

Parallel circuit - 2

Introduction

Introduction

INTRODUCTION

Recall from the discussion of series circuits that the power applied to a series resistive circuit equals the power dissipated by the resistive elements.

The same is true for parallel resistive networks.

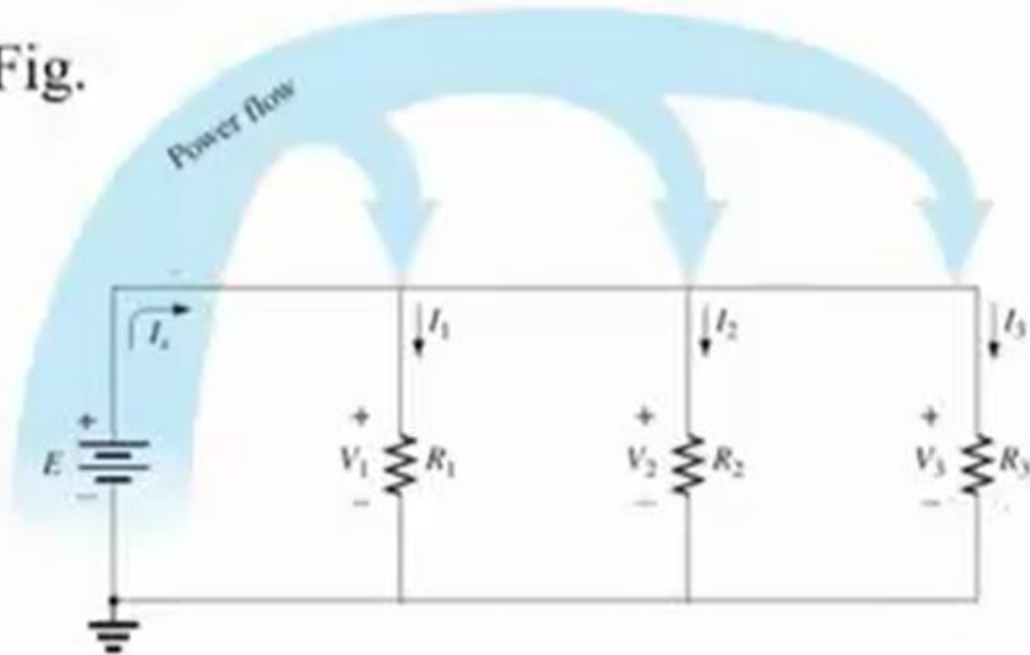
Power distribution in Parallel CKT

POWER DISTRIBUTION IN A PARALLEL CIRCUIT

for any network composed of resistive elements, the power applied by the battery will equal that dissipated by the resistive elements.

For the parallel circuit in Fig.

$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$



continue

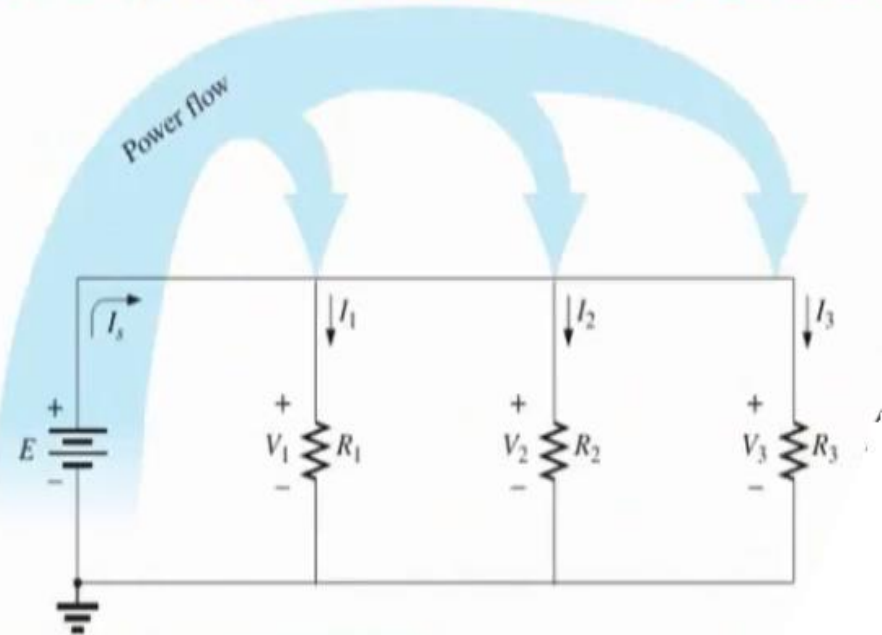
POWER DISTRIBUTION IN A PARALLEL CIRCUIT

The power delivered by the source :

$$P_E = EI_s \quad (\text{watts, W})$$

As is the equation for the power to each resistor (shown for R_1 only):

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} \quad (\text{watts, W})$$



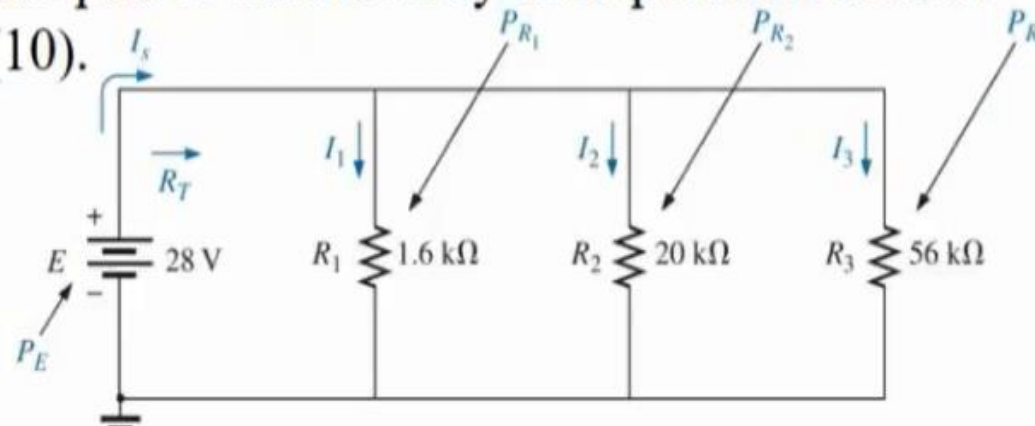
Example

POWER DISTRIBUTION IN A PARALLEL CIRCUIT

EXAMPLE 1

For the parallel network in Fig

- Determine the total resistance R_T .
- Find the source current and the current through each resistor.
- Calculate the power delivered by the source.
- Determine the power absorbed by each parallel resistor.
- Verify Eq. (10).



continue

POWER DISTRIBUTION IN A PARALLEL CIRCUIT

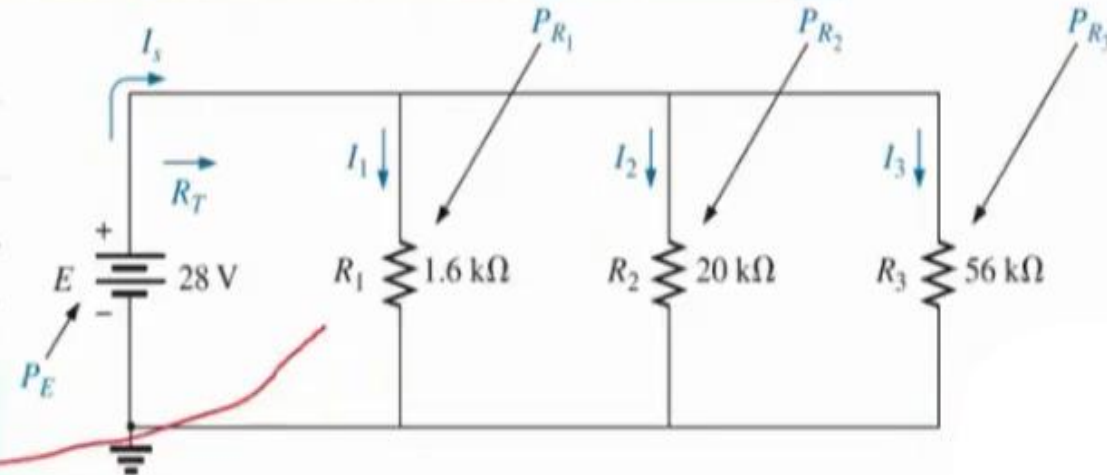
SOLUTION

a.
$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{1.6 \text{ k}\Omega} + \frac{1}{20 \text{ k}\Omega} + \frac{1}{56 \text{ k}\Omega}}$$

