

Compensation , phase–lag compensation lag–lead

part -1

Dr.Ziad Saeed Mohammed

The Compensation of the linear control systems

1. Introduction

1.1 definition of compensation

1.2 types of compensation

2. The basic controller operation analysis

2.1 phase-lead controller

2.2 phase-lag controller

2.3 phase lag-lead controller

} passive
compensation controller

2.4 PI D controller ---active compensation

3. Design compensation using method of Root loci

- We make the system stable by **increasing a component**.
- This procedure is called the **compensation** .

1.1 Definition of the compensation:

Increasing a component ,which makes the system's performance to be improved, other than only varying the system's parameters, this procedure is called the **compensation** of the system.

1. Introduction

Compensator:

The compensator is an additional component or circuit that is inserted into a control system to compensate for a deficient performance.

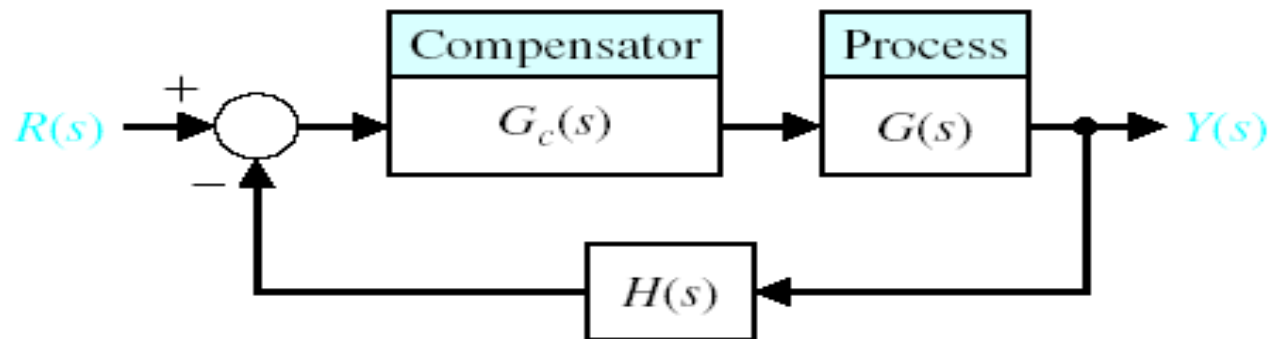
1.2 Types of the compensation

The transfer function of the compensator is designated as $G_c(s)$, and according to the location of $G_c(s)$ in the structure of the system, we can get several types :

1. Introduction

- (1) Cascade(or series) compensation
- (2) Feedback compensation
- (3) Both series and feedback compensation
- (4) Feed-forward compensation

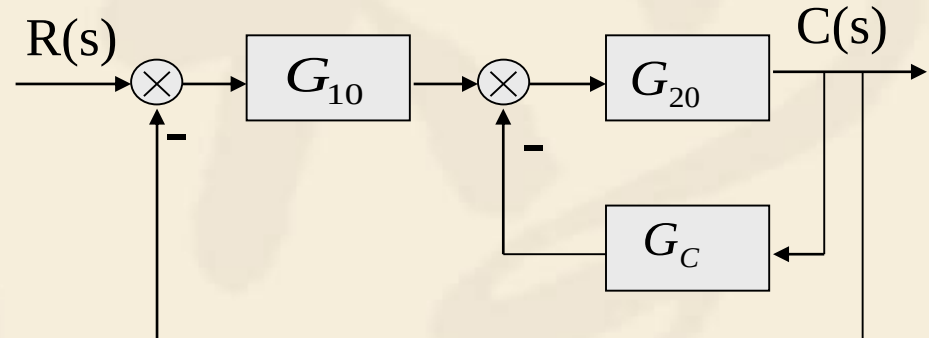
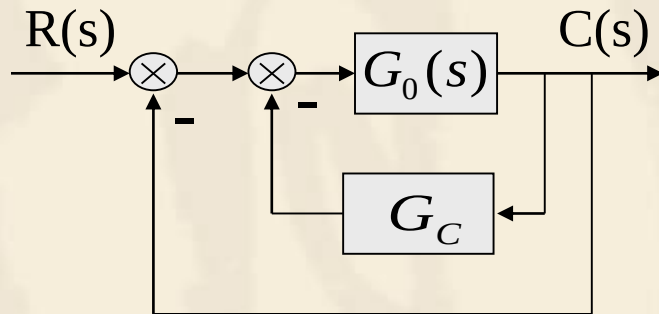
(1) Cascade(or series) compensation



Features : simple but the effects to be restricted.

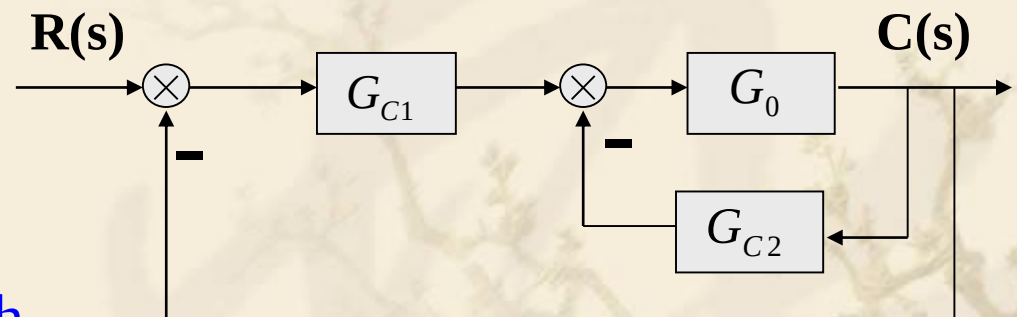
1. Introduction

(2) Feedback compensation



Features: complicated but noise limiting, the effects are more than the cascade compensation.

