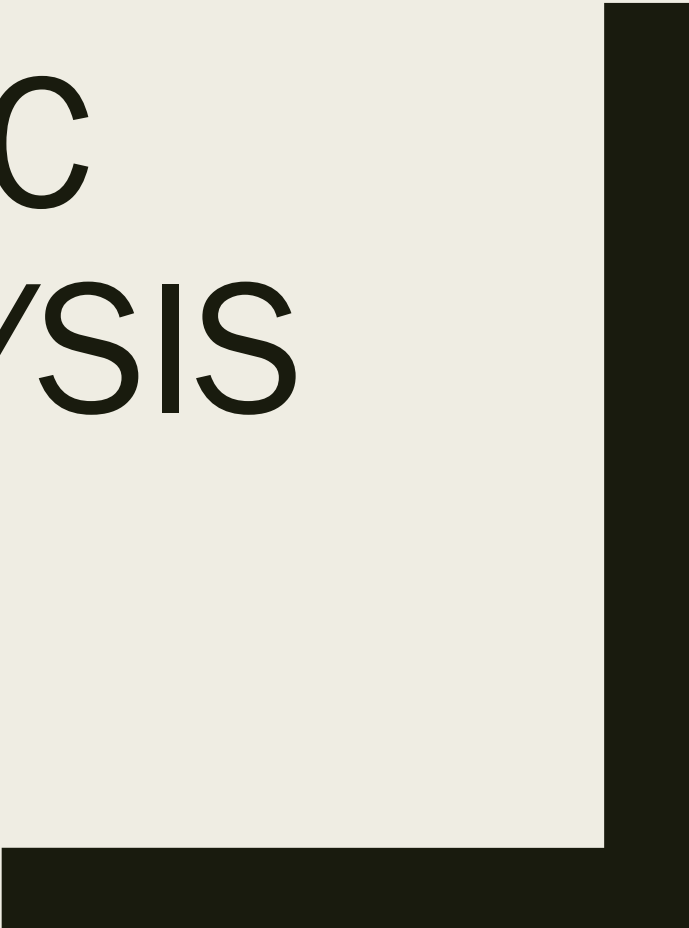




AC ELECTRIC CIRCUIT ANALYSIS

introduction



Introduction

Introduction

The response of the basic R, L, and C elements to a sinusoidal voltage and current are examined in this chapter, with special note of how frequency affects the “opposing” characteristic of each element. Phasor notation is then introduced to establish a method of analysis that permits a direct correspondence with a number of the methods, theorems, and concepts introduced in the dc chapters.