and $R_T = 1.44 \text{ k}\Omega$

b. Applying Ohm's law gives

$$I_s = \frac{E}{R_T} = \frac{28 \text{ V}}{1.44 \text{ k}\Omega} = 19.44 \text{ mA}$$



continue

POWER DISTRIBUTION IN A PARALLEL CIRCUIT

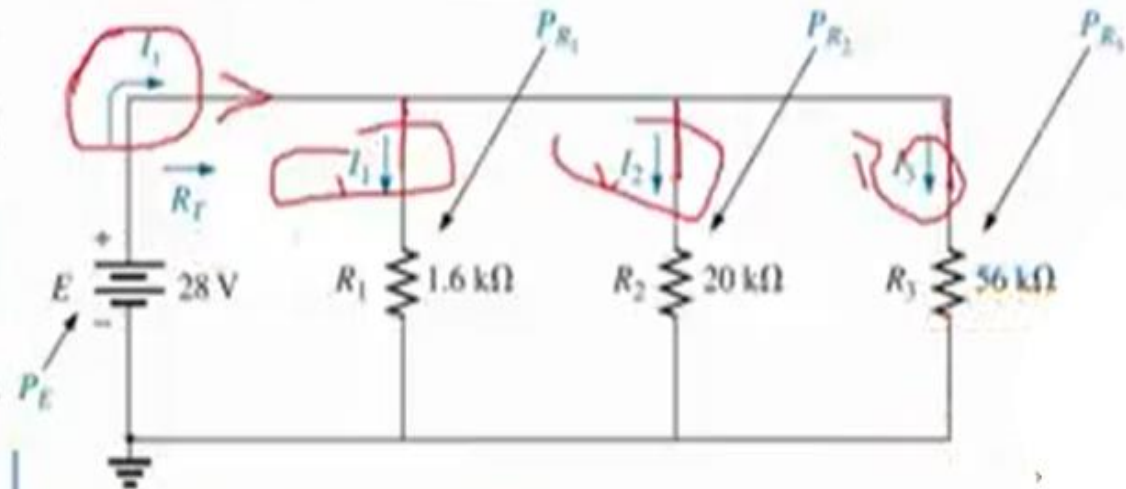
SOLUTION

b. Applying Ohm's law again gives

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{28 \text{ V}}{1.6 \text{ k}\Omega} = 17.5 \text{ mA}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{28 \text{ V}}{20 \text{ k}\Omega} = 1.4 \text{ mA}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{28 \text{ V}}{56 \text{ k}\Omega} = 0.5 \text{ mA}$$



c.

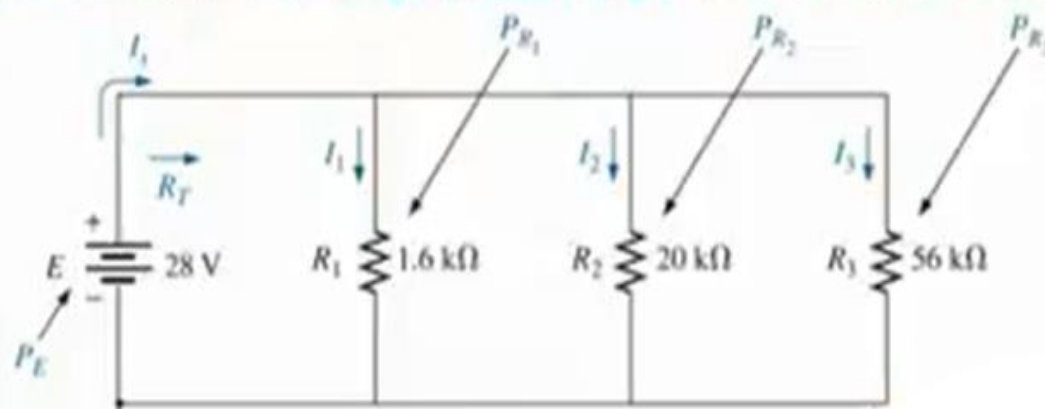
$$P_E = EI_T = (28 \text{ V})(19.4 \text{ mA}) = 543.2 \text{ mW}$$

continue

POWER DISTRIBUTION IN A PARALLEL CIRCUIT

SOLUTION

- d. Applying each form of the power equation gives



$$P_1 = V_1 I_1 = E I_1 = (28\text{ V})(17.5\text{ mA}) = \mathbf{490\text{ mW}}$$

$$P_2 = I_2^2 R_2 = (1.4\text{ mA})^2 (20\text{ k}\Omega) = \mathbf{39.2\text{ mW}}$$

$$P_3 = \frac{V_3^2}{R_3} = \frac{E^2}{R_3} = \frac{(28\text{ V})^2}{56\text{ k}\Omega} = \mathbf{14\text{ mW}}$$

e. $P_E = P_{R_1} + P_{R_2} + P_{R_3}$

$$\mathbf{543.2\text{ mW}} = 490\text{ mW} + 39.2\text{ mW} + 14\text{ mW} = \mathbf{543.2\text{ mW}} \quad (\text{checks})$$

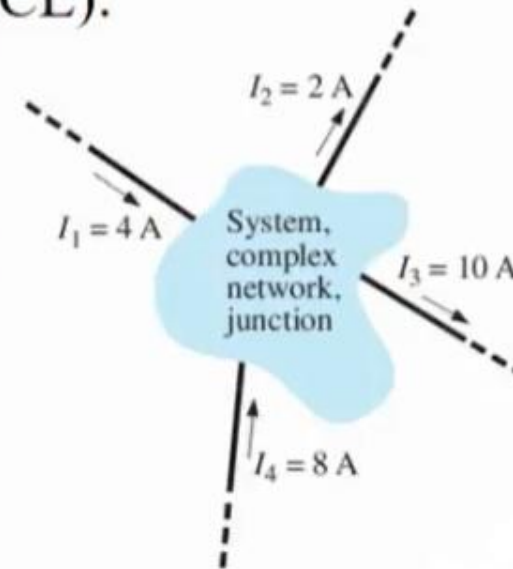
Kirchhoff's Current law

KIRCHHOFF'S CURRENT LAW (KCL)

Kirchhoff's voltage law provides a very important relationship among the voltages of a closed path. Kirchhoff is also credited with developing the following equally important relationship between the currents of a network, called Kirchhoff's current law (KCL):

The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

$$\Sigma I_i = \Sigma I_o$$



continue

KIRCHHOFF'S CURRENT LAW (KCL)

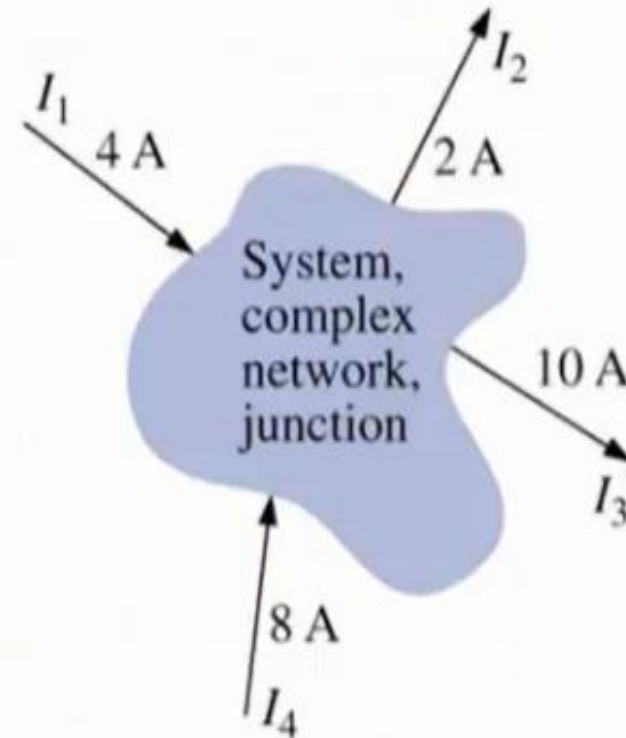
$$\Sigma I_i = \Sigma I_o$$

$$\Sigma I_i = \Sigma I_o$$

$$I_1 + I_4 = I_2 + I_3$$

$$4 \text{ A} + 8 \text{ A} = 2 \text{ A} + 10 \text{ A}$$

$$12 \text{ A} = 12 \text{ A} \quad (\text{checks})$$



example

KIRCHHOFF'S CURRENT LAW (KCL)

EXAMPLE

Determine currents I_3 and I_4 in Fig using Kirchhoff's current law.

