n+4)$

sol.

$$x(n) = \{\underset{\uparrow}{6}, 5, 4, 3, 2, 1, 0\}$$
(0 1 2 3 4 5)

$$x(n+4) = \{6, 5, 4, 3, 2, 1\}, 0$$

$b = 3$ الاختزال $= 3 - 1 = 2$

$$y(n) = x(3n+4) = \{5, \underset{\uparrow}{2}\}$$

الاختزال

$x(3n+4)$

for $n = -2$ $\quad y(-2) = x(-2) = 0$ $\quad \{3 \cdot (-2) + 4\} = -2$ (−6, +4)

for $n = -1$ $\quad y(-1) = x(+1) = 5$ $\quad \{-3 + 4\} = +1$

for $n = 0$ $\quad y(0) = x(4) = 2$ $\quad \{3 \cdot 0 + 4\} = 4$

for $n = 1$ $\quad y(1) = x(7) = 0$ $\quad \{3 \cdot 1 + 4\} =$

$$\therefore y(n) = \left[5, \underset{\uparrow}{2}\right]$$

ex/ if $x(n) = \{6, 5, 4, 3, 2, 1\}$ (↑ at $n=1$, ↑ at $n=4$)

Find

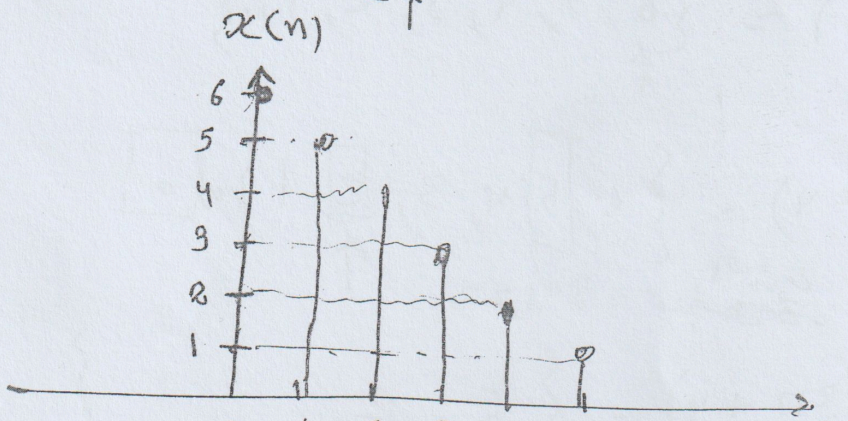
① $y(n) = x(3n+4)$

② $y(n) = x\left(\frac{n}{2} + 3\right)$

③ $y(n) = x(-n-2)$

$x(-n-2) = \{1, 2, 3, 4, 5, 6\}$ (↑ at $n=0$)

Sol. ① $x(n) = \{6, 5, 4, 3, 2, 1\}$ (↑ at $n=0$)



$0 \le 3n+4 \le 5$

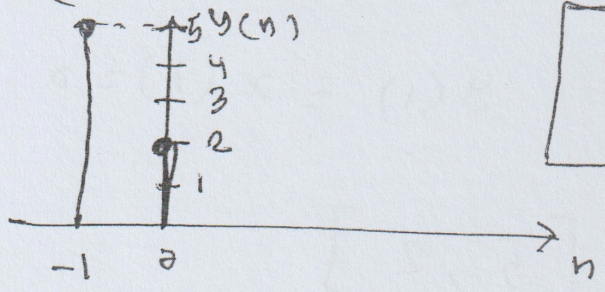
$-4 \le 3n \le 1$

$\frac{-4}{3} \le n \le \frac{1}{4}$

$n = -1$ $\quad n = 0$

$y(0) = x(3 \cdot 0 + 4) = x(4) = 2$

$y(-1) = x(3 \cdot (-1) + 4) = x(1) = 5$



$y(n) = \{5, 2\}$ (↑ at $n=0$)