The Basic Elements and Phasors

Response of basic R, L and C elements

1) Resistor

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

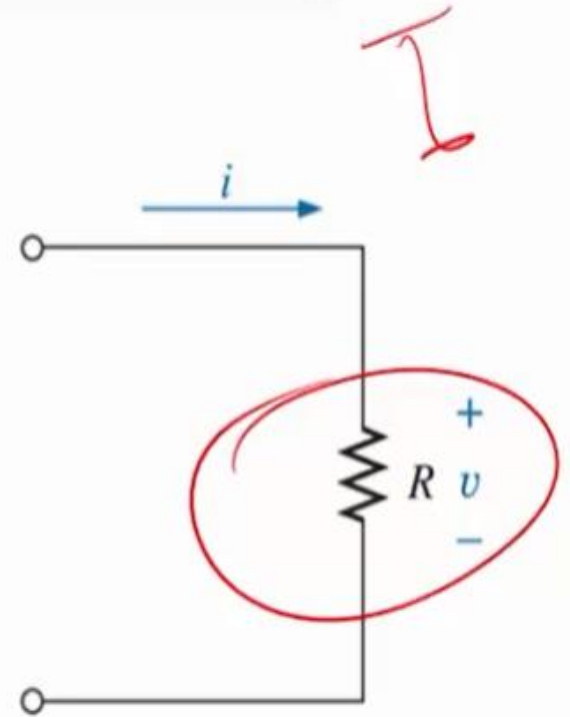
$$I_m = \frac{V_m}{R}$$

In addition, for a given i ,

$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

where

$$V_m = I_m R$$



Resistance

The Basic Elements and Phasors

Response of basic R, L and C elements

1) Resistor

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

$$I_m = \frac{V_m}{R}$$

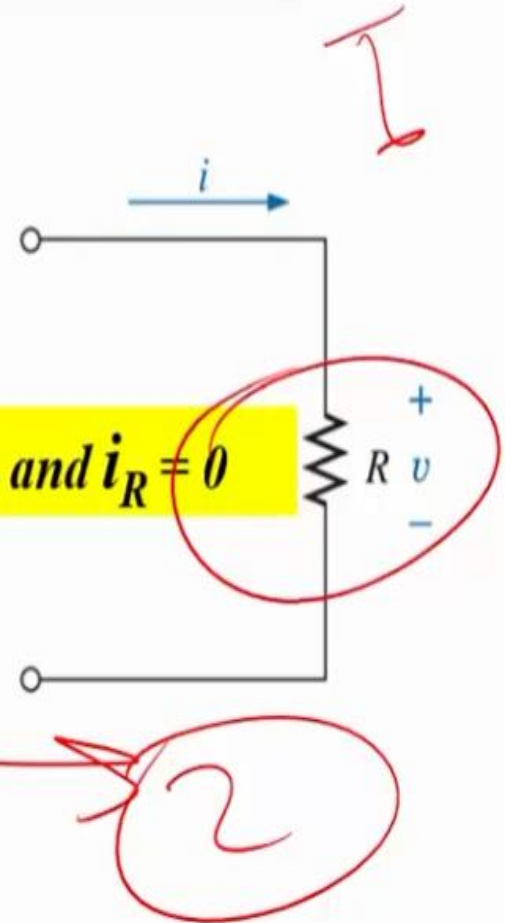
For Resistance, phase shift between v_R and $i_R = 0$