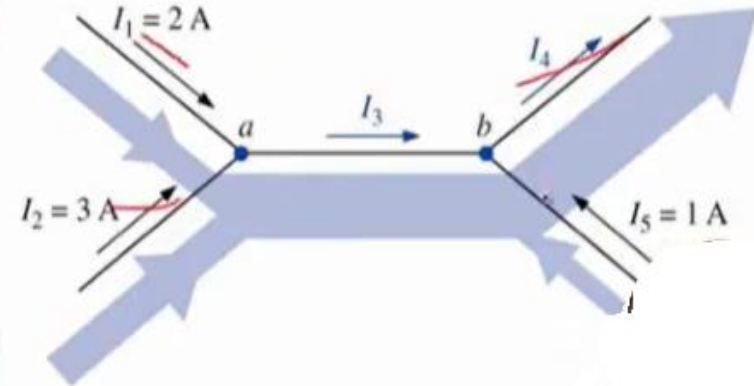
Solution

At node a

$$\begin{aligned}\sum I_i &= \sum I_o \\ I_1 + I_2 &= I_3 \\ 2\text{ A} + 3\text{ A} &= I_3 = \mathbf{5\text{ A}}\end{aligned}$$

At node b , using the result just obtained,

$$\begin{aligned}\sum I_i &= \sum I_o \\ I_3 + I_5 &= I_4 \\ 5\text{ A} + 1\text{ A} &= I_4 = \mathbf{6\text{ A}}\end{aligned}$$



Example

KIRCHHOFF'S CURRENT LAW (KCL)

EXAMPLE

Determine currents I_3 and I_5 in Fig through applications of Kirchhoff's current law.

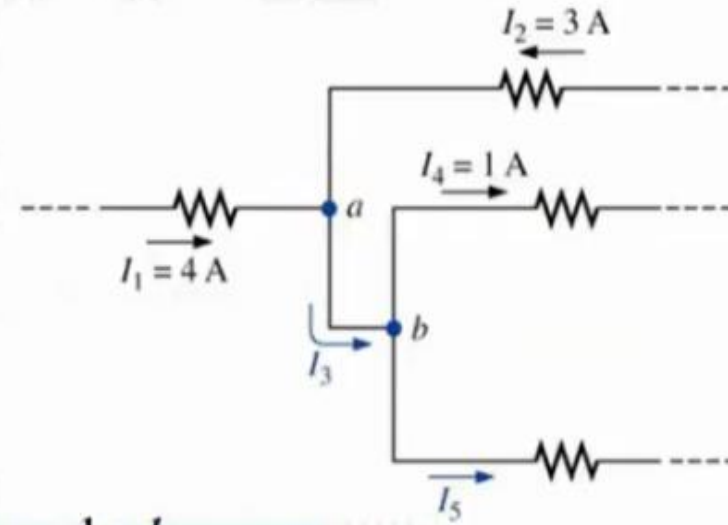
Solution

At node a

$$\begin{aligned}\sum I_i &= \sum I_o \\ I_1 + I_2 &= I_3 \\ 4\text{ A} + 3\text{ A} &= I_3 = \mathbf{7\text{ A}}\end{aligned}$$

At node b

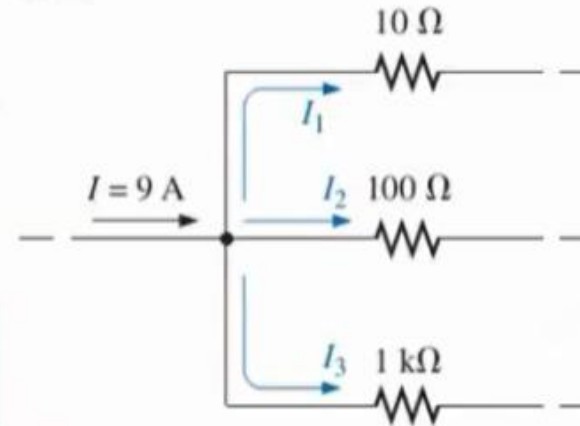
$$\begin{aligned}\sum I_i &= \sum I_o \\ I_3 &= I_4 + I_5 \\ 7\text{ A} &= 1\text{ A} + I_5 \\ I_5 &= 7\text{ A} - 1\text{ A} = \mathbf{6\text{ A}}\end{aligned}$$



Current Divider Rule (CDR)

CURRENT DIVIDER RULE (CDR)

- *For two parallel elements of equal value, the current will divide equally.*
- *For parallel elements with different values, the smaller the resistance, the greater is the share of input current.*
- *For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistance values.*

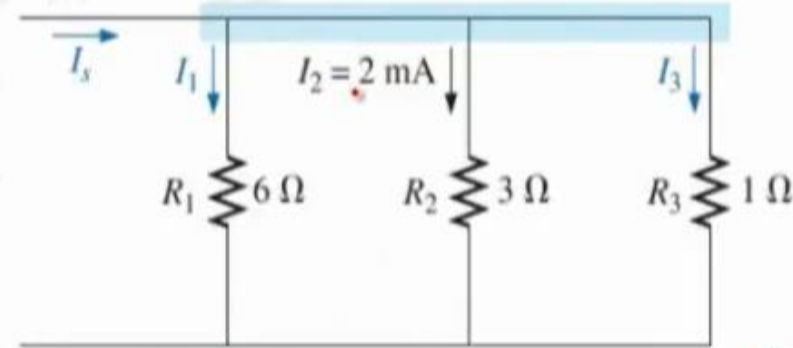


Example

CURRENT DIVIDER RULE

EXAMPLE

- Determine currents I_1 and I_3 for the network in Fig.
- Find the source current I_s .



Solution

- Since R_1 is twice R_2 , the current I_1 must be one-half I_2 , and

$$I_1 = \frac{I_2}{2} = \frac{2 \text{ mA}}{2} = \mathbf{1 \text{ mA}}$$

Since R_2 is three times R_3 , the current I_3 must be three times I_2 , and

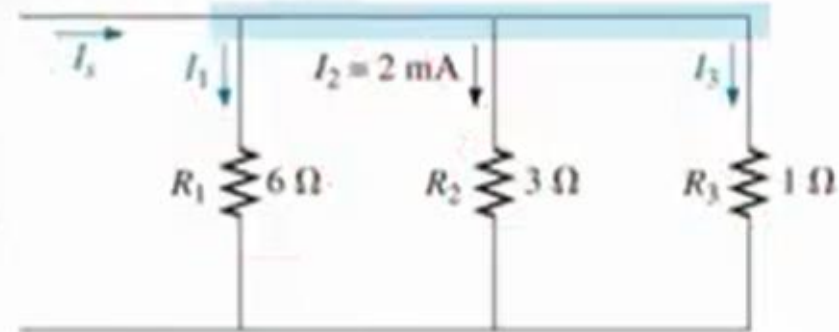
$$I_3 = 3I_2 = 3(2 \text{ mA}) = \mathbf{6 \text{ mA}}$$

Example

CURRENT DIVIDER RULE

EXAMPLE

- Determine currents I_1 and I_3 for the network in Fig.
- Find the source current I_s .



