



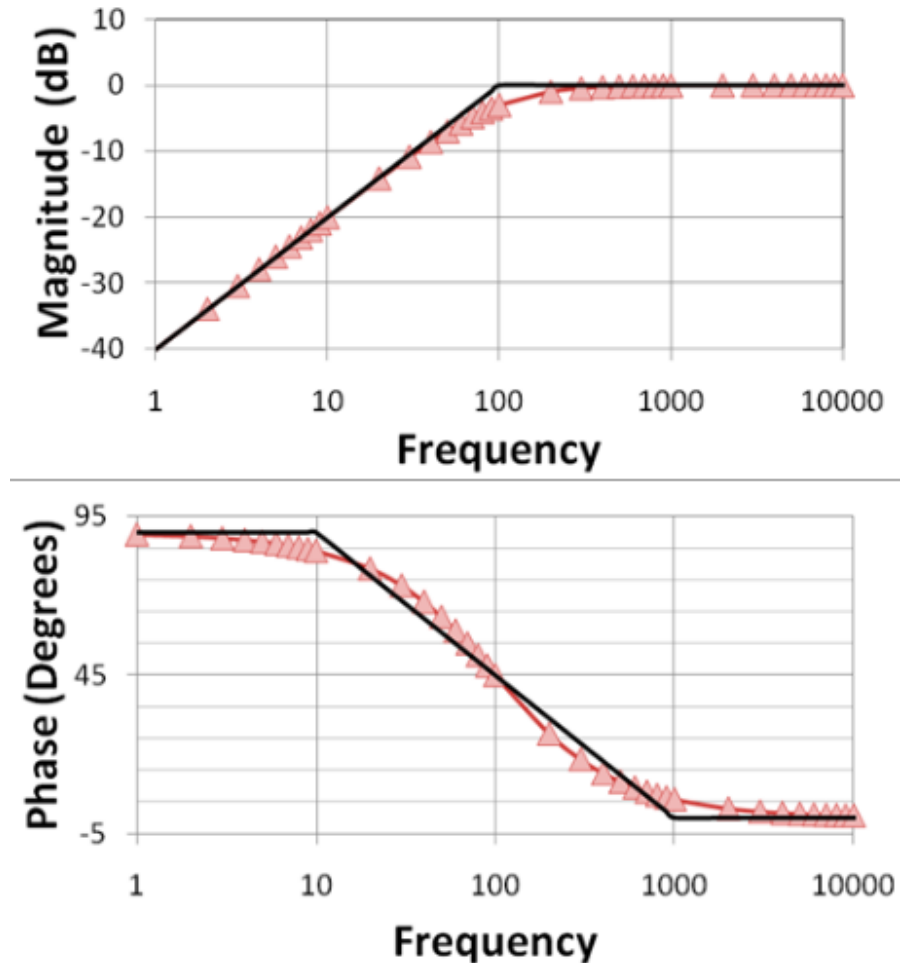
Bode Plot Diagram

what is bode plot diagram?

2024-2025

What is bode plot diagram?

- A Bode plot is *a graphical method* used to analyze the *frequency response* of a linear, time-invariant system.
- Bode plots are particularly useful in control system design and stability analysis.
- It consists of two plots:
 - **Magnitude Plot**
 - **Phase Plot**