In addition, for a given i ,

$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

where

$$V_m = I_m R$$

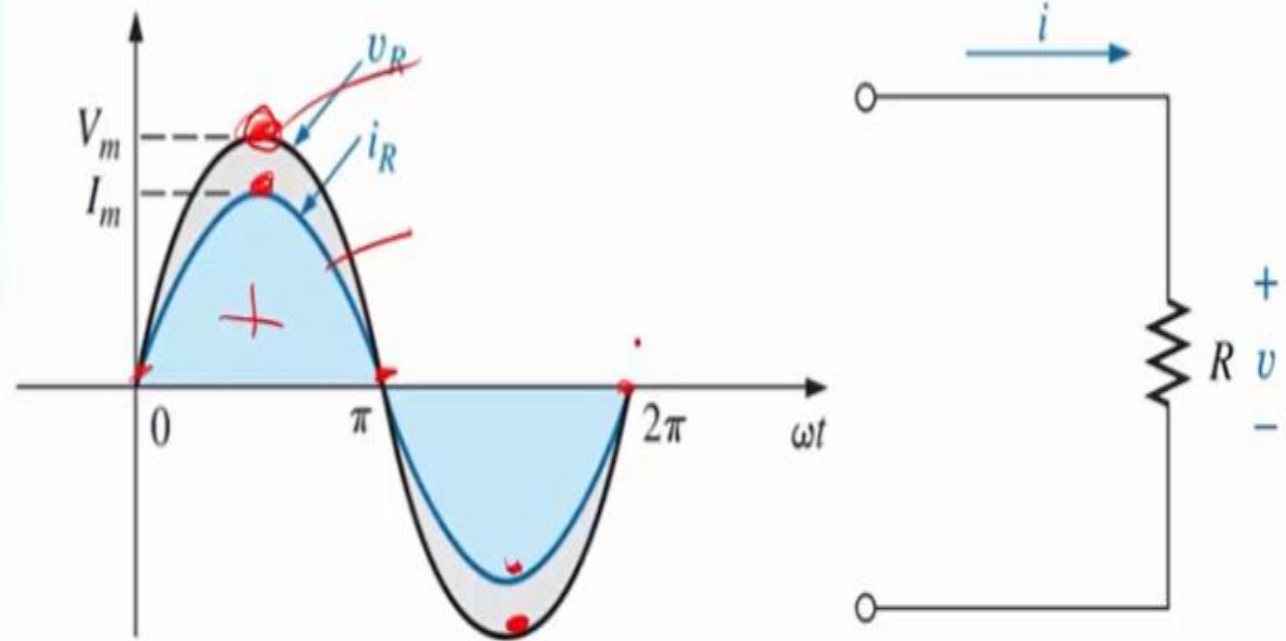


Response of basic R, L and C elements

1) Resistor

for a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.

For Resistance, phase shift between v_R and $i_R = 0$



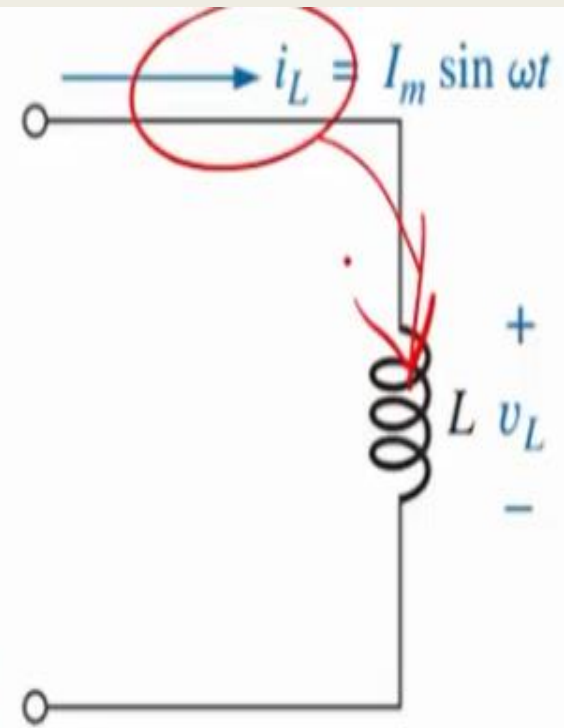
2) Inductor

$$X_L = \omega L$$

(ohms, Ω)

$$v_L = V_m \sin(\omega t + 90^\circ)$$

$$V_m = \omega L I_m$$



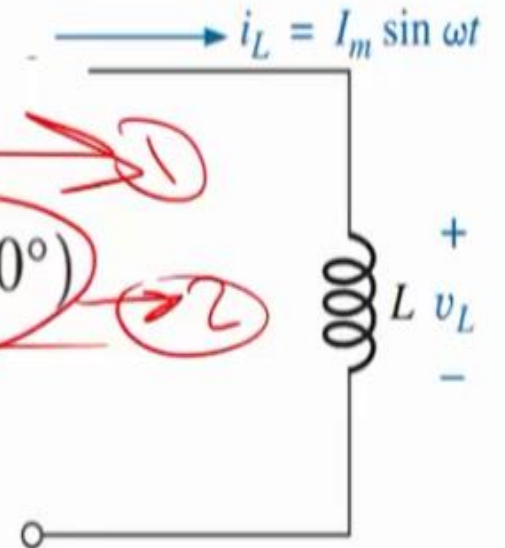
Note that the peak value of v_L is directly related to $\omega (= 2\pi f)$