Solution

- Applying Kirchhoff's current law gives

$$\sum I_i = \sum I_o$$

$$I_s = I_1 + I_2 + I_3$$

$$I_s = 1\ \text{mA} + 2\ \text{mA} + 6\ \text{mA} = \mathbf{9\ \text{mA}}$$

Current Divider Rule

CURRENT DIVIDER RULE

CDR states that:

The current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistance of the resistor of interest and multiplied by the total current entering the parallel configuration.

Solving for I_x , the final result is the current divider rule:

$$I_x = \frac{R_T}{R_x} I_T$$

Example

CURRENT DIVIDER RULE

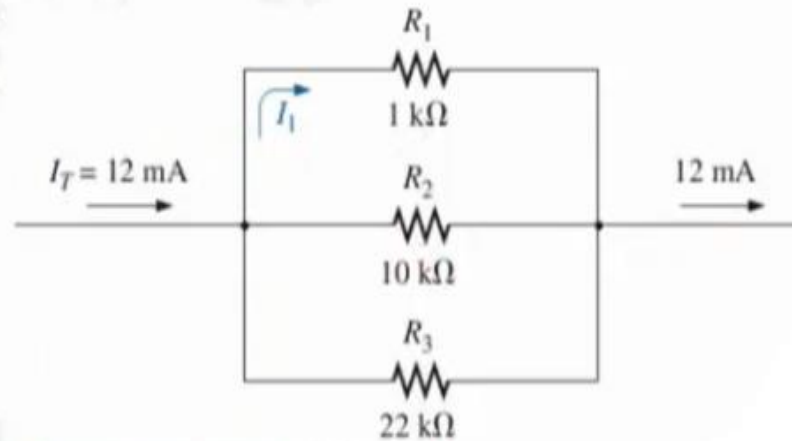
EXAMPLE

For the parallel network in Fig.
determine current I_1 using.

$$R_T = 873.01 \, \Omega$$

By using current divider rule :

$$\begin{aligned} I_1 &= \frac{R_T}{R_1} I_T \\ &= \frac{(873.01 \, \Omega)}{1 \, \text{k}\Omega} (12 \, \text{mA}) = (0.873)(12 \, \text{mA}) = \mathbf{10.48 \, \text{mA}} \end{aligned}$$



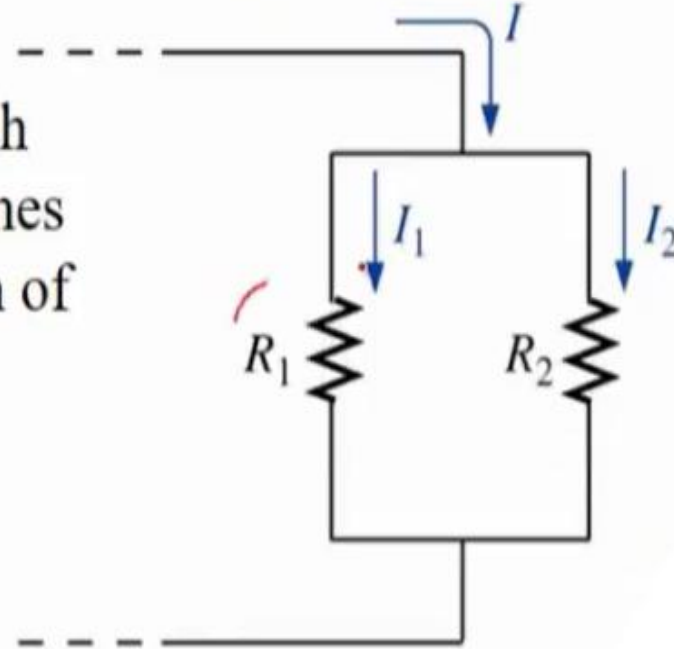
Special Case

Special Case: Two Parallel Resistors

For two parallel resistors, the current through one is equal to the resistance of the other times the total entering current divided by the sum of the two resistances.

$$I_1 = \left(\frac{R_2}{R_1 + R_2} \right) I_T$$

$$I_2 = \left(\frac{R_1}{R_1 + R_2} \right) I_T$$



Special Case

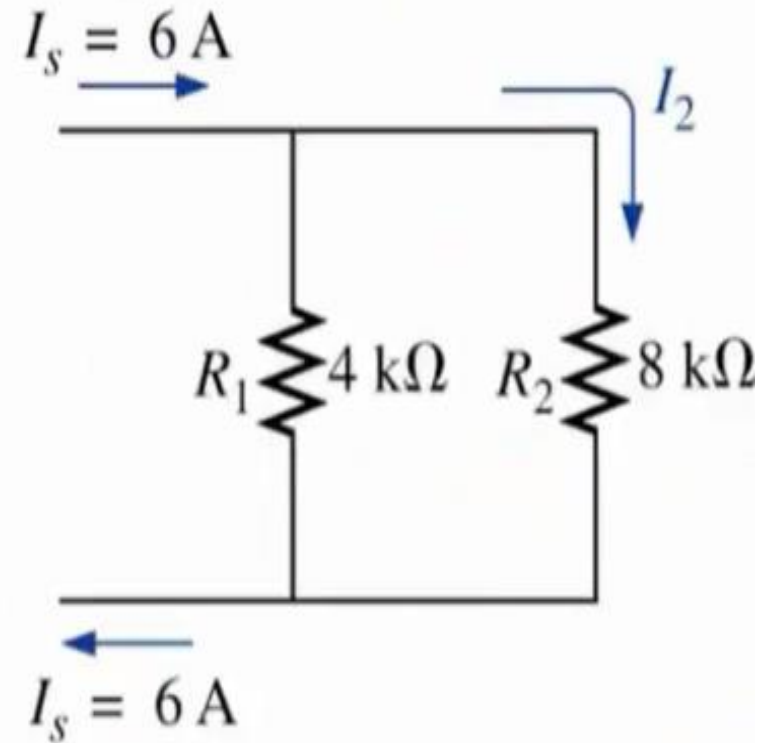
Special Case: Two Parallel Resistors

EXAMPLE

Determine current I_2 for the network in Fig using the current divider rule.

Solution

$$\begin{aligned} I_2 &= \left(\frac{R_1}{R_1 + R_2} \right) I_T \\ &= \left(\frac{4 \text{ k}\Omega}{4 \text{ k}\Omega + 8 \text{ k}\Omega} \right) 6 \text{ A} = (0.333)(6 \text{ A}) = 2 \text{ A} \end{aligned}$$

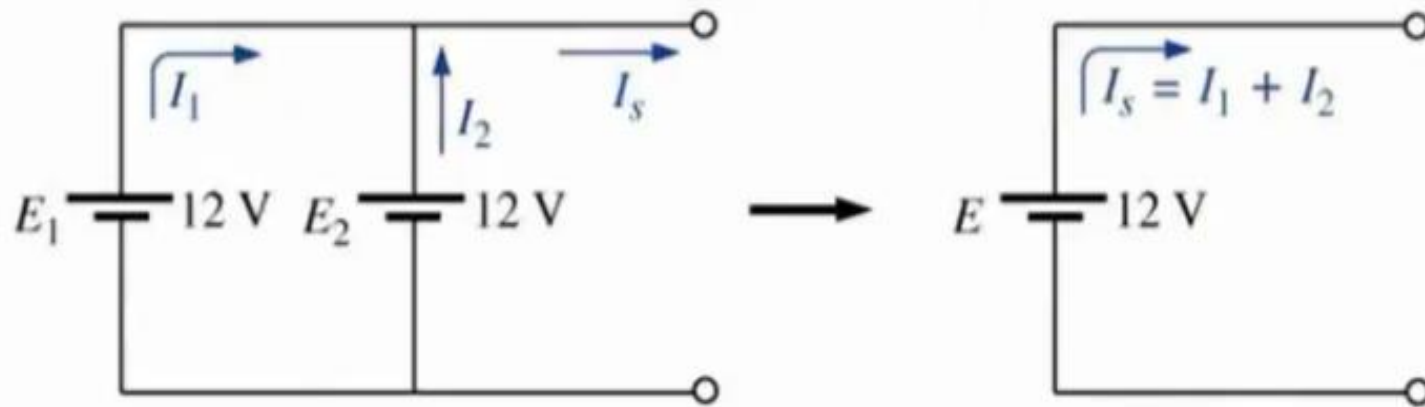


Voltage Source in Parallel

VOLTAGE SOURCES IN PARALLEL

Because the voltage is the same across parallel elements,

voltage sources can be placed in parallel only if they have the same voltage.