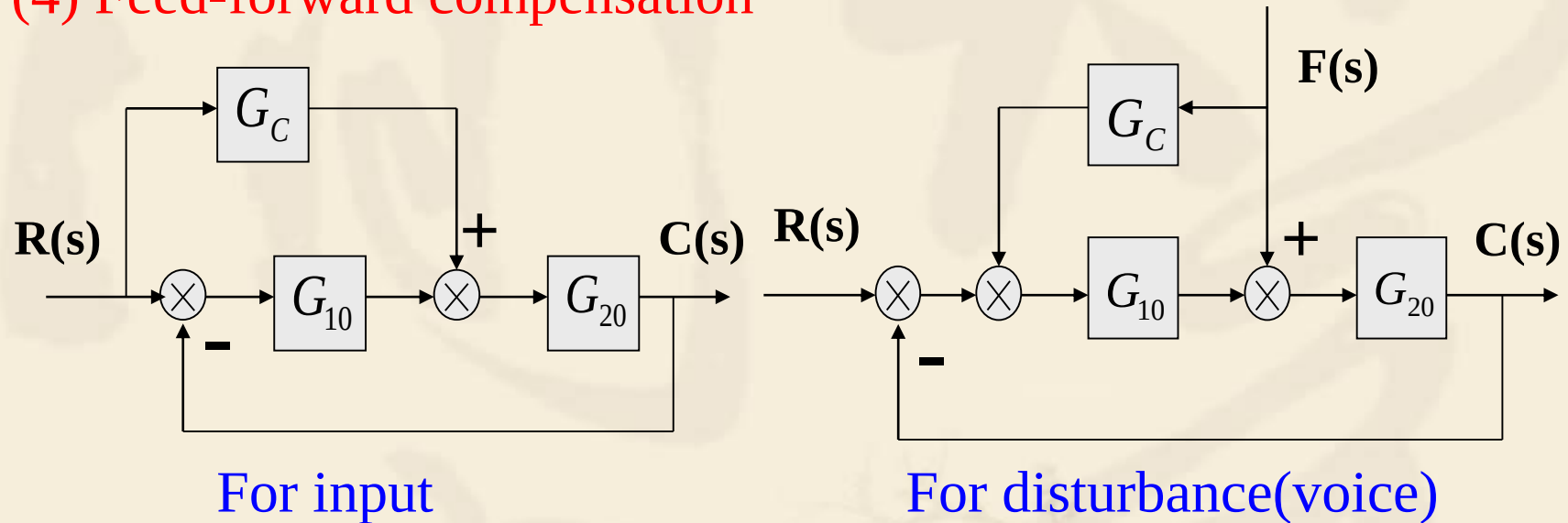
(3) Both cascade and feedback compensation



Features: have advantages both of cascade and feedback compensation.

1. Introduction

(4) Feed-forward compensation



Features: theoretically we can make the error of a system to be zero and no effects to the transient performance of the system.

compensation system classifications

Types

Using passive elements

Lag
comp.

Lead
comp.

Lag-Lead
comp.

Using active elements

P.

PI

PD-controller

PID

P = proportional controller

I = Integral

D = Differential

2.1 Passive compensation controllers

Types of passive compensation controller

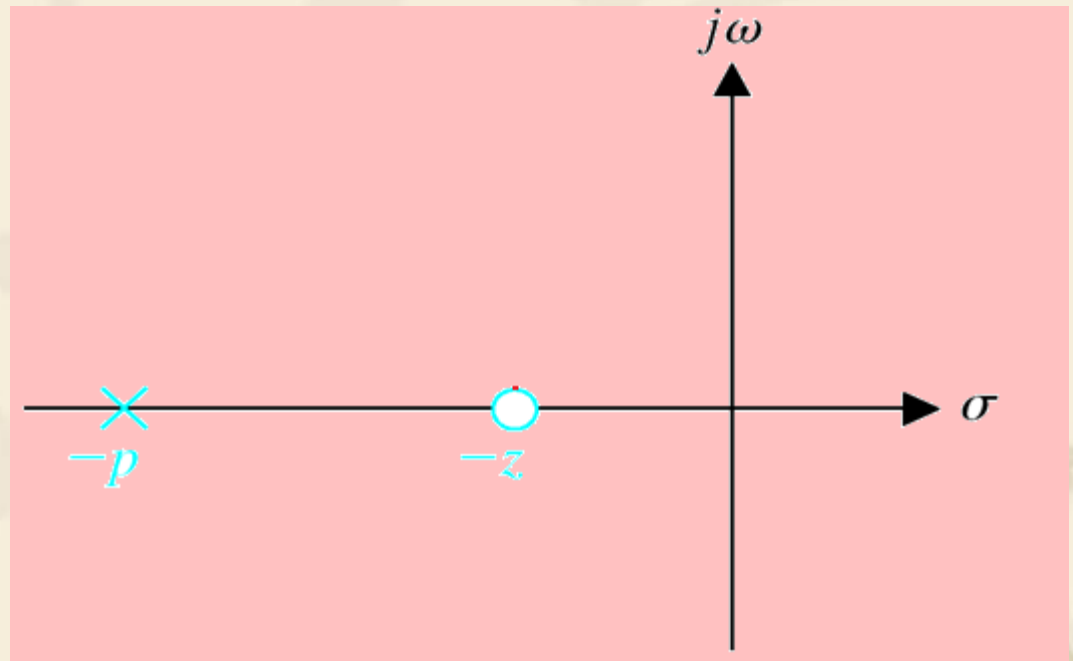
- | | |
|--------------------------------|--|
| 1) phase - lead controller | $G_c(s) = \frac{1 + \tau s}{1 + \alpha \tau s} \quad \alpha < 1$ |
| 2) phase - lag controller | $G_c(s) = \frac{1 + \tau s}{1 + \beta \tau s} \quad \beta > 1$ |
| 3) phase lag - lead controller | $G_c(s) = \frac{1 + \tau_b s}{1 + \beta \tau_b s} \cdot \frac{1 + \tau_a s}{1 + \alpha \tau_a s}$
$\alpha < 1, \quad \beta > 1 \quad \tau_b > \tau_a$ |