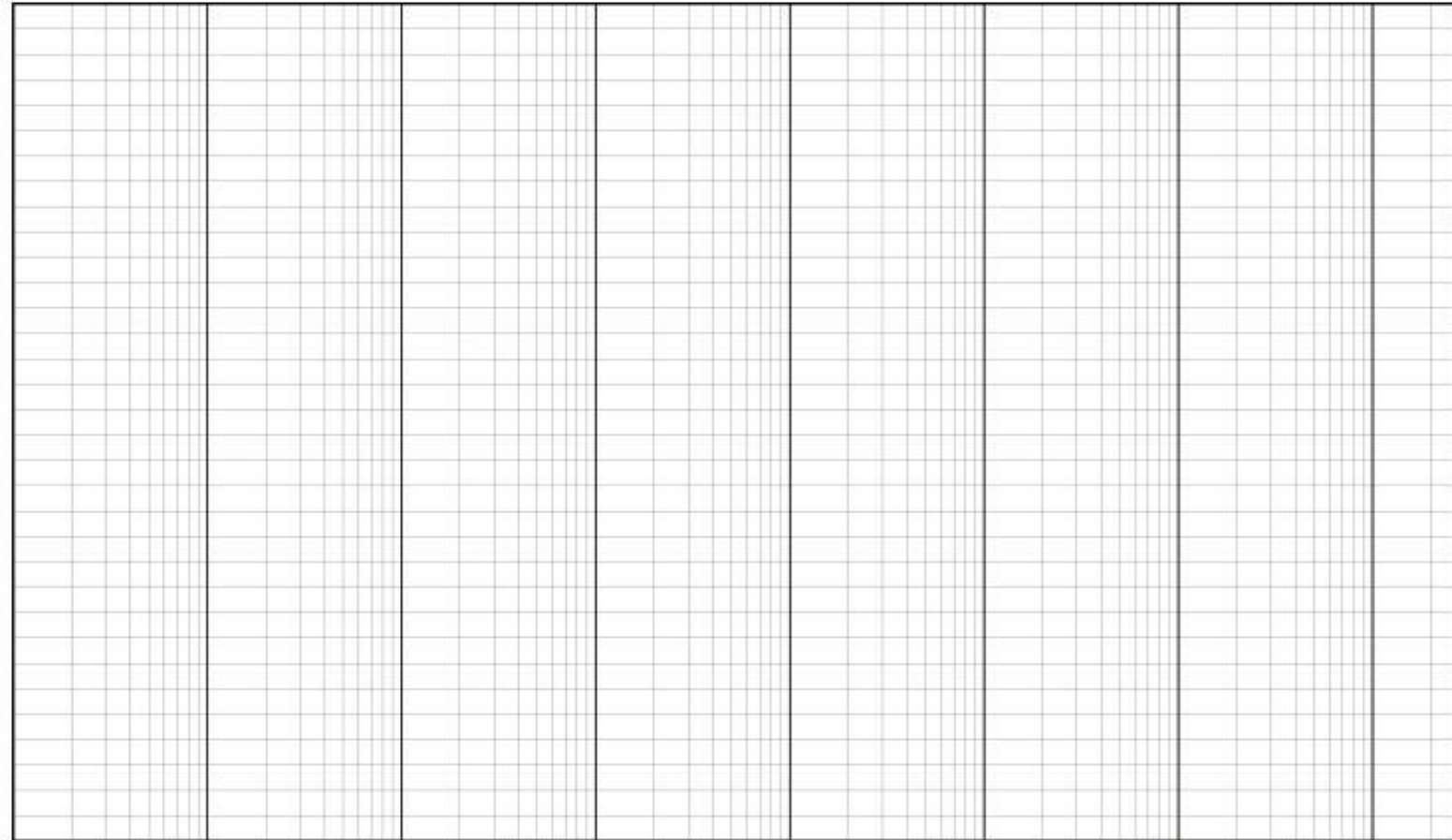
Semi log Graph Paper

Decibel

Decade

Octave

Octave



Semilog Paper

PDF Library — http://www.xuebrothers.net/pdf_lib/index.htm

Decade

Octave

Decibel

Decipol

- A **Bode plot** is a (semilog) plot of the transfer function magnitude and phase angle as a function of frequency
- The gain magnitude is many times expressed in terms of decibels (dB)

$$\text{dB} = 20 \log_{10} A$$

where A is the amplitude or gain

- a *decade* is defined as any 10-to-1 frequency range
- an *octave* is any 2-to-1 frequency range

$$20 \text{ dB/decade} = 6 \text{ dB/octave}$$

Magnitude Plot

□ Magnitude Plot *shows*:

The magnitude (or gain) of the system's frequency response on a logarithmic scale.

The x-axis represents the frequency, typically in logarithmic scale,

The y-axis represents the gain, often in decibels (dB).

□ The magnitude plot *helps*:

– in understanding how the gain of the system varies with frequency.

