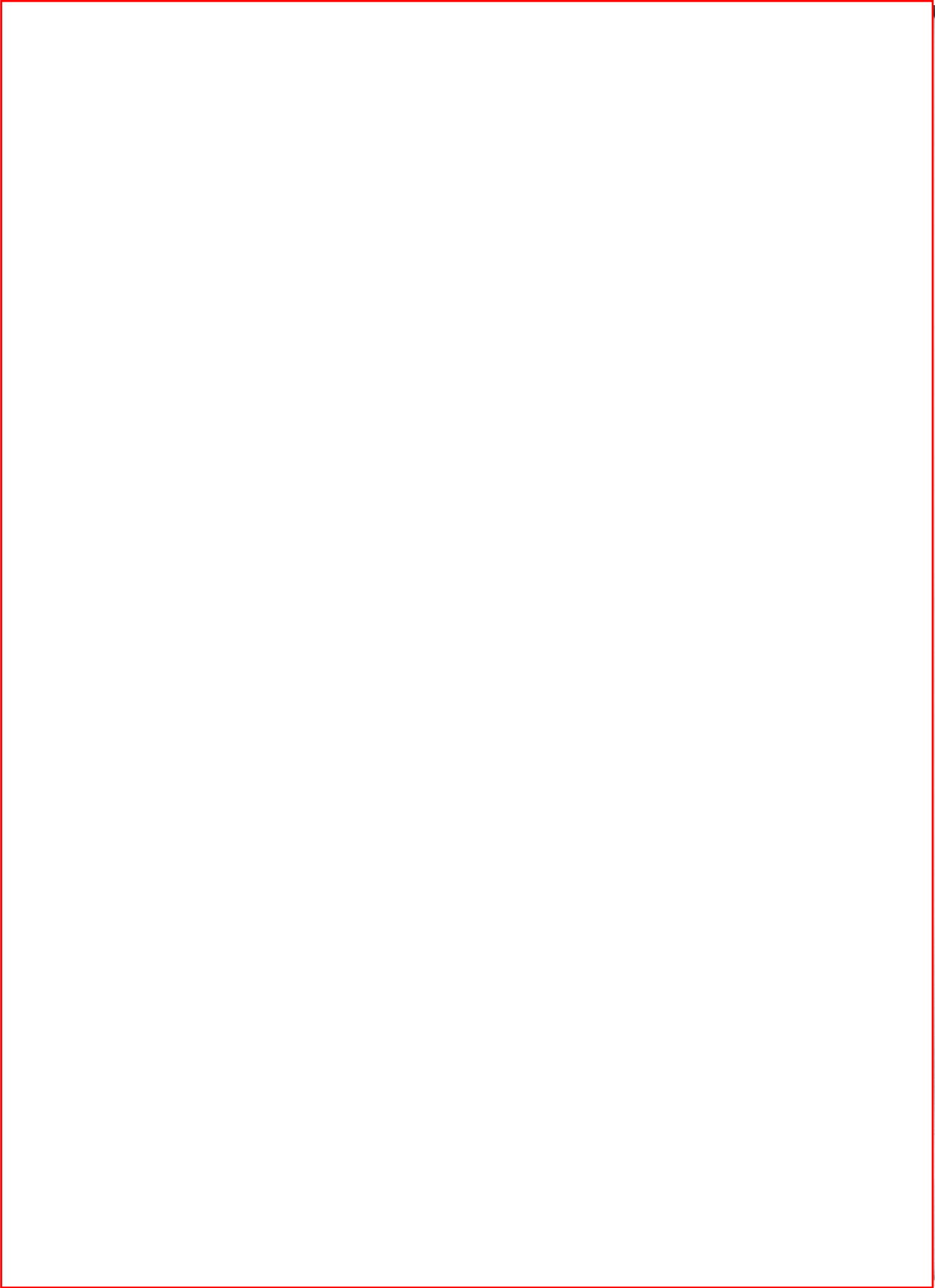




## Chapter one

### Complex numbers

Def: Complex number is a number of the form

$$Z = x + iy, \quad Z_1 = a + ib$$

$$Z = (x, y), \quad Z_1 = (a, b)$$

where  $x, a, y, b$  are real numbers and  $(i)$  is the imaginary unit, the real number  $x$  or  $a$  called the real part and  $y$  or  $b$  is the imaginary part of  $Z$  or  $Z_1$  written as:

$$\operatorname{Re}(Z) = x, \quad \operatorname{Re}(Z_1) = a$$

$$\operatorname{Im}(Z) = y, \quad \operatorname{Im}(Z_1) = b$$

Ex: (1)  $Z = 3 + 2i$

$$\operatorname{Re}(Z) = 3, \quad \operatorname{Im}(Z) = 2 \Rightarrow (3, 2)$$

$$(2) Z = \frac{1}{2} - 5i \Rightarrow \operatorname{Re}(Z) = \frac{1}{2}, \quad \operatorname{Im}(Z) = -5$$