2.1 Passive compensation controllers

1) Phase - lead controller

Transfer function : $G_c(s) = \frac{1 + \tau s}{1 + \alpha \tau s} = \frac{1}{\alpha} \frac{s + z}{s + p} \quad \alpha < 1$

$$z = \frac{1}{\tau} \quad p = \frac{1}{\alpha \tau}$$

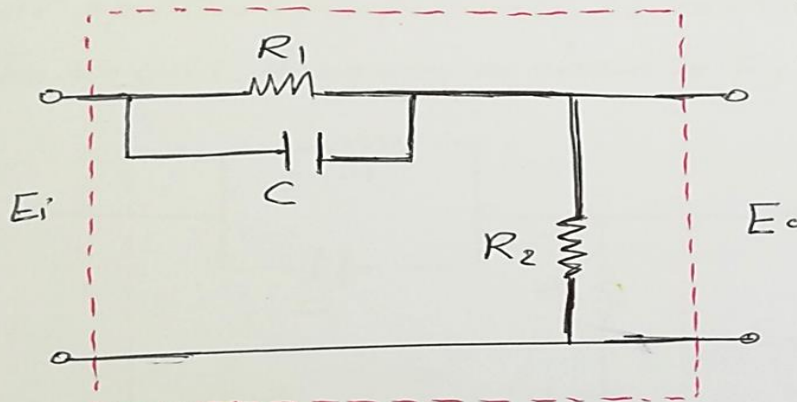


Zero and pole

Passive realization of compensators

Lead compensator

$G_c(s) = \frac{s+Z_o}{s+P_o}$, the circuit diagram as shown below:



Lead compensator circuit diagram

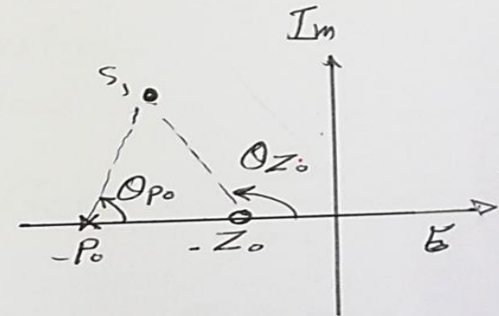
$$T.F = \frac{E_o}{E_i} = \frac{R_2}{R_2 + \frac{R_1 \cdot \frac{1}{Cs}}{R_1 + \frac{1}{Cs}}}$$

$$T.F = \frac{R_2(R_1Cs + 1)}{R_2(R_1Cs + 1) + R_1} = \left\{ G_c(s) = \frac{s + \frac{1}{\alpha\tau}}{s + \frac{1}{\tau}} \right\}$$

$$\text{where: } \alpha = 1 + \frac{R_1}{R_2}$$

$$\tau = \frac{R_1 R_2}{R_1 + R_2} C$$

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Lead-comp. (بصري)



$$P_o > Z_o$$

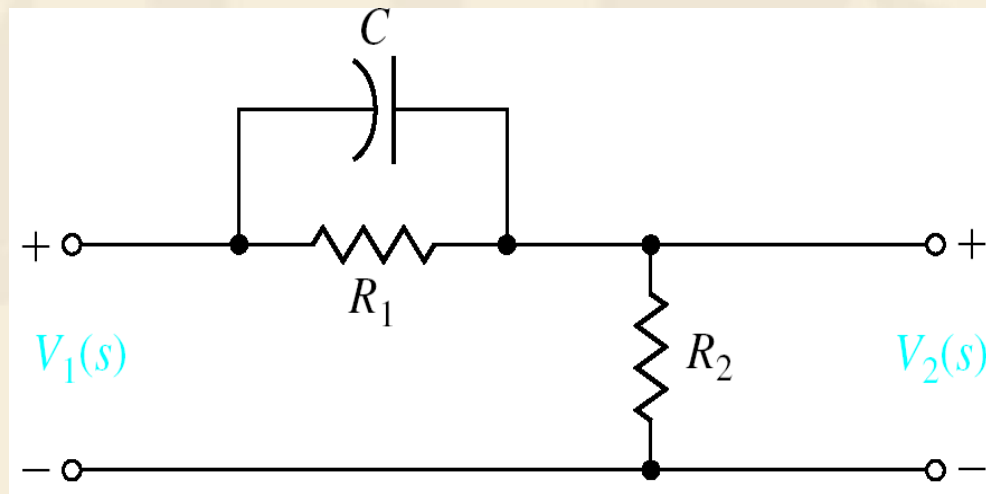
$$\theta_c = (\theta_{Z_o} - \theta_{P_o})$$

if = +ve = Lead comp.

