

Table of Laplace Transforms :

$f(t)$	$F(s)$
1	$\frac{1}{s}$
Constant K	$\frac{K}{s}$
K f(t), K is constant	K F(s)
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
e^{at}	$\frac{1}{s-a}$
$e^{-at} t^n$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{(s+a)}{(s+a)^2 + \omega^2}$
$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
$t e^{-at}$	$\frac{1}{(s+a)^2}$
$1 - e^{-at}$	$\frac{a}{s(s+a)}$

~~1-5 (a)~~

$$① \quad F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

$$② \quad L e^{-at} = \frac{1}{s+a} \quad \text{or} \quad L e^{at} = \frac{1}{s-a}$$

$$3- L 1 = \frac{1}{s}$$

$$4- L f_1(t) + f_2(t) + f_3(t) = F_1(s) + F_2(s) + F_3(s)$$

$$5) L [k \cdot f(t)] = k \cdot F(s)$$

$$6) L \frac{df(t)}{dt} = s F(s) - f(0)$$

$$7- L \int_0^t f(t) dt = \frac{F(s)}{s}$$

$$8- L (t \cdot f(t)) = \frac{-dF(s)}{ds}$$

$$9- L f(t-T) = e^{-Ts} F(s)$$

~~10-5 (s)~~

①

②

$$10) L(e^{at} \cdot f(t)) = f(s-a) \quad (2)$$

$$11) L(e^{-at} \cdot f(t)) = f(s+a)$$

$$12) L(\sin \omega t) = \frac{\omega}{s^2 + \omega^2}$$

$$13) L(\cos \omega t) = \frac{s}{s^2 + \omega^2}$$

$$14) L(e^{-at} \cdot \sin \omega t) = \frac{\omega}{(s+a)^2 + \omega^2}$$

$$15) L(e^{-at} \cdot \cos \omega t) = \frac{s}{(s+a)^2 + \omega^2}$$

$$16) L(\sinh \omega t) = \frac{\omega}{s^2 - \omega^2}$$

$$17) L(\cosh \omega t) = \frac{s}{s^2 - \omega^2}$$

$$18) L(e^{at} \cosh \omega t) = \frac{s-a}{(s-a)^2 - \omega^2}$$

$$19) L(e^{at} \sinh \omega t) = \frac{\omega}{(s-a)^2 - \omega^2}$$

$$20) \mathcal{L}(t \cdot e^{at}) = \frac{1}{(s+a)^2}$$

$$21) \mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$

$$22) \mathcal{L}(t) = \frac{1}{s^2}$$

$$23) \mathcal{L}(t^2) = \frac{1}{s^{2+1}} = \frac{2}{s^3}$$

$$24) \mathcal{L}(t^3) = \frac{6}{s^4}$$

$$25) \mathcal{L}(e^{at}) = \frac{1}{s-a}$$

$$\mathcal{L}(f(t)) = s \cdot \mathcal{L}^2$$

$$s \mathcal{L}(t^2) = \frac{2}{s^3} \Rightarrow s \cdot \frac{2}{s^3} = \frac{10}{s^3}$$

(3)

$$\mathcal{L} (e^{-2t} \cos 3t + e^t \sin 4t)$$

$$= \frac{s+2}{(s+2)^2 + (3)^2} + \frac{4}{(s-1)^2 + (4)^2} \quad \underline{\underline{Q.D}}$$

$$\text{ex } \mathcal{L} \cos 3t = \frac{s}{s^2 + 9}$$

$$\text{ex } \mathcal{L} \sin 2t = \frac{2}{s^2 + (2)^2}$$

$$\text{ex } \mathcal{L} (e^{-at} \cdot t^2) = \frac{2}{(s+a)^3}$$

$$\text{ex } \mathcal{L} (e^{-at} \cdot t^3) = \frac{6}{(s+a)^4}$$

$$\begin{aligned} \mathcal{L} (e^{-3t} \cdot \cos 4t) &= \frac{s+a}{(s+a)^2 + \omega^2} \\ &= \frac{s+3}{(s+3)^2 + (4)^2} \end{aligned}$$

$$\begin{aligned} \mathcal{L} \cos &= \frac{s}{s^2 + \omega^2} \\ \mathcal{L} e^{-at} &= \frac{1}{s+a} \end{aligned}$$