$0 \le -n-2 \le 5$

$2 \le -n \le 7$

$-7 \le n \le -2$

$-2 \ge n \ge -7$

$x(-n-2)$

$y(0)$ $\quad -2 \quad 0$

or

$$x(n) = \{6, 5, 4, 3, 2, 1\}$$

shift by four [4]

$$x(n+4) = \{6, 5, 4, 3, 2, 1\}$$

compration by 3

عدد الأصفار = 3 - 1 = 2

$$y(n) = x(3n+4) = \{5, 2\}$$

نفس الناتج

Sol of (2) $y(n) = x\left(\frac{n}{2} + 3\right)$

$$x(n) = \{6, 5, 4, 3, 2, 1\}$$

shift by (3)

$$x(n+3) = \{6, 5, 4, 3, 2, 1\}$$

no. of zero = 2 - 1 = 1

enterpolation

$$x\left(\frac{n}{2}+3\right) = \{6, 0, 5, 0, 4, 0, 3, 0, 2, 0, 1\}$$

③ $y(n) = x(-n-2)$

Sol. $y(n) = x(-(n+2))$

$x(n) = \{6, 5, 4, 3, 2, 1\}$ (↑ on 6)

$x(n+2) = \{6, 5, 4, 3, 2, 1\}$ (↑ on 4)

$y(n) = x(-(n+2)) = \{1, 2, 3, 4, 5, 6\}$ (↑ on 4)

$H(\omega)$ = Find ① $y(n) = u(-n+4)$

② $y(n) = x\left(\frac{n}{2} - 2\right)$

if $x(n) = \{1, 2, 3\}$ (↑ on 2)

Convolution
&
DE Convolution

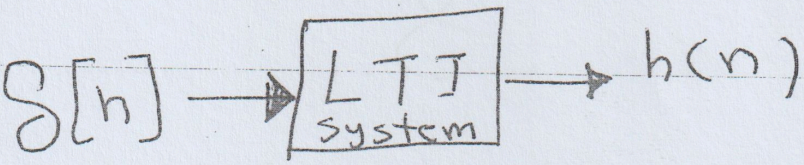
د. محمد نوري حسين

2023/11/6

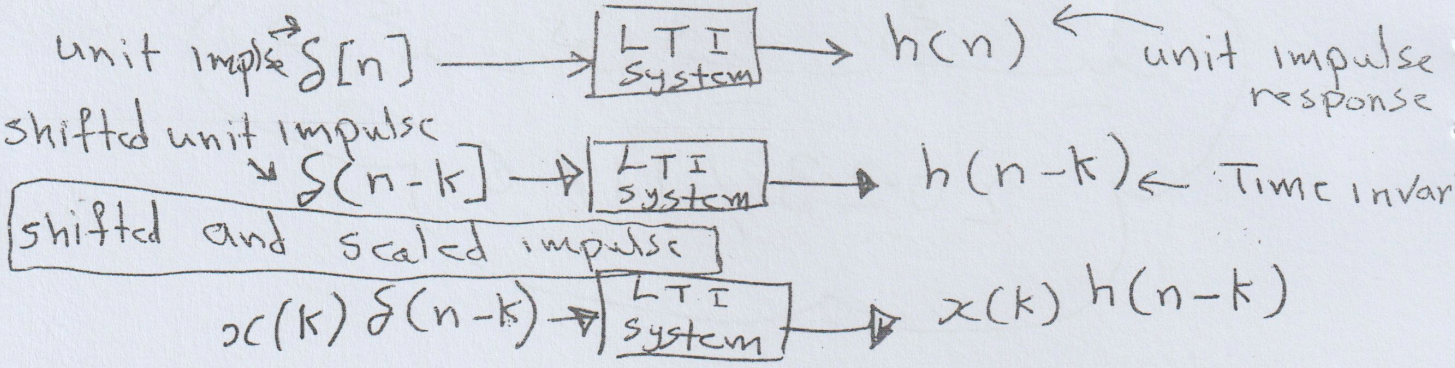
Discrete Time Signal convolution

The Simplest input Signal To a system is a single **unit impulse**

$h[n]$ = System output (response) To a unit impulse input $\delta[n]$



The importance of The unit impulse response $h(n)$ is That for L.T.I systems we can use superpasition to construct The output $Y(n)$ for any input $x[n]$.