Advantages

- * Reduce the steady-state error (ess)
- * Improve the transient response.

Circuit diagram of the phase-lead controller



$$G_c(s) = \frac{R_2}{R_1 + R_2} \frac{R_1 Cs + 1}{\frac{R_2}{R_1 + R_2} R_1 Cs + 1} = \alpha \frac{\tau s + 1}{\alpha \tau s + 1}$$

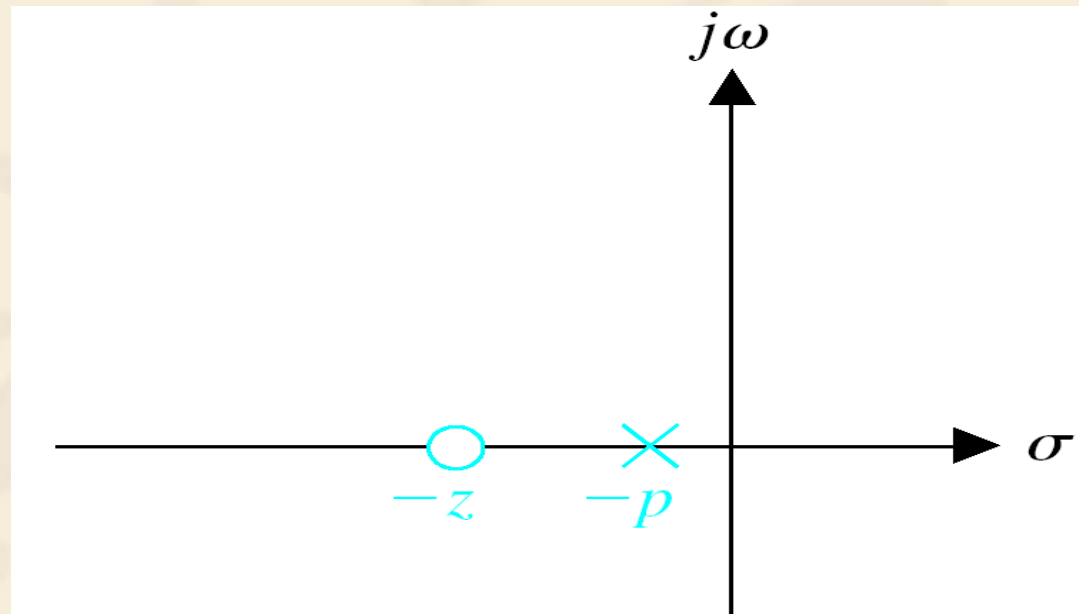
$$\tau = R_1 C \quad \alpha = \frac{R_2}{R_1 + R_2}$$

2.1 Passive compensation controllers

2) Phase - lag controller

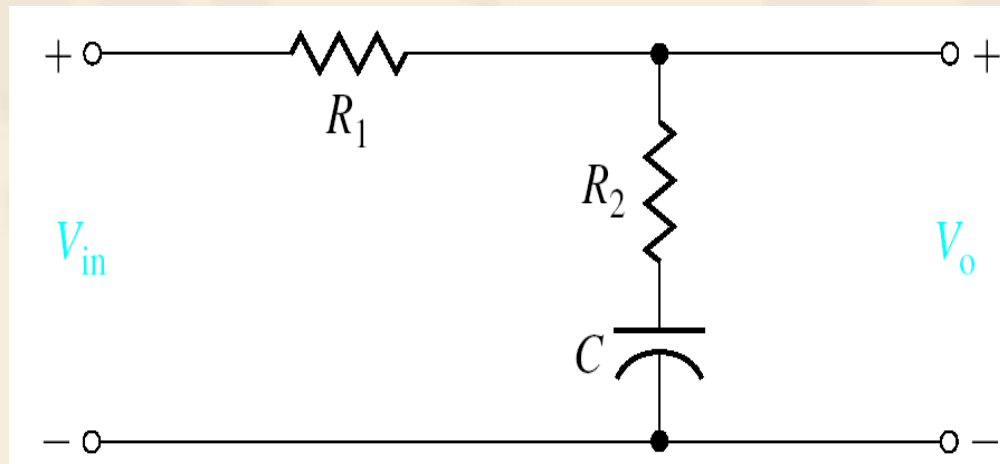
Transfer function : $G_c(s) = \frac{\tau s + 1}{\beta \tau s + 1} = \frac{1}{\beta} \cdot \frac{s + z}{s + p} \quad \beta > 1$

$$z = \frac{1}{\tau} \quad p = \frac{1}{\beta \tau}$$



Zero and pole

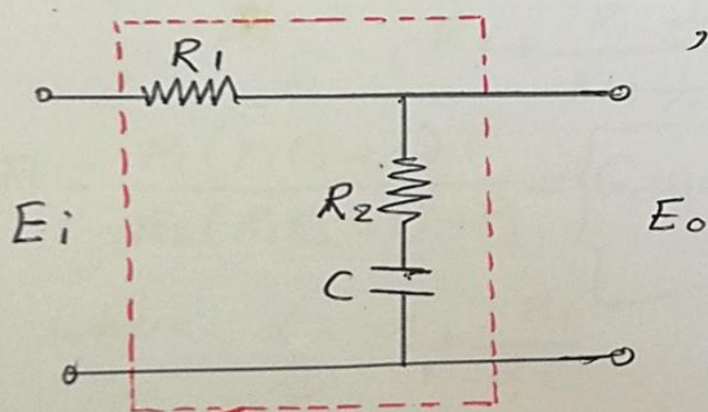
Circuit of the Phase-lag controller



$$G_c(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1 + \tau s}{1 + \beta \tau s} \quad \tau = R_2 C \quad \beta = \frac{R_1 + R_2}{R_2}$$