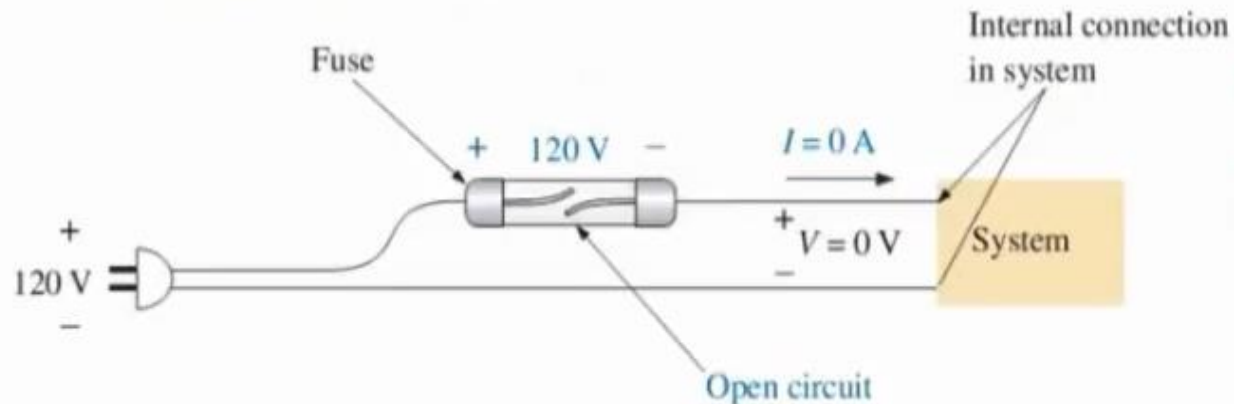
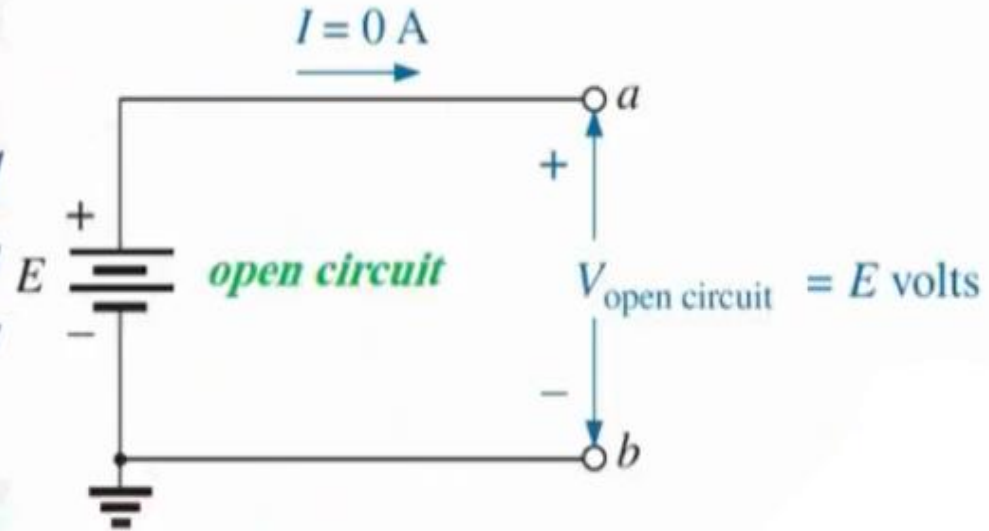
Open Circuit

OPEN AND SHORT CIRCUITS

open

circuit

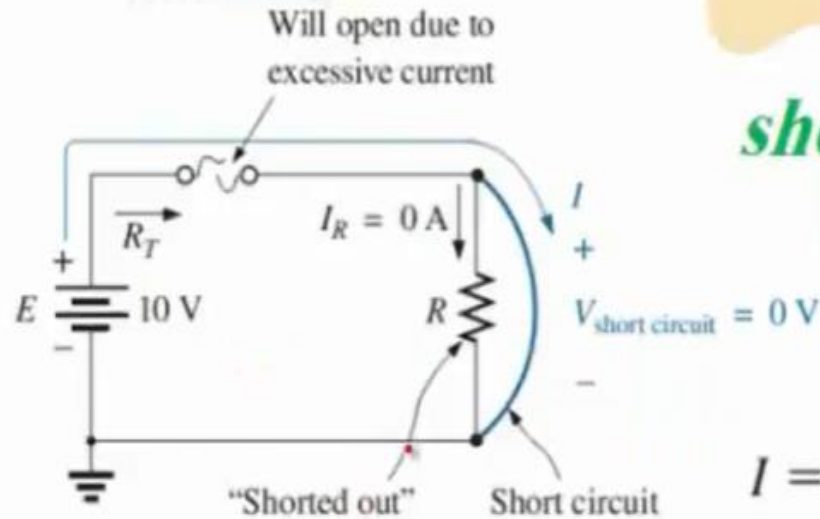
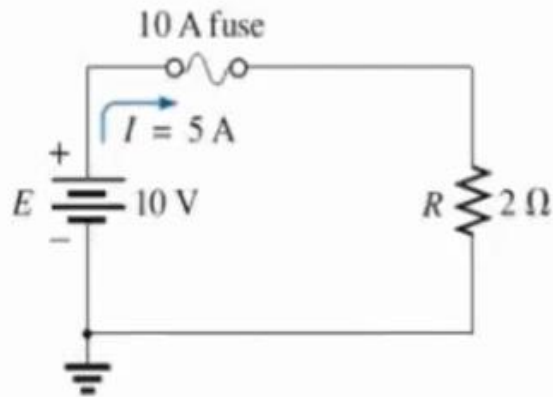
*An **open circuit** can have a potential difference (voltage) across its terminals, but the current is always zero amperes.*



Short Circuit

short circuit

A short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.



Short circuit

System

short circuit

$$I = \frac{E}{R} = \frac{10\text{ V}}{0\ \Omega} \rightarrow \infty\text{ A}$$

EXAMPLE 25 Determine voltage V_{ab} for the network in Fig. 53.

Solution: The open circuit requires that I be zero amperes. The voltage drop across both resistors is therefore zero volts since

$V = IR = (0)R = 0$ V. Applying Kirchhoff's voltage law around the closed loop gives

$$V_{ab} = E = 20 \text{ V}$$

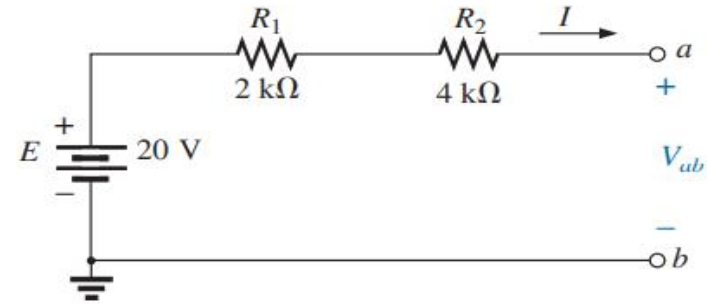


FIG. 53

Network for Example 25.

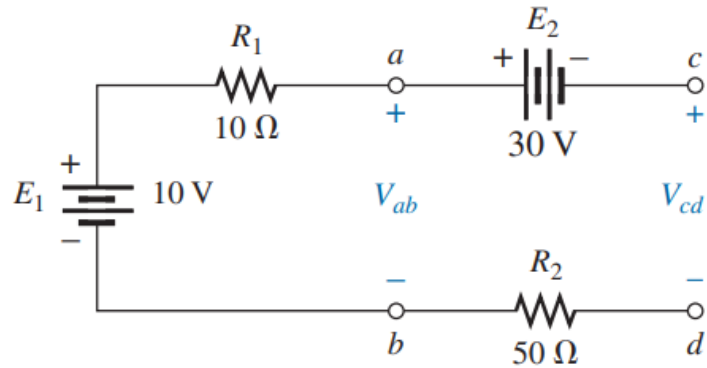


FIG. 54

Network for Example 26.

