

The powers of i !

$$i^2 = (\sqrt{-1})^2 = -1$$

$$\text{since } i = \sqrt{-1}$$

$$i^3 = i^2 \cdot i = (-1)i = \boxed{-i}$$

$$i^4 = i^2 \cdot i^2 = (-1)(-1) = \boxed{1}$$

$$i^5 = i^4 \cdot i = (1)i = \boxed{i}$$

$$i^6 = i^5 \cdot i = i \cdot i = i^2 = \boxed{-1}$$

⋮

$$\text{note that } i^n = \begin{cases} \mp 1 & \text{if } n \text{ even} \\ \mp i & \text{if } n \text{ odd} \end{cases}$$

Ex Simplify

1) i^{2n}

2) i^{4n}

3) i^{4n+3}

4) $\sqrt{-16} - i^{93}$

5) $\sqrt{-25} + i^{111} - 3i^{14} - 2\sqrt{-9}$

(use the positive root only)

6) i^{4n-5}

Sol!

$$1) i^{2n} = (i^2)^n = (-1)^n = \begin{cases} -1 & \text{if } n \text{ odd} \\ +1 & \text{if } n \text{ even} \end{cases}$$

$$2) i^{4n} = (i^4)^n = (1)^n = 1$$

$$3) i^{4n+3} = i^{4n} \cdot i^3 = (i^4)^n \cdot (i^2) \cdot i = 1 \cdot (-1)i = -i$$

$$\begin{aligned} 4) \sqrt{-16} - i^{93} &= (\sqrt{16} \cdot \sqrt{-1}) - (i^4)^{23} \cdot i \\ &= \mp 4i - (1) \cdot i = \mp 4i - i \\ &= \begin{cases} -5i \\ 3i \end{cases} \end{aligned}$$

$$\begin{aligned}
 5) \quad & \sqrt{-25} + i^{11} - 3i^{14} - 2\sqrt{-9} \\
 &= \sqrt{25} \cdot \sqrt{-1} + (i^4)^{27} \cdot i^3 - 3[(i^4)^3 \cdot i^2] - 2\sqrt{9} \cdot \sqrt{-1} \\
 &= 5i + 1 \cdot (i)^2 \cdot i - 3[(1)(-1)] - 2(3)i \\
 &= 5i - i + 3 - 6i = 3 - 2i
 \end{aligned}$$

$$\begin{aligned}
 6) \quad & i^{4n-5} = i^{4n} \cdot i^{-5} = (i^4)^n \cdot \frac{1}{i^5} \\
 &= (1)^n \cdot \frac{1}{(i^4)i} = \frac{1}{i} \Rightarrow \frac{i}{i} = i\bar{i} \\
 &\frac{1}{i} \cdot \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i
 \end{aligned}$$

operation on complex numbers

If $Z_1 = a+ib$, $Z_2 = c+id$ are complex numbers, then the following holds:

① $Z_1 = Z_2$ if $a=c$ and $b=d$

for example $Z_1 = 3+2i$, $Z_2 = a+2i$
then $a=3$

② If $Z_1 = -2-xi$, $Z_2 = y+3i$ where $Z_1 = Z_2$
then $y = -2$, $x = -3$

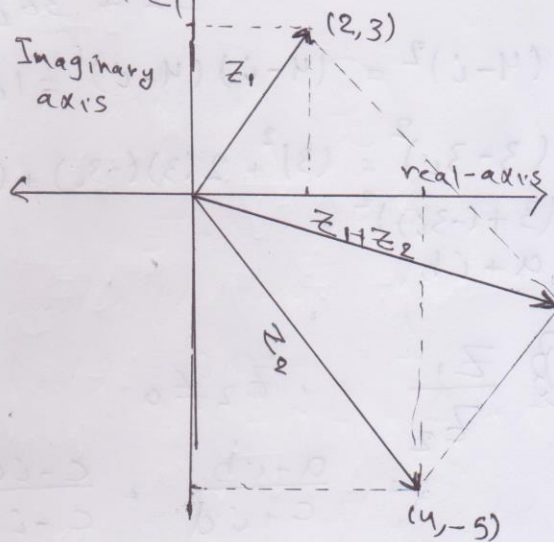
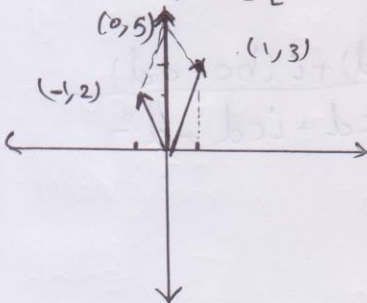
③ If $Z_1 = 0$ then $a, b = 0 \Rightarrow Z_1 = 0+0i$

$$\begin{aligned}
 3) \quad Z_1 + Z_2 &= (a+ib) + (c+id) \\
 &= (a+c) + i(b+d) = x + iy \\
 &= (c+a) + i(d+b) \\
 &= (c+id) + (a+ib) = Z_2 + Z_1
 \end{aligned}$$