sum of the previous result over all values of k since system is linear the output is also summed (superposition)

$x(k)\,\delta(n-k)$ → [LTI System] → $x(k)\,h(n-k)$

$\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$ → [LTI System] → $\sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$

The convolution Sum

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

$$y(n) = x(n) * h(n)$$

$x(n)$ → [LTI system $h(n)$] → $y(n) = x(n) * h(n)$

ex 4] for $y(n) = \sum_{m=0}^{\infty} x(n-m)$

Find $h(n)$

Sol.] $h(n) = y(n)$ for $x(n) = \delta(n)$

$$\therefore h(n) = \sum_{m=0}^{\infty} \delta(n-m)$$

$h(0) = \delta(0) + \delta(-1) + \cdots = 1$

$h(1) = \delta(1) + \delta(1-1) + \delta(1-2) + \cdots = 1$

$h(2) = 1$

$\vdots$

$h(\infty) = 1$

$$\therefore h(n) = \{1, 1, 1, \cdots\} = u(n)$$

ex 5] Find the impulse response $h(n)$ (the first three samples) for the system defined by the following difference equation.

$$y(n) = y(n-1) - 0.8\, y(n-2) + x(n)$$

Assume the system is causal. i.e

$$h(n) = 0 \quad \text{for} \quad n < 0$$

Sol. $h(n) = y(n)$ for $x(n) = \delta(n)$

$$y(n) = y(n-1) - 0.8\,y(n-2) + x(n)$$

$$h(n) = h(n-1) - 0.8\,h(n-2) + \delta(n)$$

$$h(0) = h(-1) - 0.8\,h(-2) + \delta(0) = 1$$

Causal Causal

$$h(1) = h(0) - 0.8\,h(-1) + \delta(1) = 1$$

Causal

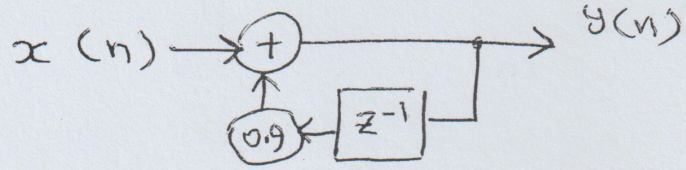
$$h(2) = h(1) - 0.8\,h(0) + \delta(2) = 1 - 0.8 \cdot 1$$

$$= 0.2$$

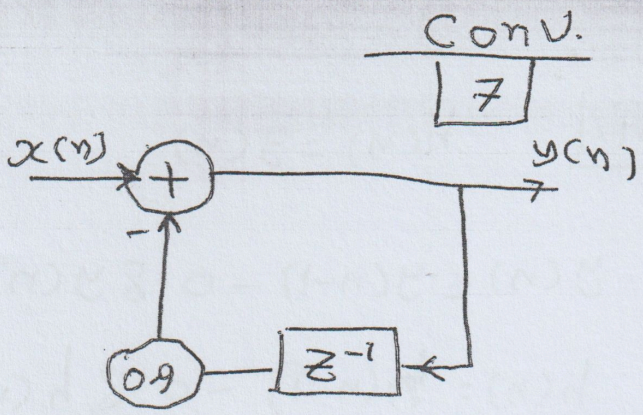
$$h(3) = h(2) - 0.8\,h(1) + \delta(3) = 0.2 - 0.8 \cdot 1$$

$$= -0.6$$

ex Find The first Three Sample value of The impulse response $h(n)$ for The system shown in figure.