Phase Plot

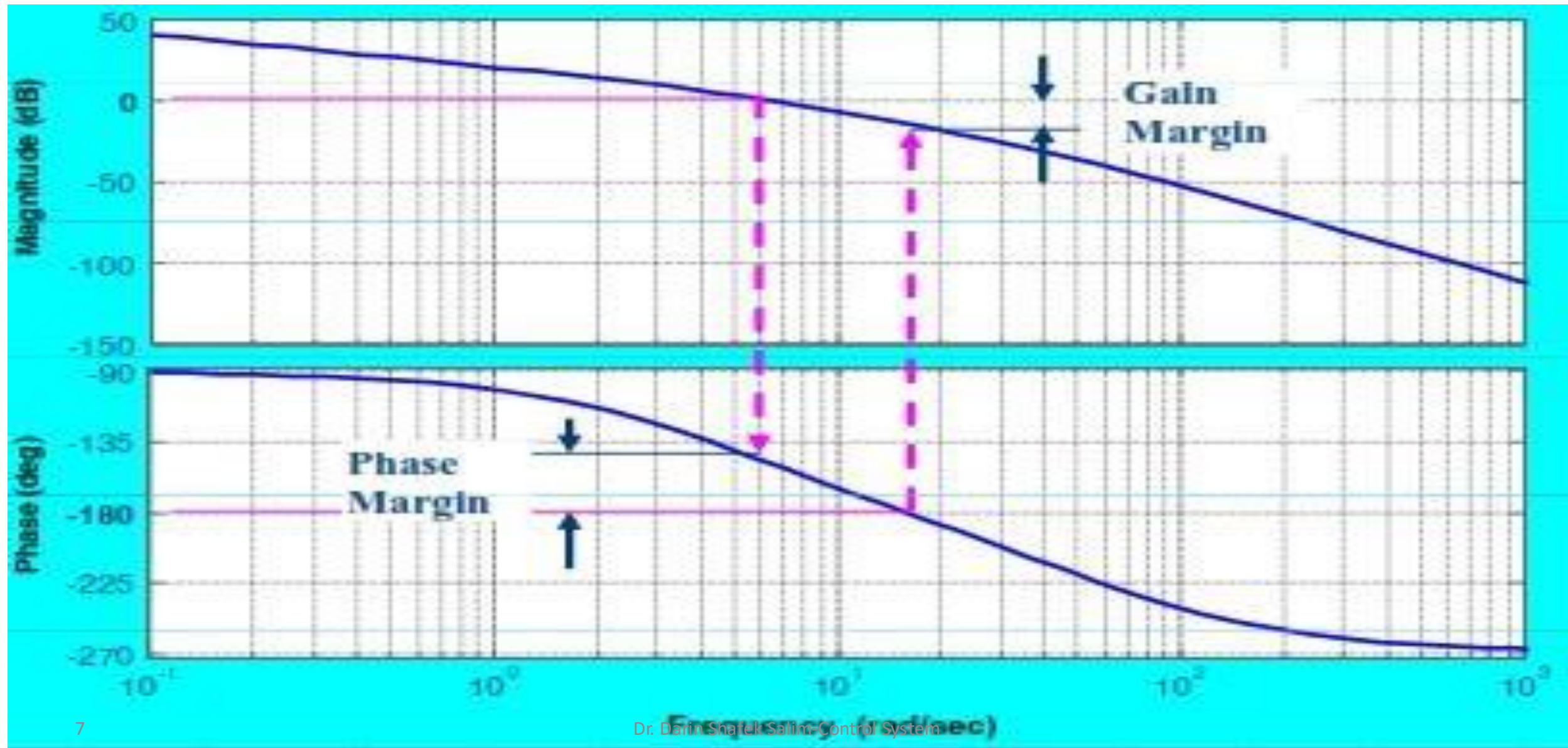
□ Phase Plot *shows*:

- The phase shift of the system's frequency response.
- The x-axis again represents the frequency on a logarithmic scale, and
- The y-axis represents the phase shift in degrees.

□ The phase plot *provides*:

- insight into how the phase of the output signal is shifted relative to the input signal at different frequencies.

Bode Plot



Example

MATLAB Code

```
% Frequency range
omega = logspace(-1, 2, 500);
tau = 1;
% Magnitude and Phase
magnitude = 20 * log10(1 ./ sqrt((tau * omega).^2 + 1));
phase = -atan(tau * omega) * (180 / pi);
% Plotting
```

% Magnitude plot

```
subplot(2, 1, 1);
semilogx(omega, magnitude);
title('Bode Plot of First-Order Low-Pass Filter');
ylabel('Magnitude (dB)');
grid on;
```

% Phase plot

```
subplot(2, 1, 2);
semilogx(omega, phase);
xlabel('Frequency (rad/s)');
ylabel('Phase (degrees)');
grid on;
```

where:

- τ is the time constant of the system.
- s is the complex frequency variable.

For this example, let's assume $\tau = 1$ second.

$$H(s) = \frac{1}{\tau s + 1}$$

Bode Plot Methods

$$H(s)G(s) = \frac{(s+2)(s+6)}{s(s+1)(s+5)(s+10)}$$

- هناك طريقتان لرسم البود بلوت :
- الطريقة الاولى: نأخذ دالة انتقال الدارة المفتوحة OPEN LOOP T.F=GH ونطبق الخطوات الآتية:
- نعوض عن كل s ب $j\omega$.
- نحسب $Lm=20\log GH$ ويقاس ب db .
- نحسب $\Phi = \text{زاوية } GH$ وتقاس ب degree .
- نعمل جدول بين ω, Lm, Φ حيث نعطي قيم ω ونحسب Lm, Φ
- الخطوة الأخيرة نرسم البود بلوت باستخدام ورقة بيانية تسمى Semi log graph paper الموضحة في الشريحة اللاحقة.

Bode Plot Methods

- الطريقة الثانية: نأخذ كل حد من حدود الـ GH ونرسم له بود بلوت.
- نجمع كل الرسومات ونحصل على الرسم النهائي او الكلي للبودبلوت .
- وكما موضح بالشريحة القادمة.

$$H(s) G(s) = \frac{(s + 2)(s + 6)}{s(s + 1)(s + 5)(s + 10)}$$

Note

If the *constant 'k'* is *positive*, then what would be its contribution on the phase plot?

1

BODE PLOT

Gain Factor K.