Response of basic R, L and C elements

2) Inductor

$$i_L = I_m \sin(\omega t \pm \theta)$$

$$v_L = \omega L I_m \sin(\omega t \pm \theta + 90^\circ)$$



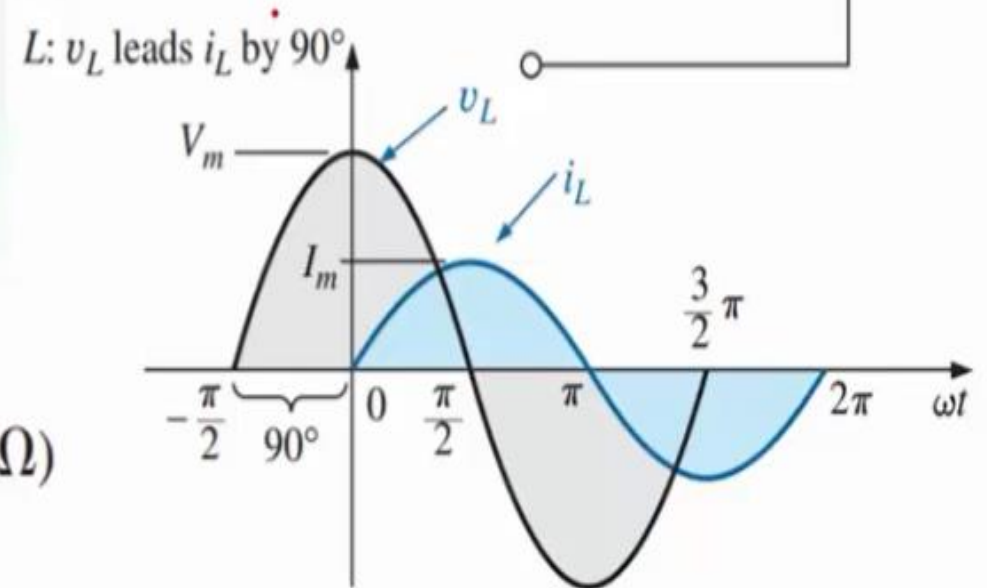
The quantity ωL , called the reactance (from the word reaction) of an inductor, is symbolically represented by X_L and is measured in ohms; that is,

$$X_L = \omega L$$

(ohms, Ω)

$$X_L = \frac{V_m}{I_m}$$

(ohms, Ω)



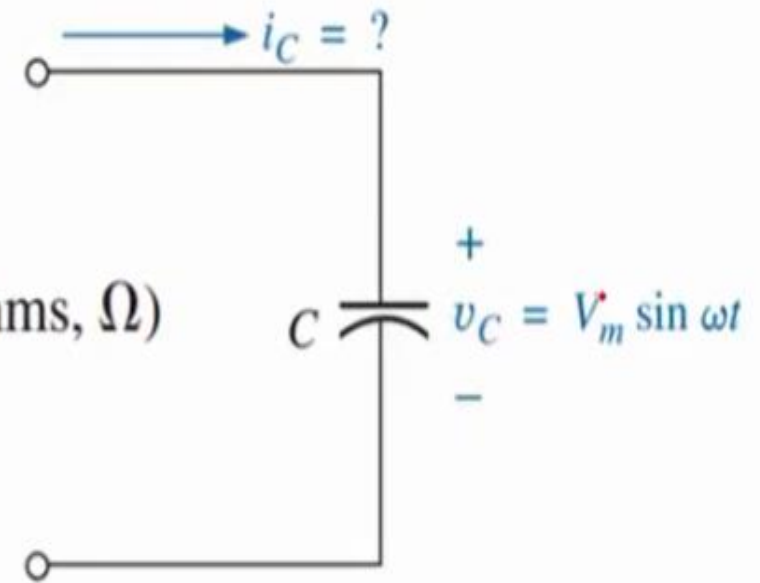
3) Capacitor

$$i_C = I_m \sin(\omega t + 90^\circ)$$

$$I_m = \omega C V_m$$

$$X_C = \frac{1}{\omega C}$$

(ohms, Ω)



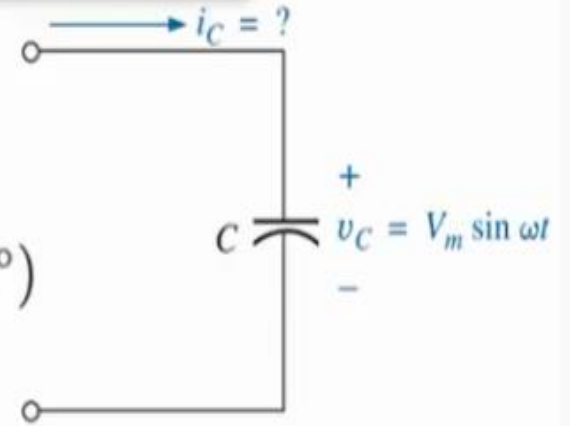
Note that the peak value of i_C is directly related to $\omega (= 2\pi f)$