Sol.



$$y(n) = x(n) - 0.9\, y(n-1)$$

The system is causal $\therefore h(n) = 0$ for $n < 0$

for $x(n) = \delta(n)$ $\quad y(n) = h(n)$

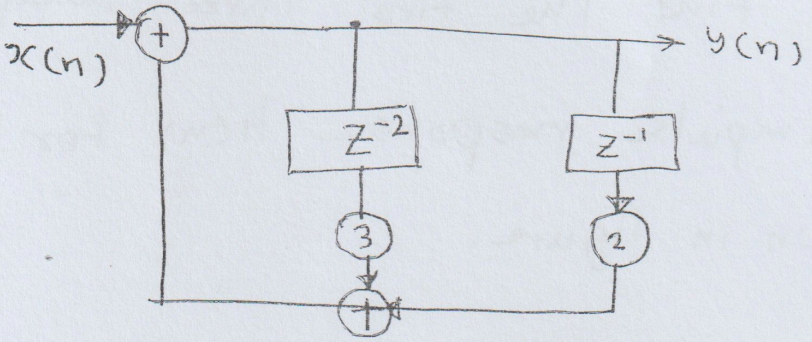
$$h(n) = \delta(n) - 0.9\, h(n-1)$$

$$h(0) = \delta(0) - 0.9\, \overset{0}{h(-1)} = 1$$

$$h(1) = \overset{0}{\delta(1)} - 0.9\, h(0) = -0.9 \times 1 = -0.9$$

$$h(2) = \delta(2) - 0.9\, h(1) = -0.9 \cdot -0.9 = 0.81$$

H.W



Find $h(n)$

Correlation and convolution

(1) Cross-correlation: function is a measure of similarities between two signals.

(2) Auto-correlation: is a special case from the cross-correlation of the function with it self.

(3) convolution: represents the relation between the i/p and the o/p of linear time invariant system.

$$y(n) = x(n) * h(n) = h(n) * x(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

convolution may classify according the sequence

(1) circular convolution: is used for periodic sequence or multiplication of two sequences in frequency domain

$$y(n) = \sum_{k=0}^{N-1} x(k)\, h(n-k)$$

N max length

(2) linear convolution: is used for aperiodic sequences or The multiplication of Two sequens in Time domain

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$$

$$N = N1 + N2 - 1$$

example of Linear convolution

ex 1/

find $y(n) = x(n) * h(n)$

if $x(n) = \{3, 1, 2\}$ and $h(n) = \{3, 2, 1\}$

by

① graphical method

② direct method

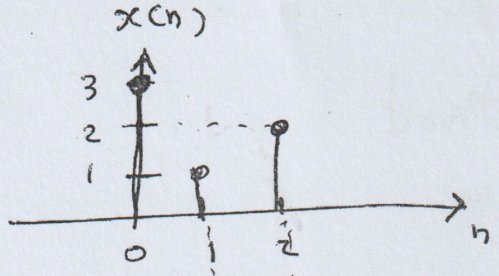
③ Table lookup method

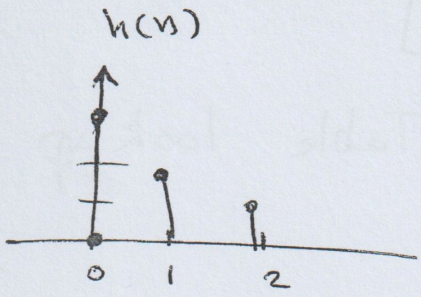
Sol.