Lag compensator :- A First order compensator having a single Zero, and pole, the T.F is; $P = \text{Differential}$
 * الإخراج يتأخر عن الإدخال بزمن معين

$$G_c(s) = \frac{s + Z_0}{s + P_0}, \text{ The circuit diagram shown below}$$



Lag compensator circuit diagram.

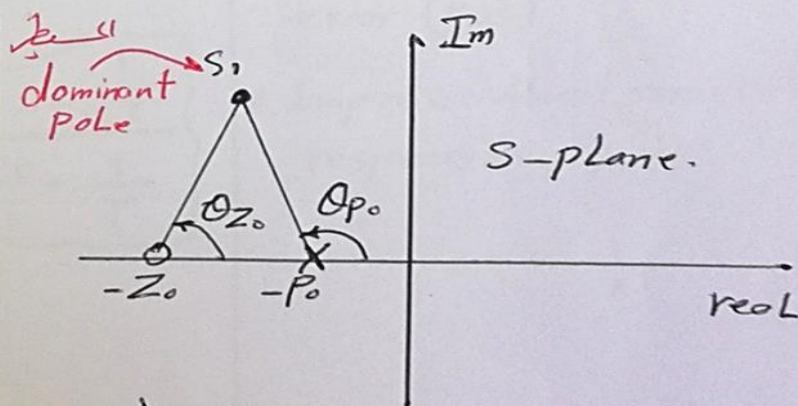
$$T.F = G_c(s) = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} = \frac{E_o}{E_i}$$

$$T.F = \frac{R_2Cs + 1}{[(R_1 + R_2)(Cs + 1)]} = \frac{\tau s + 1}{\alpha \tau s + 1}$$

$$\therefore G_c(s) = \frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha \tau}}, \text{ where } \tau = RC,$$

$$\alpha = 1 + \frac{R_1}{R_2}$$

The phase angle contribution, shown.



$$Z_0 > P_0$$

ϕ_c = compensator angle.

$$= (\phi_{Z_0} - \phi_{P_0})$$

= Negative $\Rightarrow$ Lag comp.

Advantages,

- * Improve the steady-state response.
- * Reduce the noise signals in high frequency.

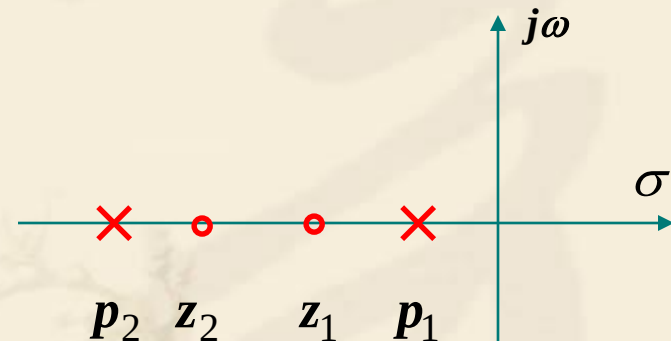
2.1 Passive compensation controllers

2) Phase lag - lead controller

$$\text{transfer function : } G_c(s) = \frac{1 + \tau_a s}{1 + \alpha \tau_a s} \cdot \frac{1 + \tau_b s}{1 + \beta \tau_b s} \quad \alpha < 1 \quad \beta > 1$$

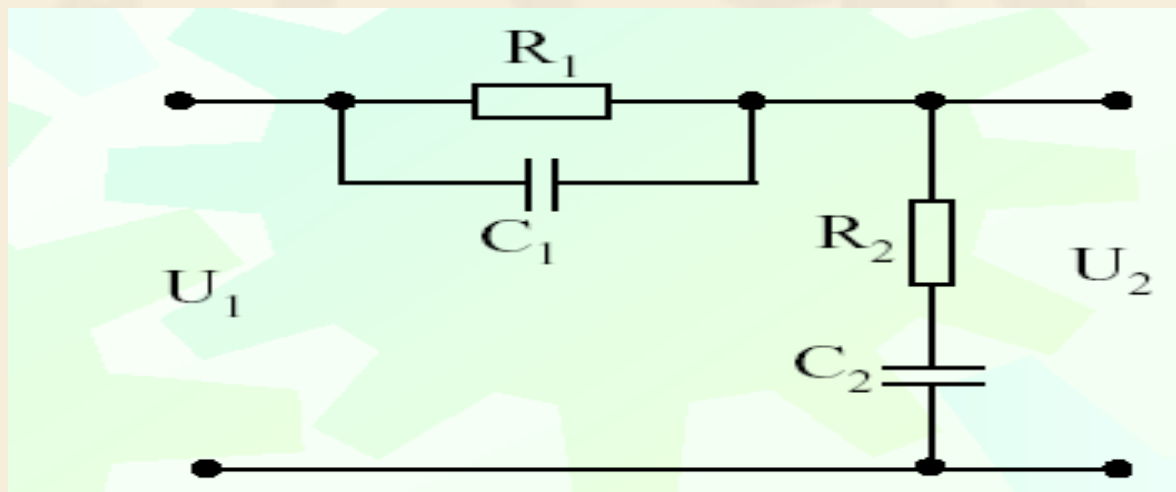
$$p_1 = -\frac{1}{\beta \tau_b} \quad z_1 = -\frac{1}{\tau_b}$$