Response of basic R, L and C elements

3) Capacitor

$$v_C = V_m \sin(\omega t \pm \theta)$$

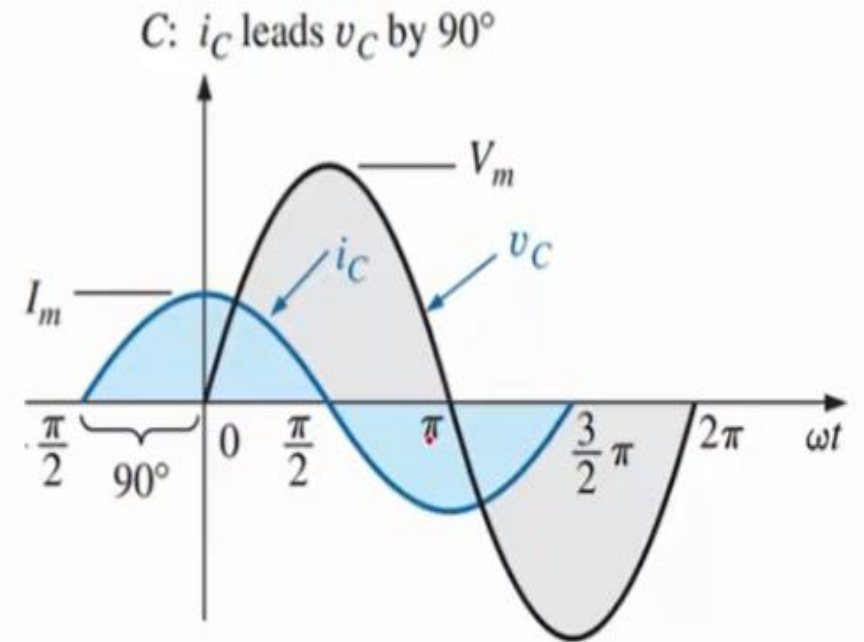
$$i_C = \omega C V_m \sin(\omega t \pm \theta + 90^\circ)$$



The quantity $(1/\omega C)$, called the reactance of a capacitor, is symbolically represented by X_C and is measured in ohms; that is :

$$X_C = \frac{1}{\omega C} \quad (\text{ohms}, \Omega)$$

$$X_C = \frac{V_m}{I_m} \quad (\text{ohms}, \Omega)$$



EXAMPLE The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is $10\ \Omega$. Sketch the curves for v and i .

a. $v = 100 \sin 377t$

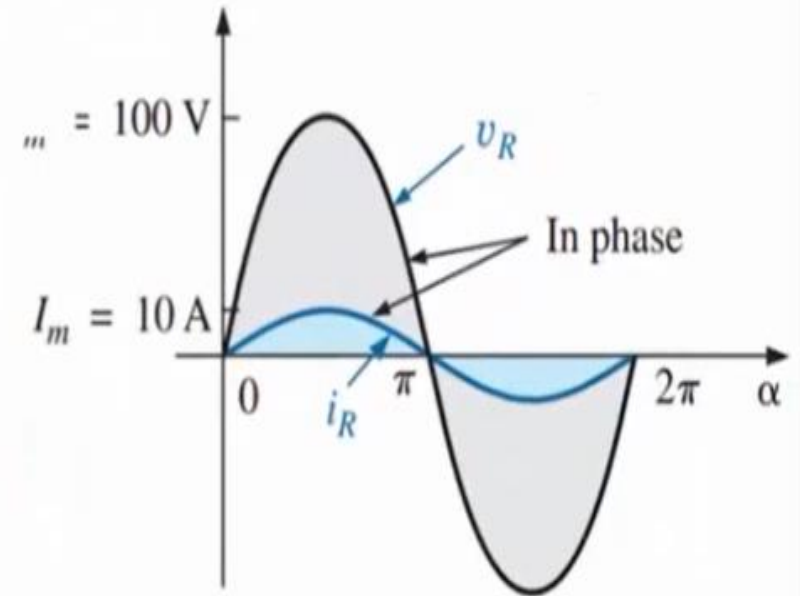
Solutio :

a. Eq. (2): $I_m = \frac{V_m}{R} = \frac{100\text{ V}}{10\ \Omega} = 10\text{ A}$

(v and i are in phase), resulting in

$$i = 10 \sin 377t$$

The curves are sketched in Fig.