- The gain factor multiplies the overall gain by a constant value for all frequencies.
- It has no effect on phase.

$$G(s) = K$$

$$G(j\omega) = K$$

$$\begin{aligned} M &= 20\log|G(j\omega)| \\ &= 20\log(K) \text{ [dB]} \end{aligned}$$

$$\phi = 0$$



M : magnitude, ϕ : phase.

Note

If *a Zero* is located at *origin*, how does it get represented on the magnitude plot?

2

BODE PLOT

Derivative Factor $j\omega$ – zero at the origin.

- Magnitude is a straight line with a slope of 20 dB/decade becoming zero at $\omega=1$ [rad/s].
- Phase is constant at 90° at all frequencies.

$$G(s) = s, \quad G(j\omega) = \omega j$$

$$M = 20 \log |G(j\omega)| \\ = 20 \log(\omega)$$

$$\phi = 90^\circ$$



Not

e

If a *Zero is s^2* , how does it get represented on the magnitude plot?

3

BODE PLOT

Double zero at the origin.

- Simply double the slope of the magnitude and the phase, i.e., 40 dB/decade becoming zero at $\omega=1$ [rad/s] and 180° phase.

$$G(s) = s^2, \quad G(j\omega) = -\omega^2$$

$$M = 20\log|G(j\omega)| \\ = 40\log(\omega)$$

$$\phi = 180^\circ$$



Note

If *a pole* is located at *origin*, how does it get represented on the magnitude plot?

4

BODE PLOT

Integral Factor $1/j\omega$ – pole at the origin.

- **Magnitude is a straight line with a slope of -20 dB/decade becoming zero at $\omega=1$ [rad/s].**
- **Phase is constant at -90° at all frequencies.**

$$G(s) = \frac{1}{s}, \quad G(j\omega) = \frac{1}{j\omega} = -\frac{1}{\omega}j$$

$$M = 20\log|G(j\omega)|$$
$$= 20\log\left(\frac{1}{\omega}\right) = -20\log\omega$$

$$\phi = -90^\circ$$



Note

If a pole is $1/s^2$, how does it get represented on the magnitude plot?

5

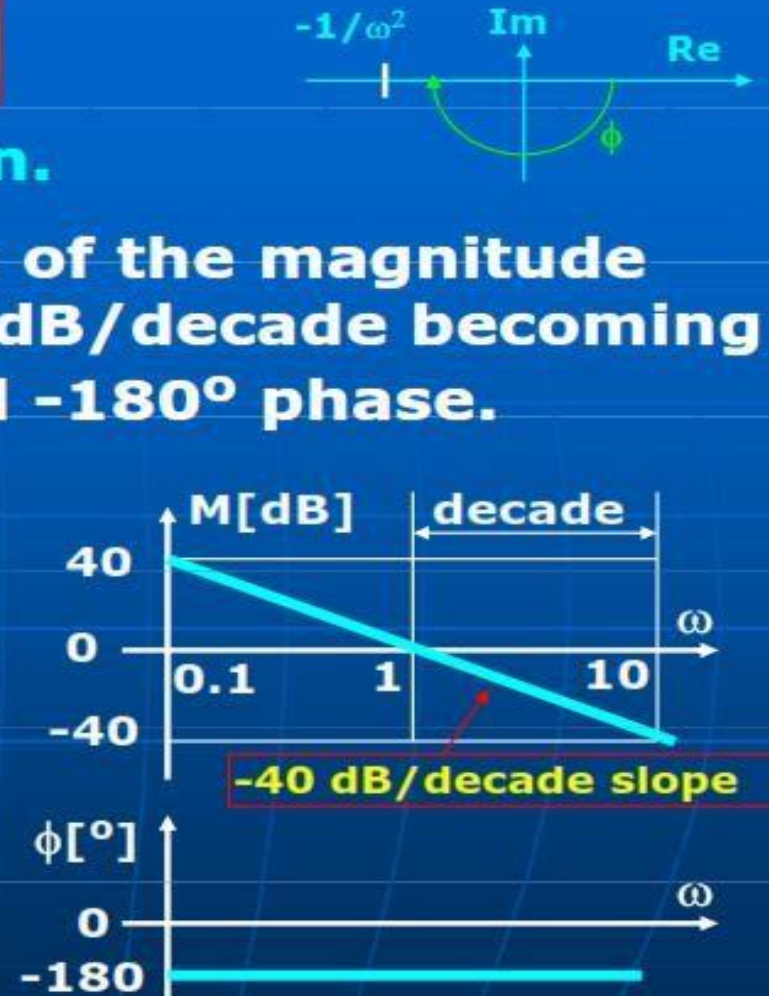
BODE PLOT

Double pole at the origin.