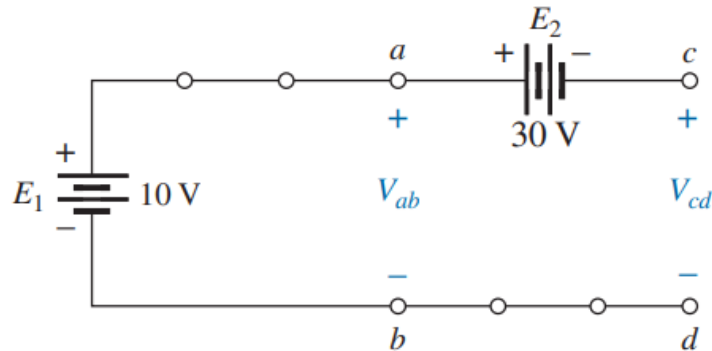


FIG. 55

Circuit in Fig. 54 redrawn.

EXAMPLE 26 Determine voltages V_{ab} and V_{cd} for the network in Fig. 54.

Solution: The current through the system is zero amperes due to the open circuit, resulting in a 0 V drop across each resistor. Both resistors can therefore be replaced by short circuits, as shown in Fig. 55. Voltage V_{ab} is then directly across the 10 V battery, and

$$V_{ab} = E_1 = \mathbf{10\text{ V}}$$

Voltage V_{cd} requires an application of Kirchhoff's voltage law:

$$+E_1 - E_2 - V_{cd} = 0$$

or
$$V_{cd} = E_1 - E_2 = 10\text{ V} - 30\text{ V} = \mathbf{-20\text{ V}}$$

The negative sign in the solution indicates that the actual voltage V_{cd} has the opposite polarity of that appearing in Fig. 54.

EXAMPLE 27 Determine the unknown voltage and current for each network in Fig. 56.

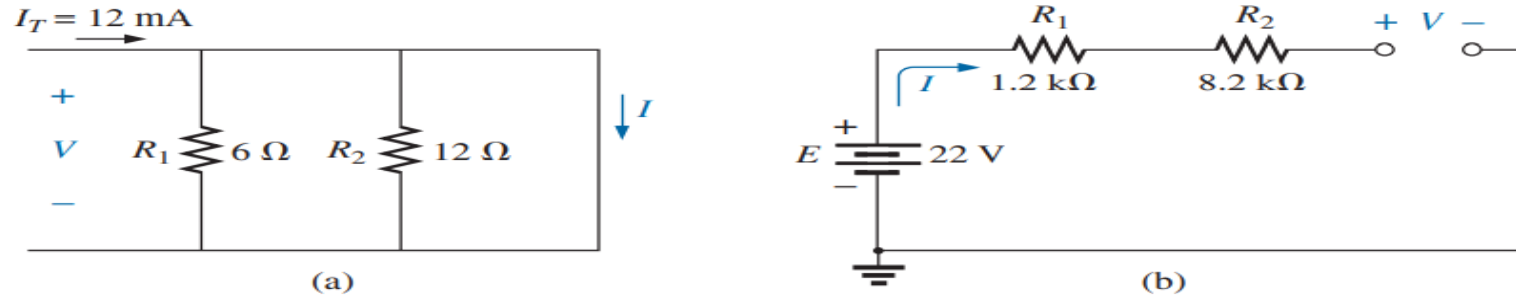


FIG. 56

Networks for Example 27.

Solution: For the network in Fig. 56(a), the current I_T will take the path of least resistance, and since the short-circuit condition at the end of the network is the least-resistance path, all the current will pass through the short circuit. This conclusion can be verified using the current divider rule. The voltage across the network is the same as that across the short circuit and will be zero volts, as shown in Fig. 57(a).

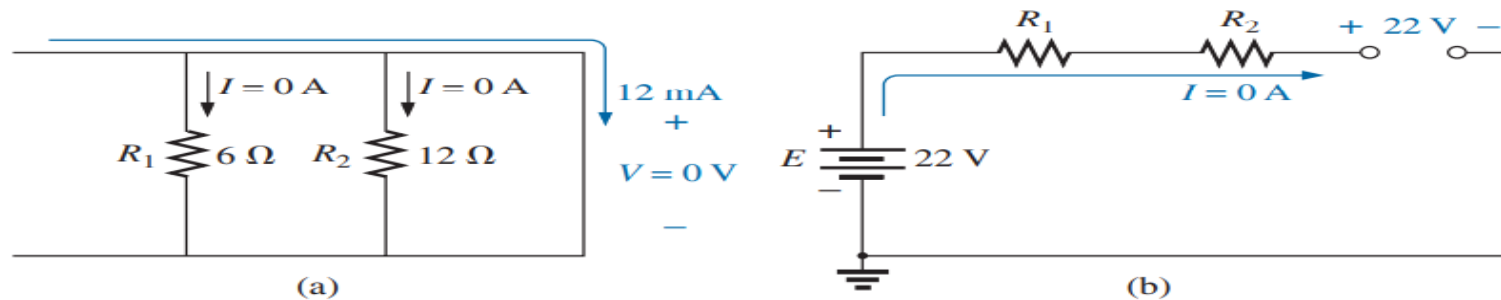


FIG. 57

Solutions to Example 27.

EXAMPLE 28 Determine V and I for the network in Fig. 58 if resistor R_2 is shorted out.

Solution: The redrawn network appears in Fig. 59. The current through the $3\ \Omega$ resistor is zero due to the open circuit, causing all the current I to pass through the jumper. Since $V_{3\Omega} = IR = (0)R = 0\text{ V}$, the voltage V is directly across the short, and

$$V = 0\text{ V}$$
$$I = \frac{E}{R_1} = \frac{6\text{ V}}{2\ \Omega} = 3\text{ A}$$

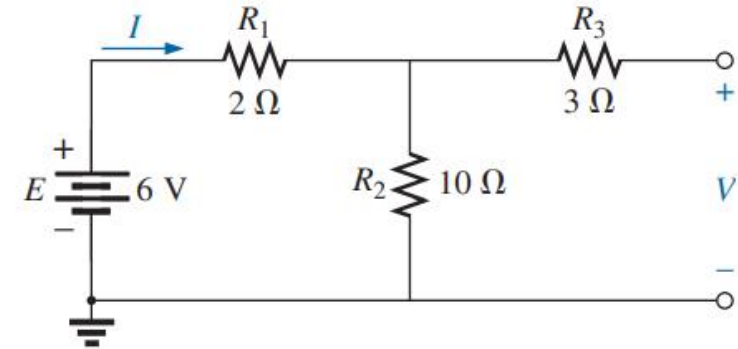


FIG. 58

Network for Example 28.

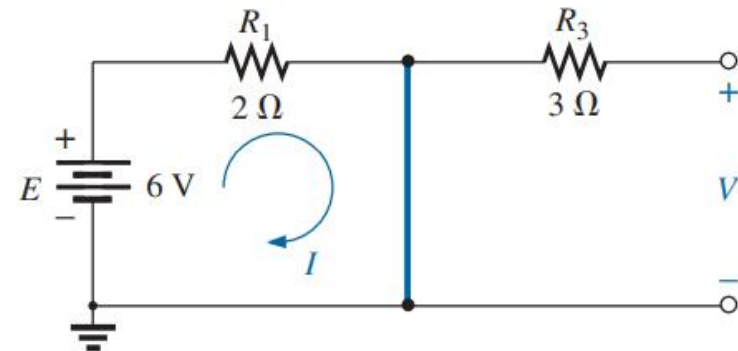


FIG. 59

Network in Fig. 58 with R_2 replaced by a jumper.

Summary table.