$$p_2 = -\frac{1}{\alpha \tau_a} \quad z_2 = -\frac{1}{\tau_a}$$



Zero and pole

Circuit of the Phase lag-lead controller



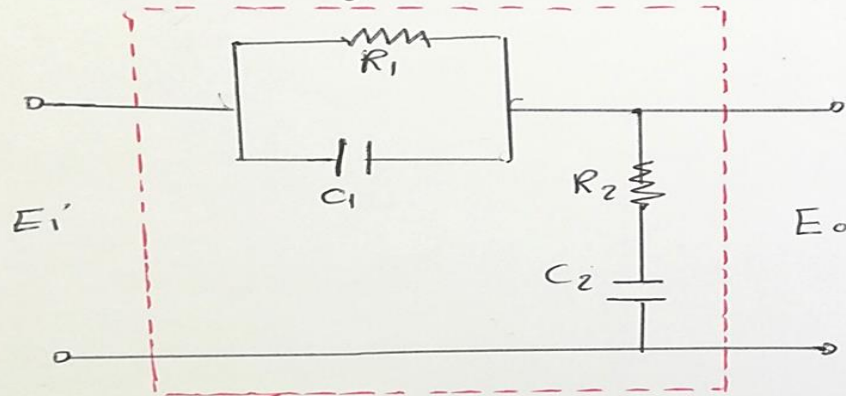
$$G_c(s) = \frac{(1 + T_a s)(1 + T_b s)}{T_a T_b s^2 + (T_a + T_b + T_{ab})s + 1}$$

$$T_a = R_1 C_1, \quad T_b = R_2 C_2, \quad T_{ab} = R_1 C_2$$

$$G_c(s) = \frac{(1 + T_a s)(1 + T_b s)}{(1 + a T_a s)(1 + \frac{T_b}{a} s)}$$

$$\alpha < 1$$

The Lead-Lag Compensator, Its the combination circuit which contain a Lag-comp. with Lead comp. to give a total or optimal performance to control system.
The circuit diagram as shown in Fig. below.



$$G_C(s) = \frac{E_o}{E_i}$$

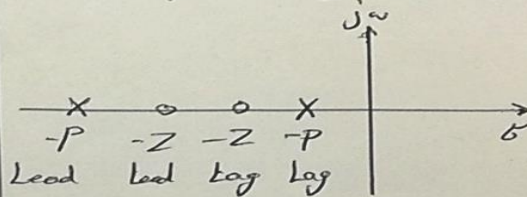
$$G_C(s) = \frac{R_2 + \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s} + \frac{R_1 \cdot \frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}}}$$

by simplification:

$$\therefore G_C(s) = \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{(R_1 C_1 s + 1)(R_2 C_2 s + 1) + R_1 C_2 s} \quad \text{and}$$

$$G_C(s) = \frac{(s + Z_{\text{Lag}})(s + Z_{\text{Lead}})}{(s + P_{\text{Lag}})(s + P_{\text{Lead}})}$$

The poles & zeros on the s-plane are shown in Fig. below



Advantages

Improve the transient and the steady-state response.

PID Controllers

PID has become almost universally used in
industrial control.

These controllers have proven to be robust and extremely
beneficial in the control of many important applications.

Proportional only:

$$G_p(s) = K_p$$

Proportional plus Integral:

$$G_{PI}(s) = K_p + \frac{K_i}{s}$$

Proportional plus derivative:

$$G_{PD} = K_p + K_d s$$

Proportional, integral and
derivative:

$$G_{PID}(s) = K_p + \frac{K_i}{s} + K_d s$$

2. Operation analysis of the basic compensators

2.2 Active Compensation

PID controller — active “compensator”.

Transfer function:

$$G_c(s) = K_p \left(1 + \frac{1}{\tau_I s} + \tau_D s \right)$$

$$= K_p + K_I \frac{1}{s} + K_D s \rightarrow K_I = \frac{K_p}{\tau_I}; K_D = K_p \tau_D$$

P -- proportion al controller → promoting sensitivity.