- Simply double the slope of the magnitude and the phase, i.e., -40 dB/decade becoming zero at $\omega=1$ [rad/s] and -180° phase.

$$G(s) = \frac{1}{s^2}, \quad G(j\omega) = \frac{1}{(j\omega)^2} = -\frac{1}{\omega^2}$$

$$\begin{aligned} M &= 20\log|G(j\omega)| \\ &= 20\log\left(\frac{1}{\omega^2}\right) = -40\log\omega \\ \phi &= -180^\circ \end{aligned}$$



Example 1: Bode plot Diagram

Draw the Bode plot of

$$H(s) G(s) = \frac{(s + 2)(s + 6)}{s(s + 1)(s + 5)(s + 10)}$$

Step 1:

Step 1. Rewrite $H(s) G(s)$ and replace s by $j\omega$

$$H(j\omega) G(j\omega) = \frac{(j\omega + 2)(j\omega + 6)}{j\omega (j\omega + 1)(j\omega + 5)(j\omega + 10)}$$

Next,

$$H(j\omega) G(j\omega) = \frac{2 \times 6}{1 \times 5 \times 10} \times \frac{(j\frac{\omega}{2} + 1)(j\frac{\omega}{6} + 1)}{j\omega (j\frac{\omega}{1} + 1)(j\frac{\omega}{5} + 1)(j\frac{\omega}{10} + 1)}$$

Thus,

$$H(j\omega) G(j\omega) = 0.24 \times \frac{(j\frac{\omega}{2} + 1)(j\frac{\omega}{6} + 1)}{j\omega (j\frac{\omega}{1} + 1)(j\frac{\omega}{5} + 1)(j\frac{\omega}{10} + 1)}$$

$$H(j\omega) G(j\omega) = 0.24 \times \frac{(j\frac{\omega}{2} + 1)(j\frac{\omega}{6} + 1)}{j\omega (j\frac{\omega}{1} + 1)(j\frac{\omega}{5} + 1)(j\frac{\omega}{10} + 1)}$$

Step 2. Draw table as below

Factor	Type	(Corner/Break) frequency	Slope
$(j\omega)^1$	pole	-	-20×1
$(j\frac{\omega}{2} + 1)$	zero	2	+20
$(j\frac{\omega}{6} + 1)$	zero	6	+20
$(j\frac{\omega}{1} + 1)$	pole	1	-20
$(j\frac{\omega}{5} + 1)$	pole	5	-20
$(j\frac{\omega}{10} + 1)$	pole	10	-20

Step 3 and 4:

Factor	Type	(Corner/Break) frequency	Slope
$(j\omega)^1$	pole	-	-20×1 (Starting slope)
$(j\frac{\omega}{2} + 1)$	zero	2	+20
$(j\frac{\omega}{6} + 1)$	zero	6	+20
$(j\frac{\omega}{1} + 1)$	pole	1	-20
$(j\frac{\omega}{5} + 1)$	pole	5	-20
$(j\frac{\omega}{10} + 1)$	pole	10	-20

Step 3. Starting frequency

$$\omega_{st} = \frac{\text{Least value of corner frequency}}{10} = \frac{1}{10} = 0.1$$

Step 4. Starting magnitude

$$\begin{aligned} &= 20 \log(K) + 20 \times (k_2 - k_1) \log(\omega_{st}) \\ &= 20 \log(0.24) - 20 \times 1 \times \log(0.1) = 7.6 \text{ dB} \end{aligned}$$

Step 5:

$$H(j\omega) G(j\omega) = 0.24 \times \frac{(j\frac{\omega}{2} + 1)(j\frac{\omega}{6} + 1)}{j\omega (j\frac{\omega}{1} + 1)(j\frac{\omega}{5} + 1)(j\frac{\omega}{10} + 1)}$$

Step 5. Phase angle

$$\varphi = \tan^{-1} \left(\frac{0}{K} \right) - \tan^{-1} \left(\frac{\omega}{0} \right) + \tan^{-1} \left(\frac{\omega}{2} \right) + \tan^{-1} \left(\frac{\omega}{6} \right) - \tan^{-1} \left(\frac{\omega}{1} \right) - \tan^{-1} \left(\frac{\omega}{5} \right) - \tan^{-1} \left(\frac{\omega}{10} \right)$$

$$\varphi = 0 - 90 + \tan^{-1} \left(\frac{\omega}{2} \right) + \tan^{-1} \left(\frac{\omega}{6} \right) - \tan^{-1} \left(\frac{\omega}{1} \right) - \tan^{-1} \left(\frac{\omega}{5} \right) - \tan^{-1} \left(\frac{\omega}{10} \right)$$