Ex $S(t) = 1$

$$F(s) = \int_0^{\infty} f(t) \cdot e^{-st} dt$$

$$= \int_0^{\infty} 1 \cdot e^{-st} dt$$

$$= \left[-\frac{1}{s} e^{-st} \right]_0^{\infty}$$

$$= \left[-\frac{1}{s} \cdot e^{-s\infty} \right] - \left[-\frac{1}{s} \cdot e^{-s \cdot 0} \right]$$

$$= [0] - \left[-\frac{1}{s} \times 1 \right]$$

$$= \left[0 + \frac{1}{s} \right] \Rightarrow \frac{1}{s}$$

* ملاحظة

$$\int e^{at} \cdot dt = \frac{1}{a} e^{at}$$

$$\int e^{-st} \cdot dt = -\frac{1}{s} e^{-st}$$

imp $f(t) = 1 \rightarrow F(s) = \frac{1}{s}$

$$f(t) = \frac{1}{s} \rightarrow \underset{\substack{\uparrow \\ \text{laplace}}}{L} - S(s) = 1$$

* $e^{\infty} = 0$
 $e^0 = 1$

Ex $L(t^3) = L(t * t^2) = - \frac{dF(s)}{ds}$

$$= - \frac{d(2s^3)}{ds}$$

$$= - (2 * -3 s^{3-1}) = - (-6 s^{-4})$$

$$= \frac{6}{s^4}$$

* Imp

$$L(t) = \frac{1}{s^2}$$

$$L(t^2) = \frac{2}{s^3}$$

$$L(t^3) = \frac{6}{s^4}$$

$$L(t^n) = \frac{n!}{s^{n+1}}$$

Ex $L(t \cdot f(t)) = - \frac{d(F(s))}{ds}$ imp

$$f(t) = 1 \Rightarrow F(s) = \frac{1}{s}$$

$$L(t \underset{f(t)}{\ast} 1) = \frac{-d(F(s))}{ds}$$

$$\frac{ds^n}{ds} = n \cdot s^{n-1}$$

$$= \frac{-d(\frac{1}{s})}{ds} = -(-1 \cdot s^{-2})$$

$$= \frac{+1}{s^2}$$

Ex $L(t^2) = L(t \underset{f(t)}{\ast} t) = \frac{-d(F(s))}{ds}$

$$= \frac{-d(\frac{1}{s^2})}{ds} = \frac{-d(s^{-2})}{ds}$$

$$= -(-2 \cdot s^{-3})$$

$$= \frac{2}{s^3}$$

Ex $f(t) = e^{-at}$, $t > 0$

$$F(s) = \int_0^{\infty} f(t) \cdot e^{-st} dt$$

$$= \int_0^{\infty} e^{-at} \cdot e^{-st} dt$$

$$= \int_0^{\infty} e^{-(a+s)t} dt$$

$$= \left[\frac{1}{-(a+s)} e^{-st} \right]_0^{\infty}$$

$$= \left[\frac{1}{-(a+s)} e^{-st} \right]_0^{\infty}$$

$$= \left[\frac{1}{-(a+s)} \cdot e^{-s\infty} \right] - \left[\frac{1}{-(a+s)} \cdot e^{-s+0} \right]$$

$$= [0] - \left[\frac{1}{-(a+s)} \times 1 \right]$$

$$= \frac{1}{a+s}$$

*Imp

$$f(t) = 1 \rightarrow F(s) = \frac{1}{s}$$

$$f(t) = e^{-at} \rightarrow F(s) = \frac{1}{s+a}$$

$$f(t) = e^{-3t} \rightarrow F(s) = \frac{1}{s+3}$$

$$f(s) = e^{st} \rightarrow F(s) = \frac{1}{s-5}$$