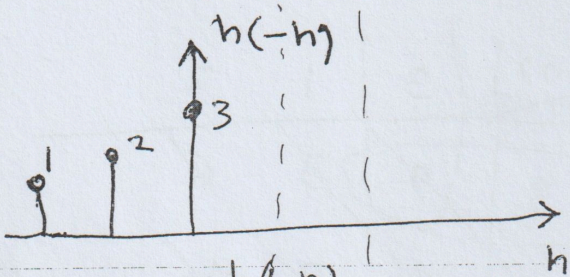
$N = 3+3-1 = 5$

$0 \le n \le 4$

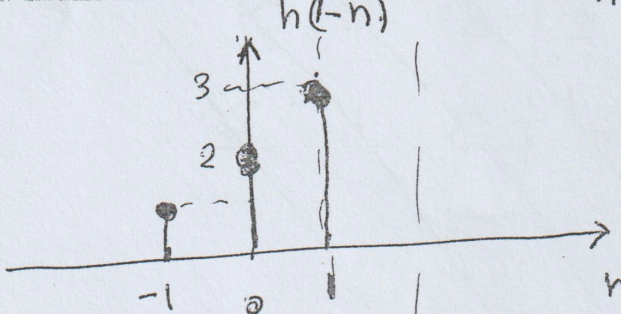
① graphical method



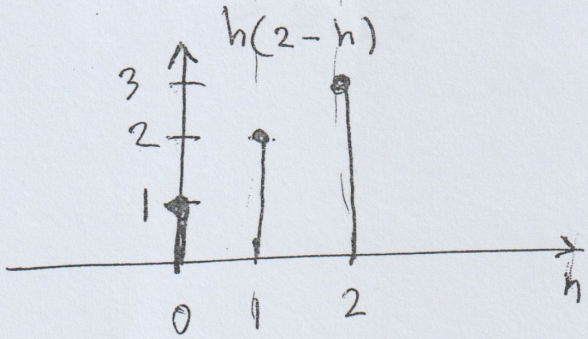




$$y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(-k-0) = 3\times 3 = 9$$



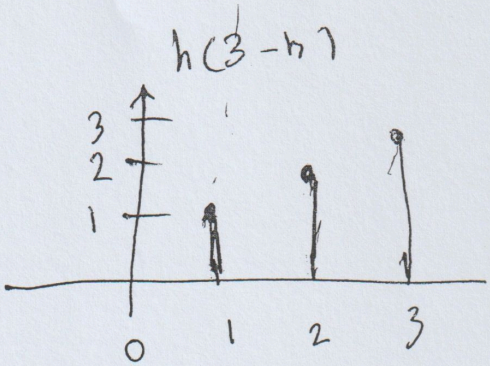
$$y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k) = 3\times 1 + 3\times 2 = 9$$