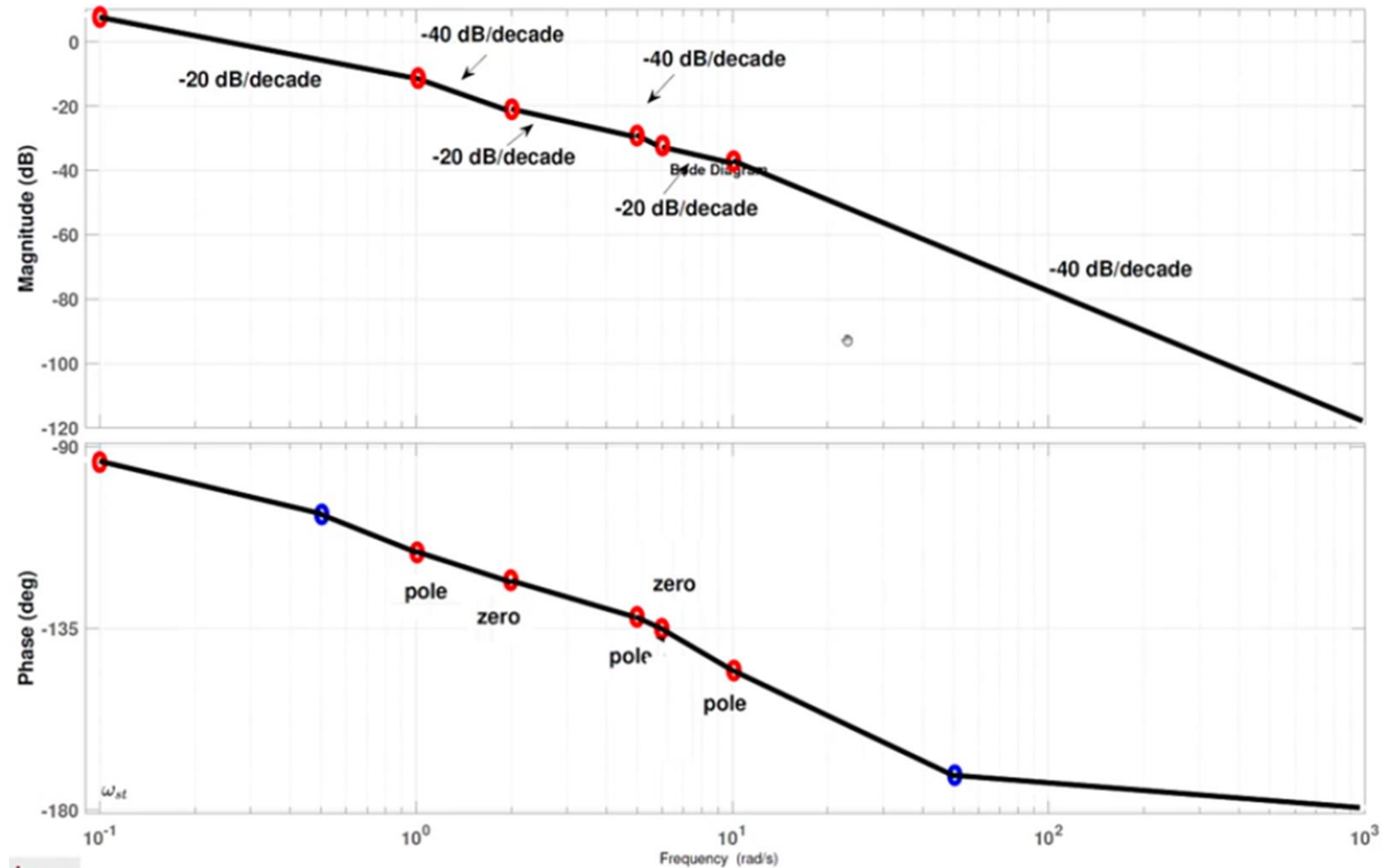
$$\varphi = -90 + \tan^{-1} \left(\frac{\omega}{2} \right) + \tan^{-1} \left(\frac{\omega}{6} \right) - \tan^{-1} \left(\frac{\omega}{1} \right) - \tan^{-1} \left(\frac{\omega}{5} \right) - \tan^{-1} \left(\frac{\omega}{10} \right)$$

ω (rad/sec)	0.1	0.5	1	2	5	6	10	50
φ (deg)	-93.6	-106	-116	-123	-132	-135	-145	-171