Series and Parallel Circuits		
Series	Duality	Parallel
$R_T = R_1 + R_2 + R_3 + \cdots + R_N$	$R \rightleftharpoons G$	$G_T = G_1 + G_2 + G_3 + \cdots + G_N$
R_T increases (G_T decreases) if additional resistors are added in series	$R \rightleftharpoons G$	G_T increases (R_T decreases) if additional resistors are added in parallel
Special case: two elements		
$R_T = R_1 + R_2$	$R \rightleftharpoons G$	$G_T = G_1 + G_2$
I the same through series elements	$I \rightleftharpoons V$	V the same across parallel elements
$E = V_1 + V_2 + V_3$	$E, V \rightleftharpoons I$	$I_T = I_1 + I_2 + I_3$
Largest V across largest R	$V \rightleftharpoons I$ and $R \rightleftharpoons G$	Greatest I through largest G (smallest R)
$V_x = \frac{R_x E}{R_T}$	$E, V \rightleftharpoons I$ and $R \rightleftharpoons G$	$I_x = \frac{G_x I_T}{G_T}$
$P = EI_T$	$E \rightleftharpoons I$ and $I \rightleftharpoons E$	$P = I_T E$
$P = I^2 R$	$I \rightleftharpoons V$ and $R \rightleftharpoons G$	$P = V^2 G$
$P = V^2 / R$	$V \rightleftharpoons I$ and $R \rightleftharpoons G$	$P = I^2 / G$

Parallel CKT

Applications

13 APPLICATIONS

One of the most important advantages of the parallel configuration is that *if one branch of the configuration should fail (open circuit), the remaining branches will still have full operating power.*

In a home, the parallel connection is used throughout to ensure that if one circuit has a problem and opens the circuit breaker, the remaining circuits still have the full 120 V. The same is true in automobiles, computer systems, industrial plants, and wherever it would be disastrous for one circuit to control the total power distribution.

Another important advantage is that *branches can be added at any time without affecting the behavior of those already in place.*

In other words, unlike the series connection, where an additional component reduces the current level and perhaps affects the response of some of the existing components, an additional parallel branch will not affect the current level in the other branches. Of course, the current demand from the supply increases as determined by Kirchhoff's current law, so you must be aware of the limitations of the supply.

The following are some of the most common applications of the parallel configuration.

Car System

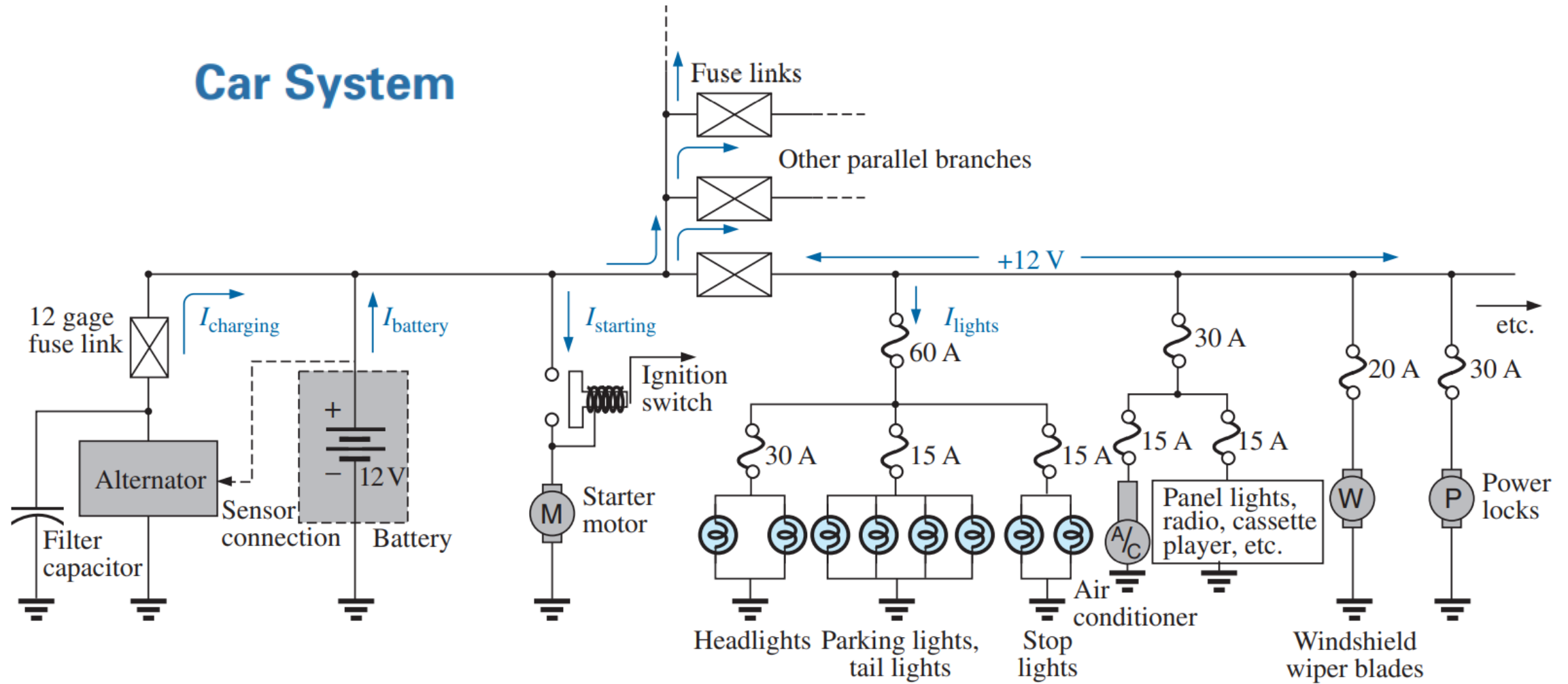


FIG. 64

Expanded view of an automobile's electrical system.

House Wiring

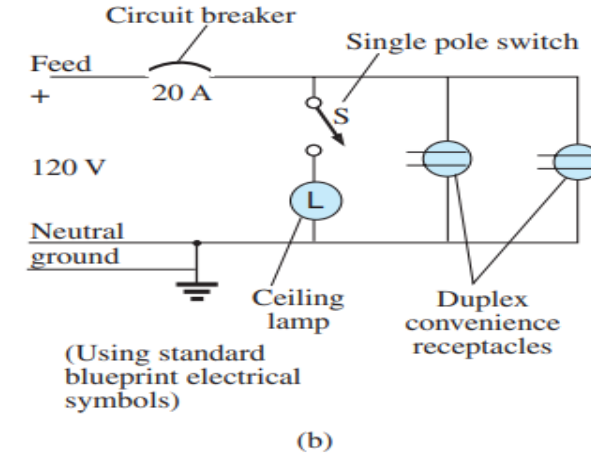
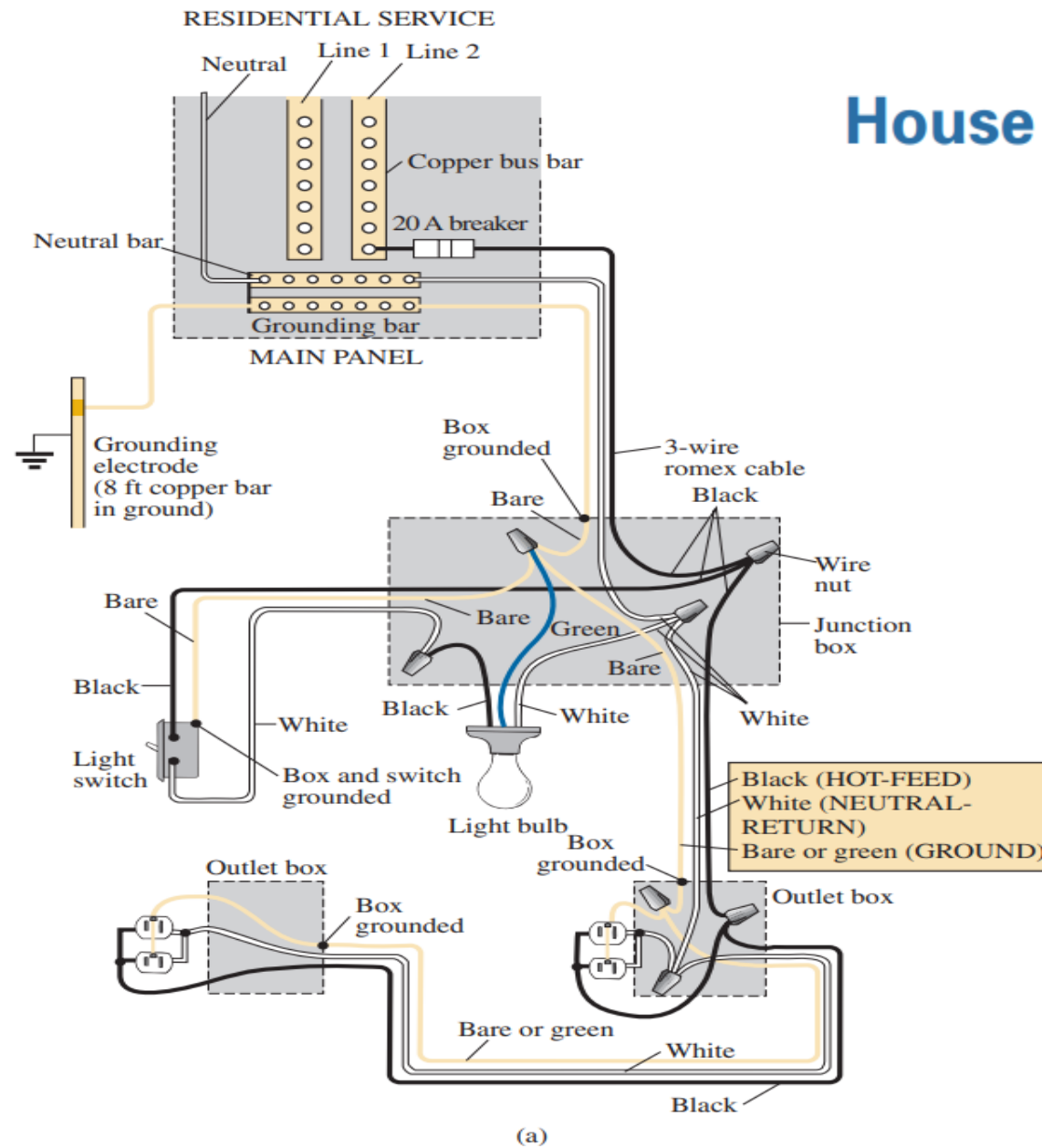