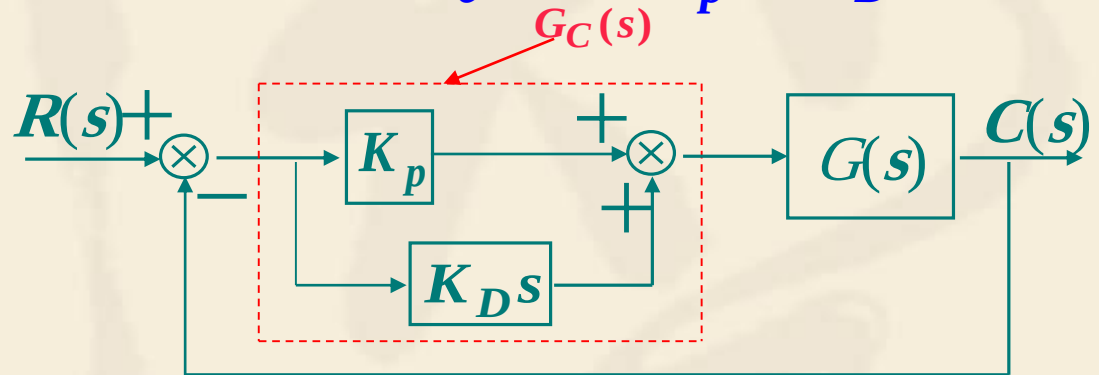
I -- integratin g controller → clearing e_{ss} .

D -- differenti al controller → improving stability.

PD controller transfer function : $G_c(s) = K_p + K_D s$

Assuming :

$$G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)}$$



The open – loop transfer function of the compensated system is : $G_c(s)G(s) = \frac{\omega_n^2 (K_p + K_D s)}{s(s + 2\xi\omega_n)}$

It shows that the PD controller is equivalent to adding a open – loop zero at : $s = -\frac{K_p}{K_D}$

2 . Operation analysis of the basic compensators

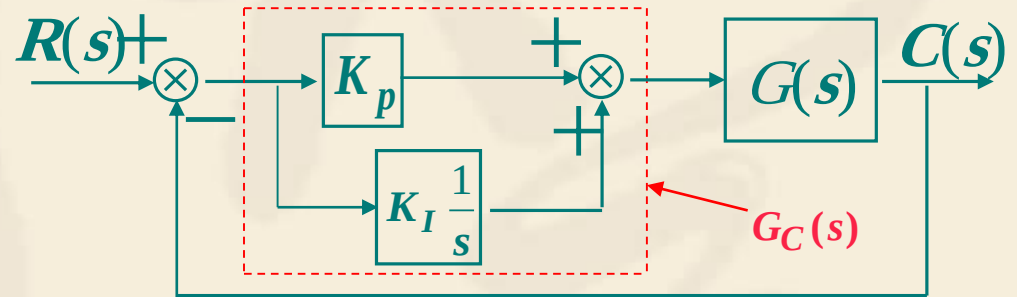
Effects of PD controller:

- 1) PD controller does not alter the system type;
- 2) PD controller improve the system's stability (to increase damping and reduce maximum overshoot);
- 3) PD controller reduce the rise time and settling time;
- 4) PD controller increase BW (Band Width) and improve $GM(K_g)$, $PM(\gamma_c)$, and M_r .
— bring in the noise !

2. Operation analysis of the basic compensators

PI controller Transfer function : $G_c(s) = K_p + K_I \frac{1}{s}$

Assuming : $G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)}$



The open – loop transfer function of the compensated system is :

$$G_c(s)G(s) = \frac{\omega_n^2 (K_p + K_I \frac{1}{s})}{s(s + 2\xi\omega_n)} = \frac{\omega_n^2 (K_p s + K_I)}{s^2 (s + 2\xi\omega_n)}$$

It shows that the PI controller is equivalent to adding a open – loop

zero at : $s = -\frac{K_I}{K_p}$ and a pole at : $s = 0$

2. Operation analysis of the basic compensators

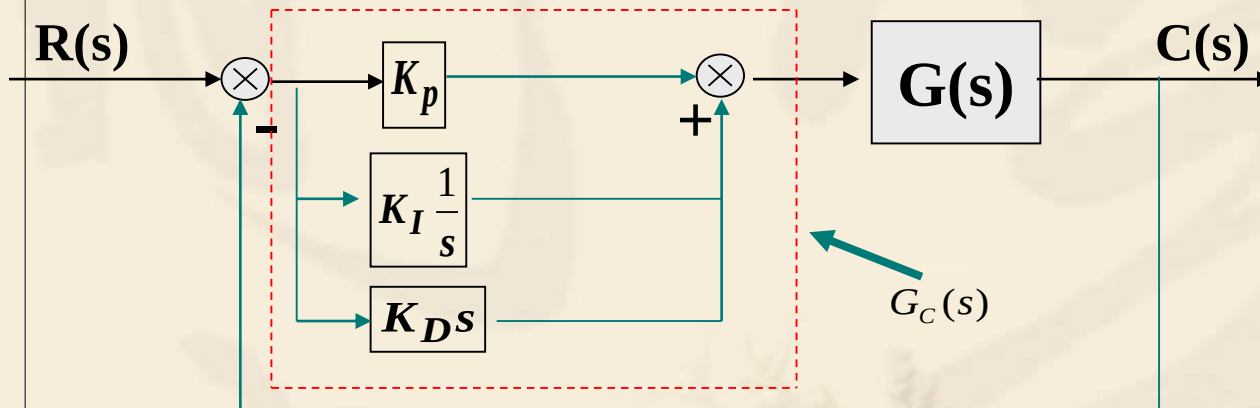
Effects of PI controller:

- 1) Increase the system's type — clear the steady-state error ;
- 2) reduce BW (Band Width) and $GM(K_g)$, $PM(\gamma_c)$ and M_r ;

beneficial to the noise limiting ,
not beneficial to the system's stability.