EXAMPLE The current through a 0.1 H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a. $i = 10 \sin 377t$

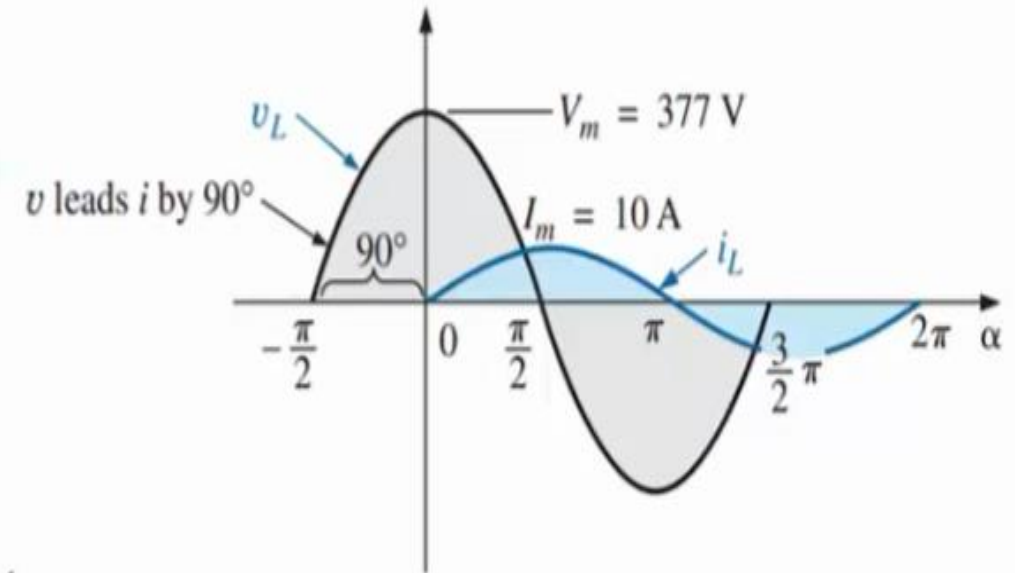
Solutions:

a. Eq. (4): $X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7 \, \Omega$

Eq. (5): $V_m = I_m X_L = (10 \text{ A})(37.7 \, \Omega) = 377 \text{ V}$

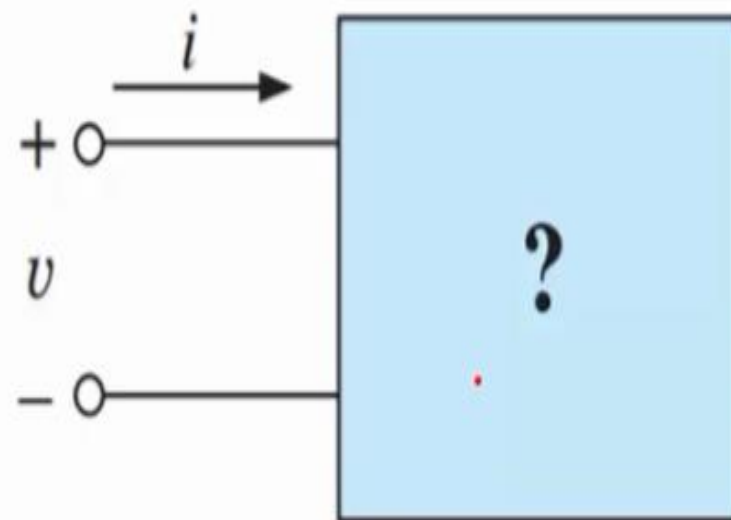
and we know that for a coil v leads i by 90° . Therefore,

$$v = 377 \sin(377t + 90^\circ)$$



EXAMPLE For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor. Determine the value of C , L , or R if sufficient data are provided (Fig. 18):