$$y(2) = \sum_{k=-\infty}^{\infty} x(k)\, h(2-k)$$

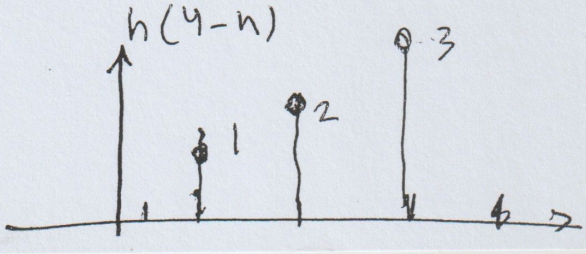
$$= 1\times 3 + 2\times 1 + 3\times 2$$

$$= 11$$



$$y(3) = \sum_{k=-\infty}^{\infty} x(k)\, h(3-k)$$

$$= 1\cdot 1 + 2\times 2 = 5$$



$$y(4) = \sum_{k=-\infty}^{\infty} x(k)\, h(4-k)$$

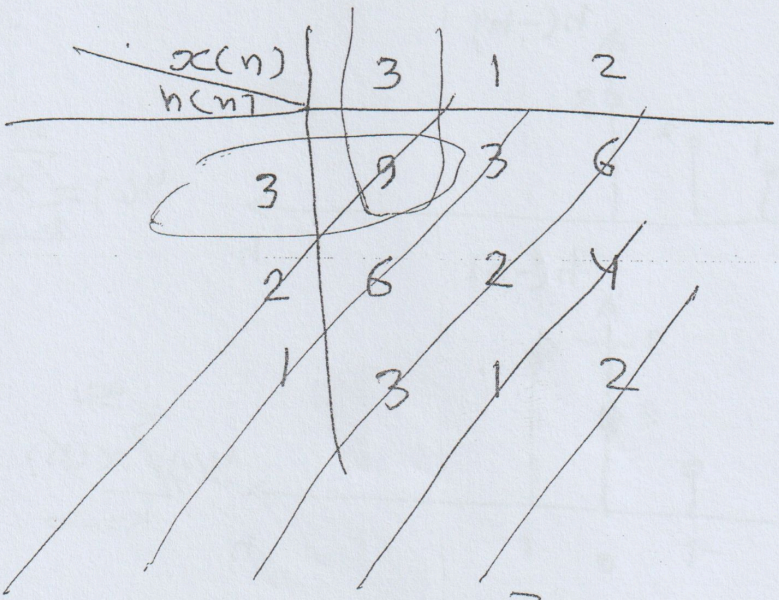
$$= 0\times 0 = 4$$

$y(n) = \{ 9, 9, 11, 5, 4 \}$

conv
11

ex1

(3) Table look up method



$y(n) = \{ 9 \quad 9 \quad 11 \quad 5 \quad 23 \}$

ex1

② Direct method

Sol. $x(n) = \{3, 1, 2\}$ (↑ on 3) $\quad h(n) = \{3, 2, 1\}$ (↑ on 3)

$N = n_1 + n_2 - 1 = 3 + 3 - 1 = 5$

$0 \le n \le N-1$

$0 \le n \le 4$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$y(0) = x(0)h(0) + x(1)h(-1) + x(2)h(-2) = 9$

$y(1) = x(0)h(1) + x(1)h(0) + x(2)h(-1) = 9$

$y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0) = 11$

$y(3) = x(0)h(3) + x(1)h(2) + x(2)h(1) = 5$

$y(4) = x(0)h(4) + x(1)h(3) + x(2)h(2) = 2$

$\therefore y(n) = \{9, 9, 11, 5, 2\}$ (↑ on 9)

ex(2) Solve The previous example use

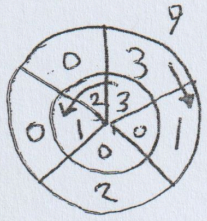
Concentric circle

$x(n) = \{3, 1, 2\}$ $\quad h(n) = \{3, 2, 1\}$

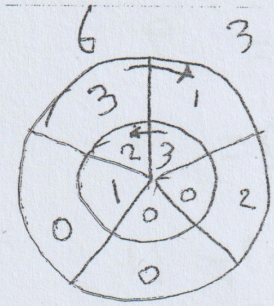
$n_1 = 3 \qquad n_2 = 3 \qquad N = 3+3-1 = 5$

$x(n) = \{3, 1, 2, 0, 0\}$

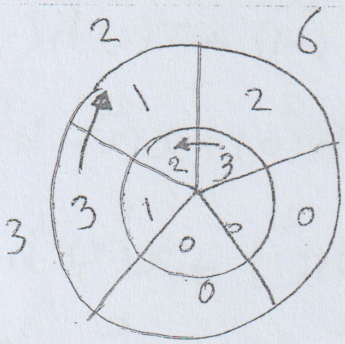
$h(n) = \{3, 2, 1, 0, 0\}$



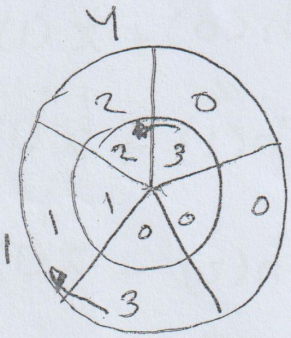
$y(0) = 9$



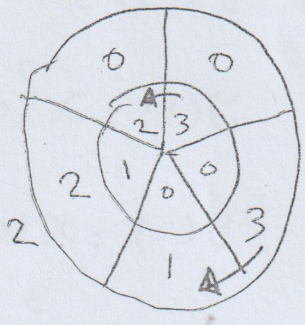
$y(1) = 9$



$y(2) = 11$



$y(3) = 5$



$y(4) = 2$

$\therefore y(n) = \{9, 9, 11, 5, 2\}$

matrix by vector method

ex| if $x(n)=\{3,2,1\}$ and $h(n)=\{3,1,2\}$

Find $y(n)$ $\quad N=n_1+n_2-1=3+3-1=5$

sol.|

$$y(n)=\begin{bmatrix}3&0&0\\2&3&0\\1&2&3\\0&1&2\\0&0&1\end{bmatrix}\begin{bmatrix}3\\1\\2\end{bmatrix}=[9,9,11,5,2]$$