We have the algebraic equation  $x^2 + 1 = 0$  --- (1)  
to solve this eq.  $\Rightarrow x^2 = -1$

So we can write  $x = -\sqrt{-1}$  or  $x = \sqrt{-1}$

Since  $x$  is not real (imaginary) we write  $i = \sqrt{-1}$

So  $i$  and  $-i$  are solutions of (1)

Ex Find the value of (1)  $i^2$ , (2)  $i^3$ , (3)  $i^4$ , (4)

$$(1) i^2 = (\sqrt{-1})^2 = -1$$

$$(2) i^3 = i^2 \times i = -1 \times i = (-\sqrt{-1})$$

$$(3) i^4 = i^2 \times i^2 = (-1) \times (-1) = 1$$

Simplify (1)  $\sqrt{-9}$ , (2)  $\sqrt{-25}$ , (3)  $\sqrt{-18}$

$$(1) \sqrt{-9} = \sqrt{9 \times (-1)} = \sqrt{9} \sqrt{-1} = 3\sqrt{-1} = 3i$$

note Any real or imaginary number is a complex number

$$\text{Ex } (1) Z = -2i \Rightarrow Z = 0 - 2i \Rightarrow \text{Re}(Z) = 0, \text{Im}(Z) = -2 \Rightarrow (0, -2)$$

$$(2) Z = 5 \Rightarrow Z = 5 + i0 \Rightarrow \text{Re}(Z) = 5, \text{Im}(Z) = 0 \Rightarrow (5, 0)$$

The complex numbers can be represented in the xy-plane by points or vectors

Ex Find the solution of

$$(1) Z^2 + 2Z + 10 = 0$$

$$a = 1, b = 2, c = 10$$

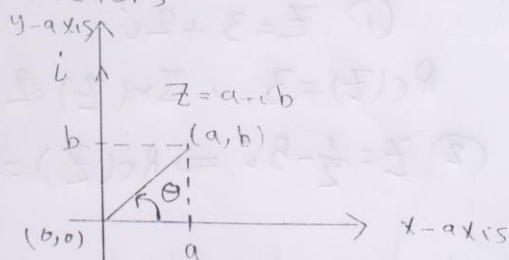
$$Z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore Z_1 = -1 + 3i, Z_2 = -1 - 3i$$

$$(2) -\frac{1}{2} Z^2 + 4Z - 9 = 0$$

$$(3) Z^2 + 6Z + \frac{3}{4} = 0$$

$$(4) x^2 + 3x + 3 = 0$$



H.W

### Operations on Complex Numbers

(3)

For two complex numbers  $Z_1 = a+ib$ ,  $Z_2 = c+id$   
the following holds

(1)  $Z_1 = Z_2$  if  $a=c$ ,  $b=d$

(2) if  $Z_1 = 0$  then  $a, b = 0$

(3)  $Z_1 + Z_2 = (a+ib) + (c+id) = \underbrace{(a+c)}_x + i \underbrace{(b+d)}_y = x+iy = Z_3$   
 $= (c+a) + i(d+b) = Z_2 + Z_1$

(4)  $Z_1 - Z_2 = (a+ib) - (c+id) = (a-c) + i(b-d)$

(5)  $Z_1 Z_2 = Z_2 Z_1 = (a+ib)(c+id) = (ac-bd) + i(cb+ad)$

(6)  $\frac{Z_1}{Z_2} = \frac{a+ib}{c+id} = \frac{(ac+bd) + i(bc-ad)}{c^2 + d^2}$

(7) The identity element for the addition is  $0 = 0+0 = (0,0)$   
The identity element for the multiplication is  $1 = 1+0 = (1,0)$

Ex Solve

(1)  $(-2+3i) + (7-4i) = (5-i)$

(2)  $-(1-2i) = -1+2i$

(3)  $-(4-i) + (4-i) = (-4+i) + (4-i) = 0+0 = 0$

(4)  $(-2+6i)(8-6i) = 20+60i$

(5)  $(4-i)^2 = 15-8i$

(6) let  $Z_1 = -2+3i$ ,  $Z_2 = 4-i$ ,  $Z_3 = -1-18i$  Find  
 $Z_2 Z_3 - Z_1 Z_3$

(H.W)