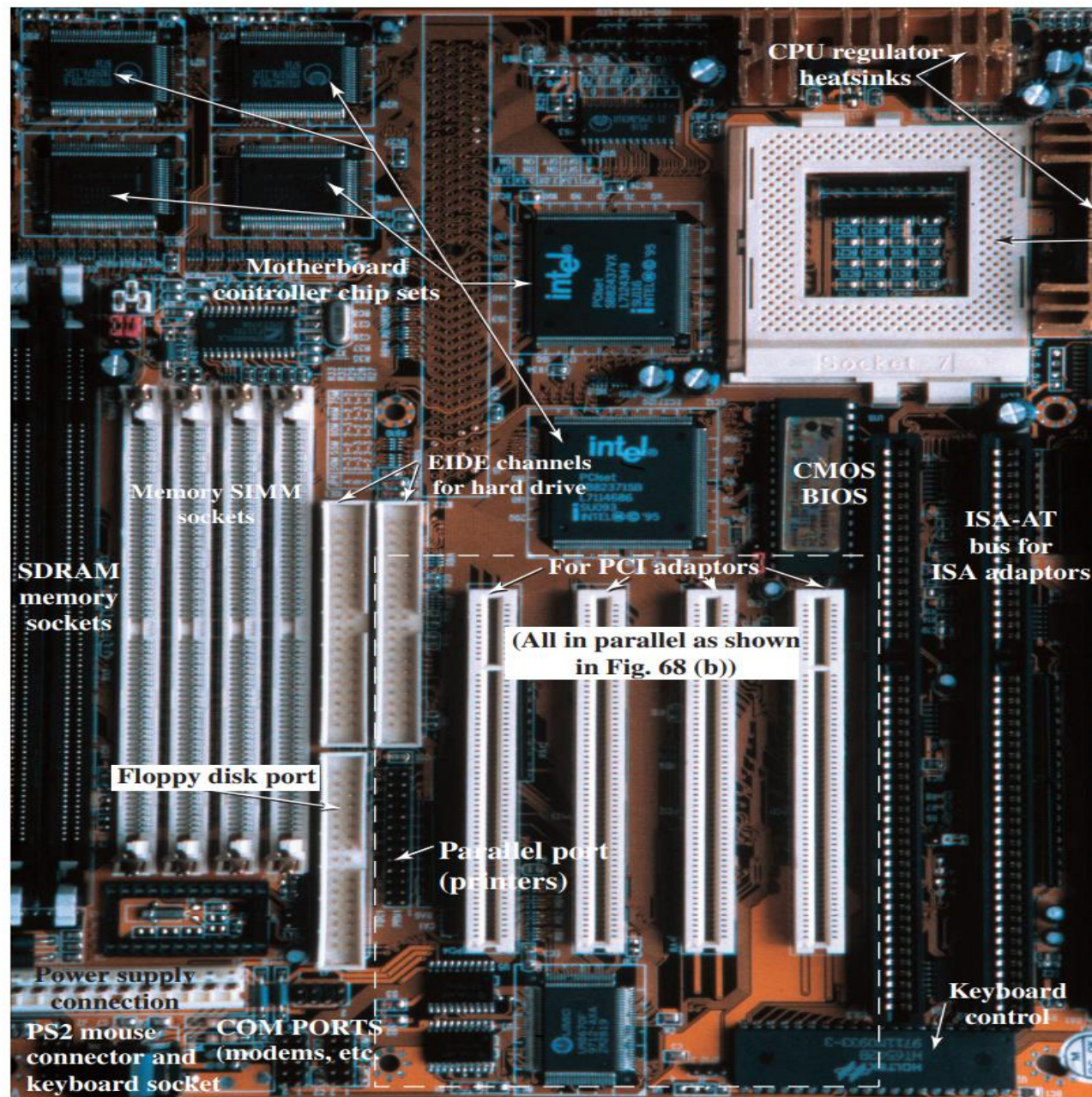
FIG. 66

Single phase of house wiring: (a) physical details; (b) schematic representation.

FIG. 67

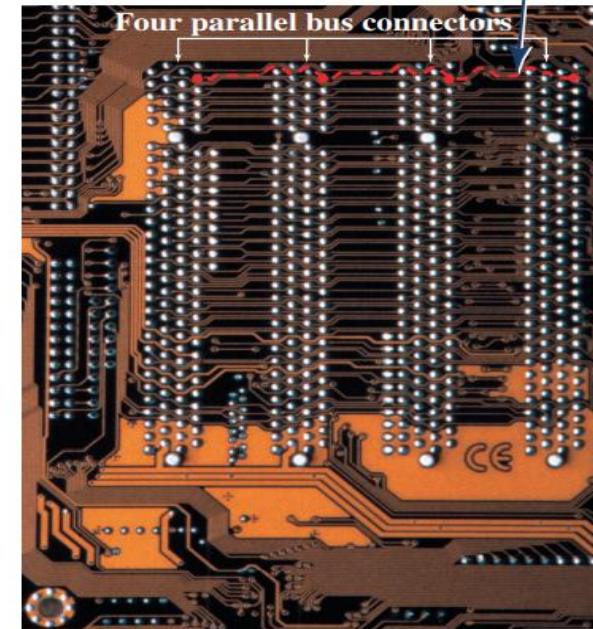
*Continuous ground connection
in a duplex outlet.*

Parallel Computer Bus Connections



(a)

Dashed line shows parallel
connection between bus
connections for one
pin connection



(b)

FIG. 68

(a) Motherboard for a desktop computer; (b) the printed circuit board connections for the region indicated in part (a).

Conclusion

Parallel dc Circuits (II)

Conclusion

Kirchhoff's voltage law provides a very important relationship among the voltages of a closed path. Kirchhoff is also credited with developing the following equally important relationship between the currents of a network, called Kirchhoff's current law (KCL).