2. Operation analysis of the basic compensators

PID controller Transfer function: $G_c(s) = K_p + K_I \frac{1}{s} + K_D s$

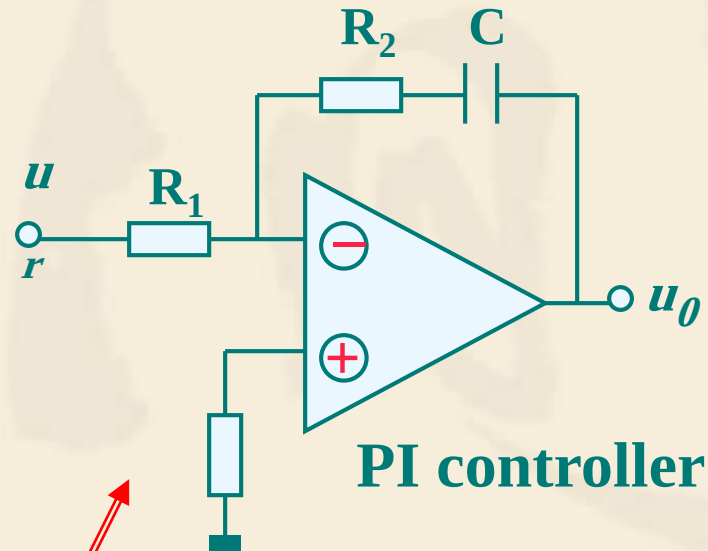


PID controller have advantages both of PI and PD.



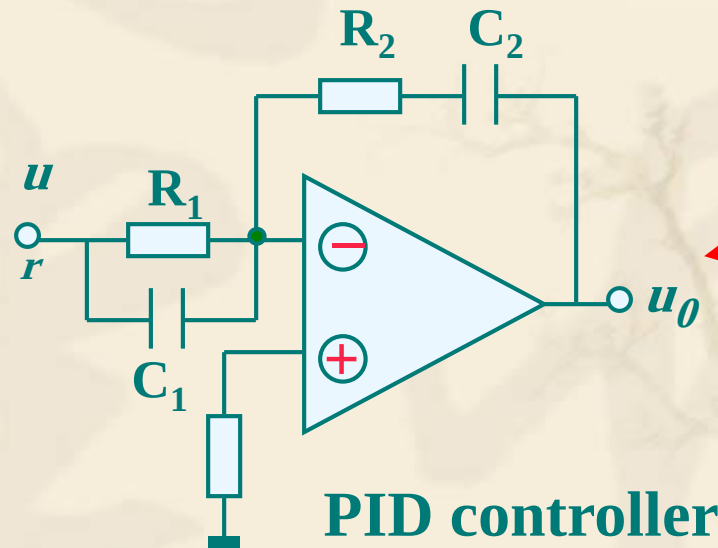
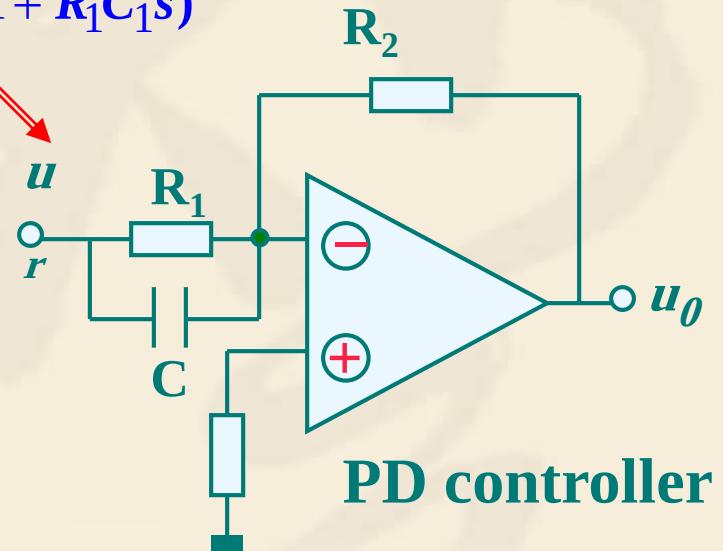
Active realization of compensators

Circuits of PID



$$\frac{U_0(s)}{U_R(s)} = \frac{R_2}{R_1} \left(1 + \frac{1}{R_2 C s} \right)$$

$$\frac{U_0(s)}{U_R(s)} = \frac{R_2}{R_1} (1 + R_1 C_1 s)$$



$$\frac{U_0(s)}{U_R(s)} = ?$$

2.3 Comparing active compensation controllers and passive compensation controllers

$$\text{PD : } K_p(1 + \tau_D s) \quad \rightarrow \text{phase - lead : } \frac{\tau s + 1}{\alpha \tau s + 1}$$

$$\text{PI : } \begin{aligned} & K_p \left(1 + \frac{1}{\tau_I s}\right) \\ &= \frac{K_p(\tau_I s + 1)}{\tau_I s} \end{aligned} \quad \rightarrow \text{phase - lag : } \frac{\tau s + 1}{\beta \tau s + 1}$$

$$\text{PID : } \begin{aligned} & K_p \left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \\ &= \frac{K_p(\tau_I \tau_D s^2 + \tau_I s + 1)}{\tau_I s} \end{aligned} \quad \rightarrow \text{phase lag - lead : } \frac{(\tau_a s + 1)(\tau_b s + 1)}{(\alpha \tau_a s + 1)(\tau_b s / \alpha + 1)}$$



THANKS