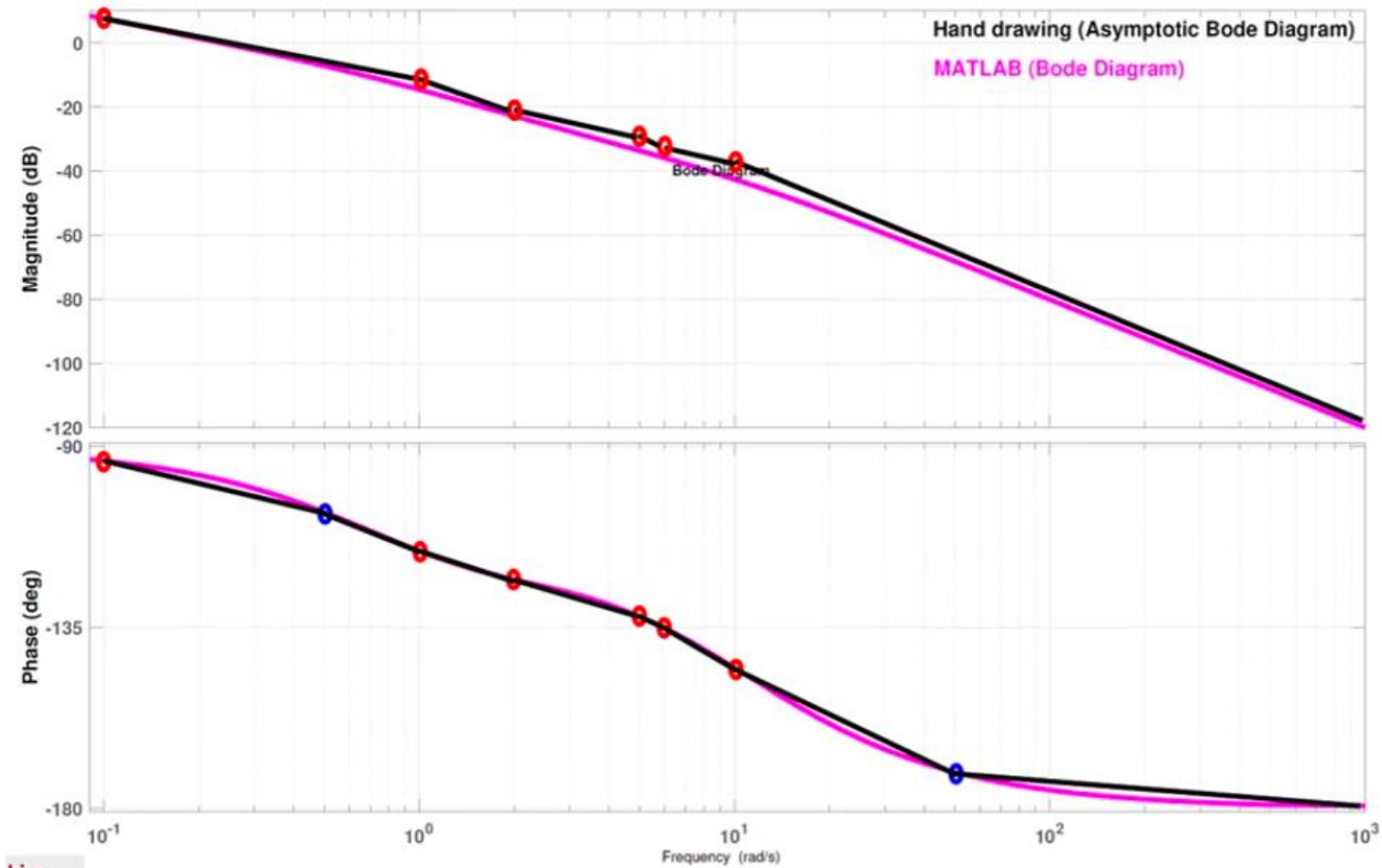
Factor	Type	(Corner/Break) frequency	Slope
$(j\omega)^1$	pole	-	-20×1 (Starting slope)
$(j\frac{\omega}{2} + 1)$	zero	2	+20
$(j\frac{\omega}{6} + 1)$	zero	6	+20
$(j\frac{\omega}{1} + 1)$	pole	1	-20
$(j\frac{\omega}{5} + 1)$	pole	5	-20
$(j\frac{\omega}{10} + 1)$	pole	10	-20