- a. $v = 100 \sin(\omega t + 40^\circ)$
 $i = 20 \sin(\omega t + 40^\circ)$
- b. $v = 1000 \sin(377t + 10^\circ)$
 $i = 5 \sin(377t - 80^\circ)$
- c. $v = 500 \sin(157t + 30^\circ)$
 $i = 1 \sin(157t + 120^\circ)$



Response of basic R, L and C elements

Solutions:

- a. Since v and i are *in phase*, the element is a *resistor*, and

$$R = \frac{V_m}{I_m} = \frac{100 \text{ V}}{20 \text{ A}} = 5 \Omega$$

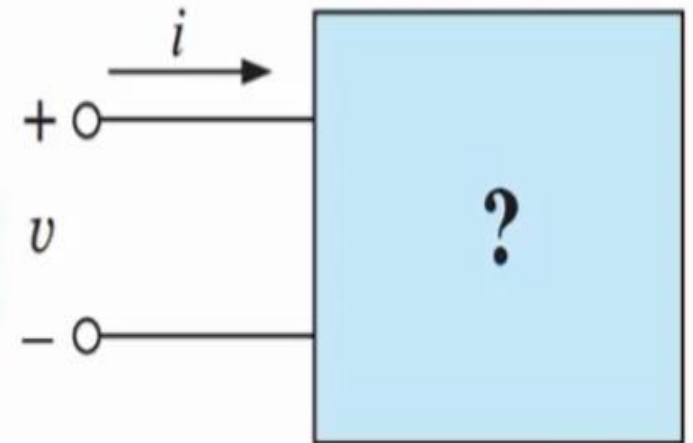
- b. Since v *leads* i by 90° , the element is an *inductor*, and

$$X_L = \frac{V_m}{I_m} = \frac{1000 \text{ V}}{5 \text{ A}} = 200 \Omega$$

so that $X_L = \omega L = 200 \Omega$ or

$$L = \frac{200 \Omega}{\omega} = \frac{200 \Omega}{377 \text{ rad/s}} = 0.53 \text{ H}$$

a. $v = 100 \sin(\omega t + 40^\circ)$
 $i = 20 \sin(\omega t + 40^\circ)$
b. $v = 1000 \sin(377t + 10^\circ)$
 $i = 5 \sin(377t - 80^\circ)$



Response of basic R, L and C elements

Solutions:

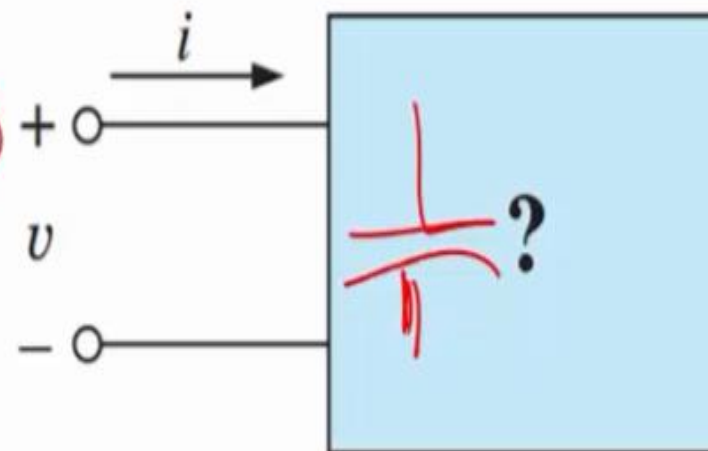
$$\begin{aligned} \text{c. } v &= 500 \sin(157t + 30^\circ) \\ i &= 1 \sin(157t + 120^\circ) \end{aligned}$$

c. Since i leads v by 90° , the element is a *capacitor*, and

$$X_C = \frac{V_m}{I_m} = \frac{500 \text{ V}}{1 \text{ A}} = 500 \Omega$$

so that $X_C = \frac{1}{\omega C} = 500 \Omega$ or

$$C = \frac{1}{\omega 500 \Omega} = \frac{1}{(157 \text{ rad/s})(500 \Omega)} = \mathbf{12.74 \text{ F}}$$