Ex

$$\begin{aligned}
 1) \quad (2+3i) + (4-5i) \\
 &= (2+4) + i(3-5) \\
 &= 6-2i
 \end{aligned}$$

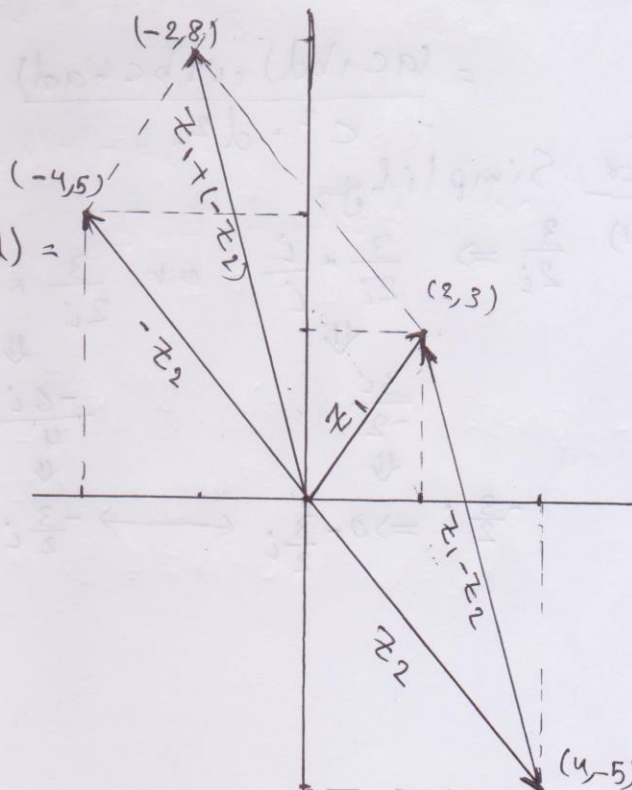
$$\begin{aligned}
 2) \quad (1+3i) + (-1+2i) \\
 &= (1+(-1)) + i(3+2) \\
 &= 0+5i = 5i
 \end{aligned}$$



$$\begin{aligned}
 4) \quad Z_1 - Z_2 &= (a+ib) - (c+id) \\
 &= (a-c) + i(b-d)
 \end{aligned}$$

Ex

$$\begin{aligned}
 1) \quad (2+3i) - (4-5i) \\
 &= (2-4) + i(3-(-5)) \\
 &= -2+8i
 \end{aligned}$$



$$\textcircled{5} \quad z_1 + z_2 = z_2 + z_1 = (a+ib) + (c+id) \\ = ac + iad + ibc + i^2 bd = (ac - bd) + i(ad + bc)$$

Ex

$$1) (2+6i)(8-6i) = 16 - 12i + 48i - 36i^2 \\ = 52 + 36i$$

$$2) (4-i)^2 = (4-i)(4-i) = 16 - 4i - 4i + i^2 = 15 - 8i$$

$$3) (3-3i)^2 = (3)^2 + 2(3)(-3i) + (-3i)^2 = 9 - 18i - 9 = -18i$$

$a+ib$

$$\textcircled{6} \quad \frac{z_1}{z_2}, \quad z_2 \neq 0 \\ = \frac{a+ib}{c+id} \cdot \frac{c-id}{c-id} = \frac{(ac+bd) + i(bc-ad)}{c^2 - icd + icd + d^2} \\ = \frac{(ac+bd) + i(bc-ad)}{c^2 + d^2}$$

Ex Simplify

$$1) \frac{3}{2i} \Rightarrow \frac{3}{2i} \cdot \frac{i}{i} \quad \text{or} \quad \frac{3}{2i} \cdot \frac{-2i}{-2i} \\ \downarrow \qquad \qquad \downarrow \\ \frac{3i}{-2} \qquad \qquad \frac{-6i}{4} \\ \downarrow \qquad \qquad \downarrow \\ -\frac{3}{2}i \Rightarrow 0 - \frac{3}{2}i \quad \longleftrightarrow \quad -\frac{3}{2}i$$

$$2) \frac{5+i}{2-3i} \times \frac{2+3i}{2+3i} = \frac{10+15i+2i+3}{4+9} = \frac{7+17i}{13}$$

$$= \frac{7}{13} + \frac{17}{13}i$$

$$3) \frac{2-3i}{6-i} \times \frac{6+i}{6+i} = \frac{12+2i-18i+3}{36+1} = \frac{15}{37} - \frac{16}{37}i$$

7. The identity element for the addition is

$$0 = 0+0i \quad (0,0)$$

$$Z+0 = 0+Z = Z$$

the identity element for the multiplication is

$$1 = 1+0i = (1,0)$$

$$Z \times 1 = 1 \times Z = Z$$

H.W. let $Z_1 = 2+3i$

$$Z_2 = 4-i$$

$$Z_3 = -1-18i$$

Find $Z_2 Z_3 - Z_1 Z_3$