The Bode pole diagram Drawing

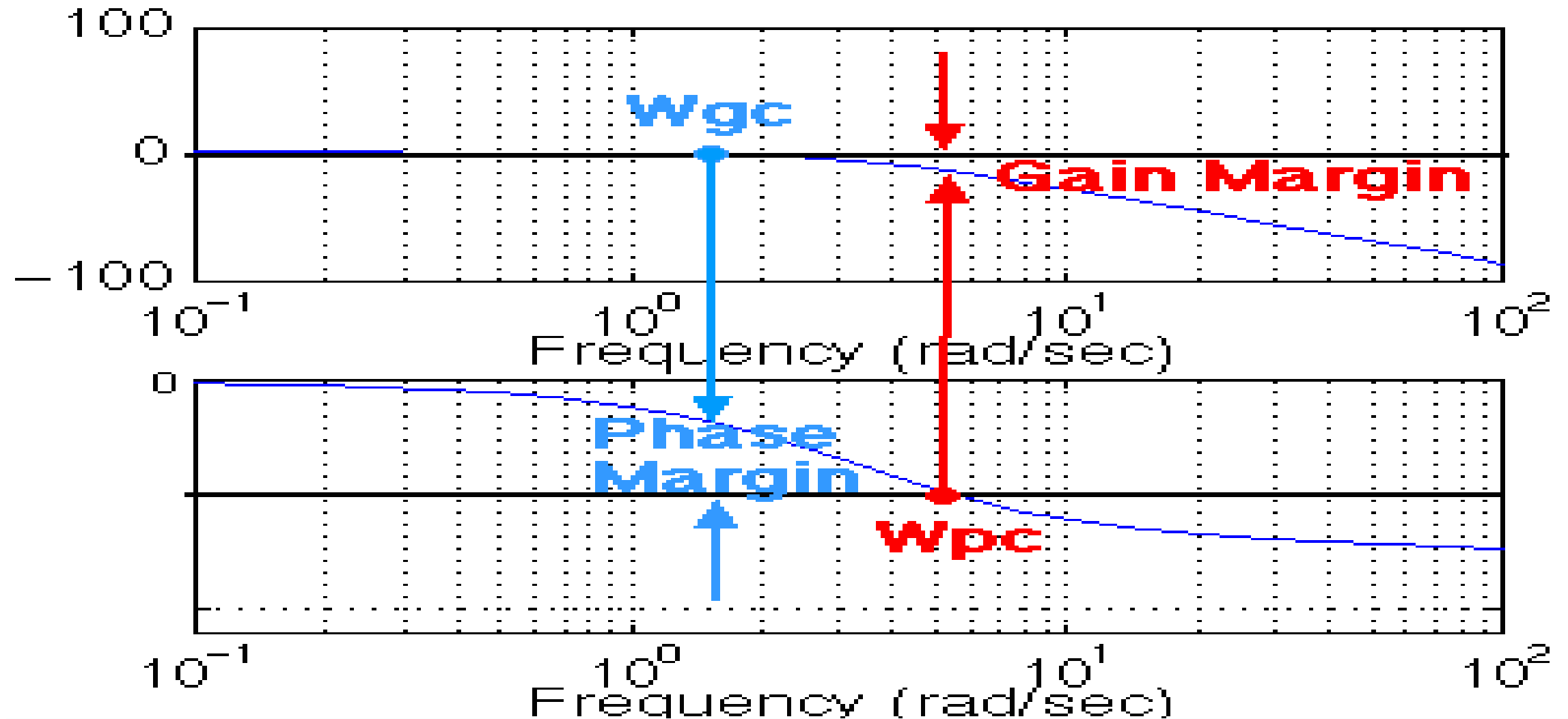
ω (rad/sec)	0.1	0.5	1	2	5	6	10	50
φ (deg)	-93.6	-106	-116	-123	-132	-135	-145	-171



MATLAB (The Bode pole diagram Drawing)



Bode Plot



Stability

- ❑ For a system to be **stable**, both the gain margin and the phase margin must be *positive*.
- ❑ For a system to be **Marginally stable**, the gain margin **or** the phase margin must be *Zero*.
- ❑ For a system to be **unstable**, the gain margin **or** the phase margin must be *negative*.

Example

1. If the gain margin is 10 dB and the phase margin is 30 degrees, then the system is stable.
2. If the gain margin is -5 dB or the phase margin is -10 degrees, then the system is unstable.
3. If the gain margin is 0 dB or the phase margin is 0 degrees, then the system is marginally stable.

Question 1

- If the unity feedback system is given by the open loop transfer function $G(s) = ks^2 / [(1 + 0.3s)(1 + 0.05s)]$, what would be the initial slope of magnitude plot?
 - a. 20 dB/decade
 - b. 40 dB/decade
 - c. 60 dB/decade
 - d. Unpredictable

Solution:

Question 2

- The system is said to be marginally stable, if gain margin is

a. 0

b. 1

c. $+\infty$

d. None of the above

Solution:

Question 3

If the phase angle at gain crossover frequency is estimated to be -105° , what will be the value of phase margin of the system?

- a. 23°
- b. 45°
- c. 60°
- d. 75°

Solution:

Question 4

- At which frequency does the magnitude of the system becomes zero dB?
 - a. Resonant frequency
 - b. Cut-off frequency
 - c. Gain crossover frequency
 - d. Phase crossover frequency

Solution:

Question 5

- If a pole is located at origin, how does it get represented on the magnitude plot?
 - a. $-10 \log (\omega)$ dB
 - b. $-20 \log (\omega)$ dB
 - c. $-40 \log (\omega)$ dB
 - d. $-60 \log (\omega)$ dB

Solution:

Question 6

- The frequency at which the phase of the system acquires _____ is known as 'Phase crossover frequency'.
 - a. 90°
 - b. -90°
 - c. 180°
 - d. -180°

Solution:

Question 7

- In an octave frequency band, the ratio of f_2 / f_1 is equivalent to

- a. 2
- b. 4
- c. 8
- d. 10

Solution:

Questions

- Which unit is adopted for magnitude measurement in Bode plots?
 - a. Degree
 - b. Decimal
 - c. Decibel
 - d. Deviation

Solution:

Question

- According to the principle of log-scales, if the ratio between two points is same, then the two points get _____ equally.
 - a. United
 - b. Separated
 - c. Multiplexed
 - d. Mixed

Solution:

Question 10

- If the constant 'k' is positive, then what would be its contribution on the phase plot?
 - a. 0°
 - b. 45°
 - c. 90°
 - d. 180°

Solution:

*Thank
you*