---

ex| if $x(n)=[1,1,2,1,2,2,1,1]$

$h(n)=[1,2,-1,1]$ Find $y(n)$

sol| $n_1=8 \quad n_2=4 \quad N=8+4-1=11$

$$y(n)=\begin{bmatrix}1&0&0&0\\1&1&0&0\\2&1&1&0\\1&2&1&1\\2&1&2&1\\2&2&1&2\\1&2&2&1\\1&1&2&2\\0&1&1&2\\0&0&1&1\\0&0&0&1\end{bmatrix}\begin{bmatrix}1\\2\\-1\\1\end{bmatrix}=[1,3,3,5,3,7,4,3,3,0,1]$$

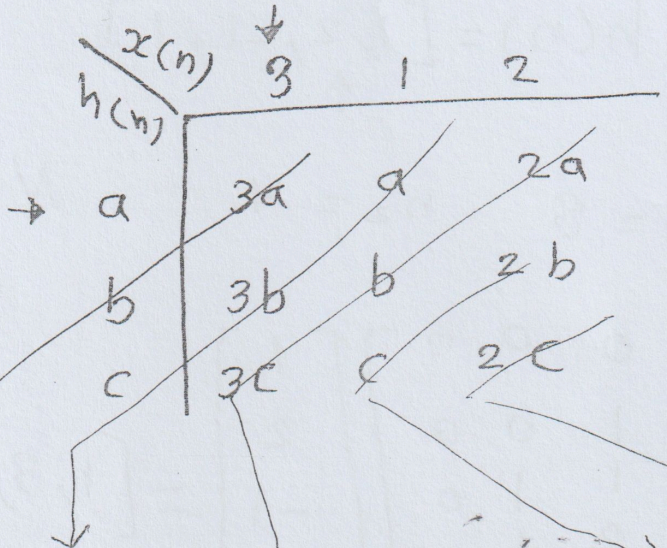
De convolution

Ex if a signal $x(n)=[\underset{\uparrow}{3},1,2]$ is convol with an unkown signal $h(n)$. The convolution result is noted as $\{\underset{\uparrow}{9},9,11,5,2\}=y(n)$ find $h(n)$

sol $n_1=3 \qquad N=5$

$N=n_1+n_2-1 \quad \therefore 5=3+n_2-1 \Rightarrow n_2=3$

$n_2=5-3+1=3$



$y(n)=[3a,\ a+3b,\ 2a+b+3c,\ c+2b,\ 2c]$

$y(n)=[9,\ 9,\ 11,\ 5,\ 2]$

$3a=9 \quad \therefore a=\frac{9}{3}=3$

$2c=2 \quad c=\frac{2}{2}=1$

$a+3b=9$

$3+3b=9$

$3b=6 \Rightarrow b=2$

Deconvolution Division Method

ex| Let $x(n) = \{\underset{\uparrow}{3}, 1, 2\}$ and $y(n) = \{\underset{\uparrow}{9}, 9, 11, 5, 2\}$

Find $h(n)$ by Division method.

Sol.|| $x(n) = \{\underset{\uparrow}{3}, 1, 2\} = 3 + x + 2x^2$

$y(n) = \{9, 9, 11, 5, 2\} = 9 + 9x + 11x^2 + 5x^3 + 2x^4$

$h(n) = y(n)/x(n)$

$$
\begin{array}{r}
3 + 2x + x^2 \\
\hline
3 + x + 2x^2 \;\big|\; 9 + 9x + 11x^2 + 5x^3 + 2x^4 \\
\underline{-9 \mp 3x \mp 6x^2} \\
0 + 6x + 5x^2 + 5x^3 \\
\underline{-6x \mp 2x^2 \mp 4x^3} \\
0 + 3x^2 + x^3 + 2x^4 \\
\underline{-3x^2 \mp x^3 \mp 2x^4} \\
0 \quad 0 \quad 0
\end{array}
$$

$\therefore h(n) = \{\underset{\uparrow}{3}, 2, 1\}$