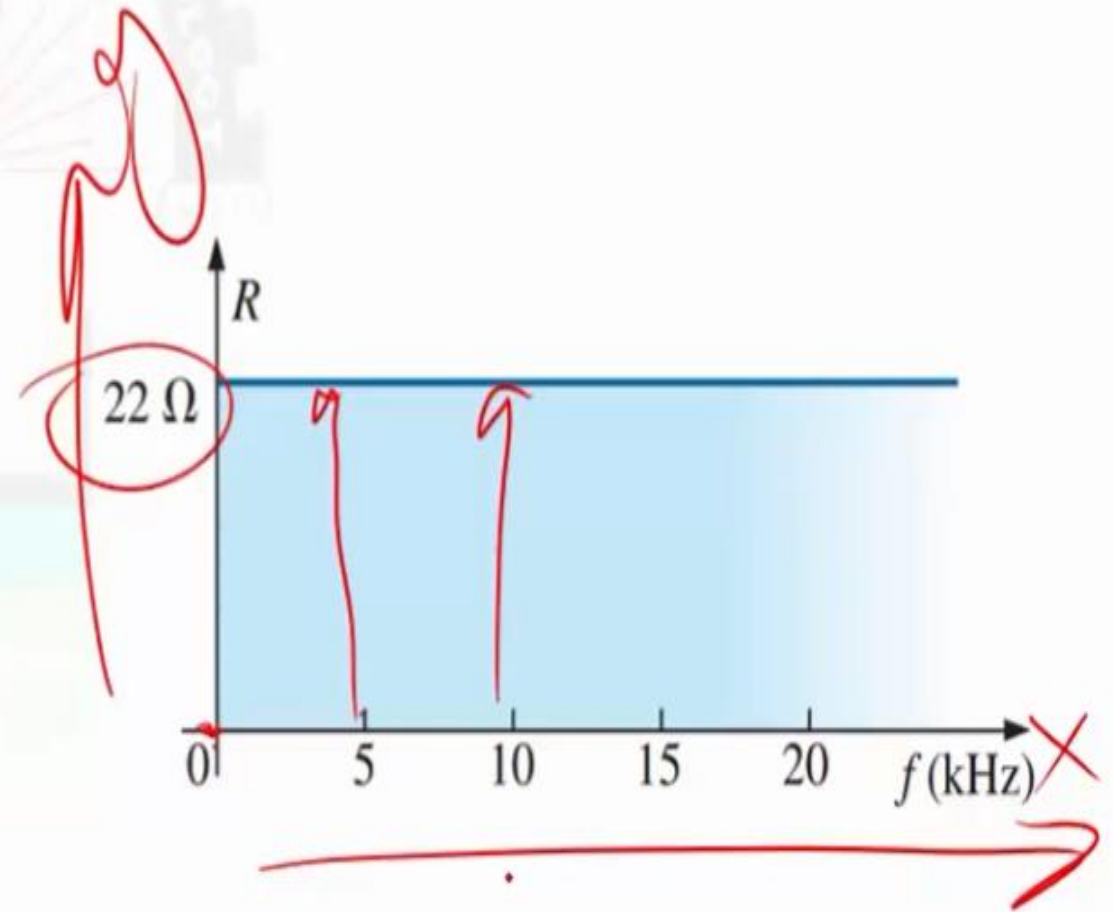
Frequency Response of The Basic Elements

Ideal Response

Resistor R For an ideal resistor, you can assume that frequency will have absolutely no effect on the impedance level, as shown by the response in Fig. Note that at 5 kHz or 20 kHz, the resistance of the resistor remains

at 22Ω ; there is no change whatsoever.

$$R = V/I.$$

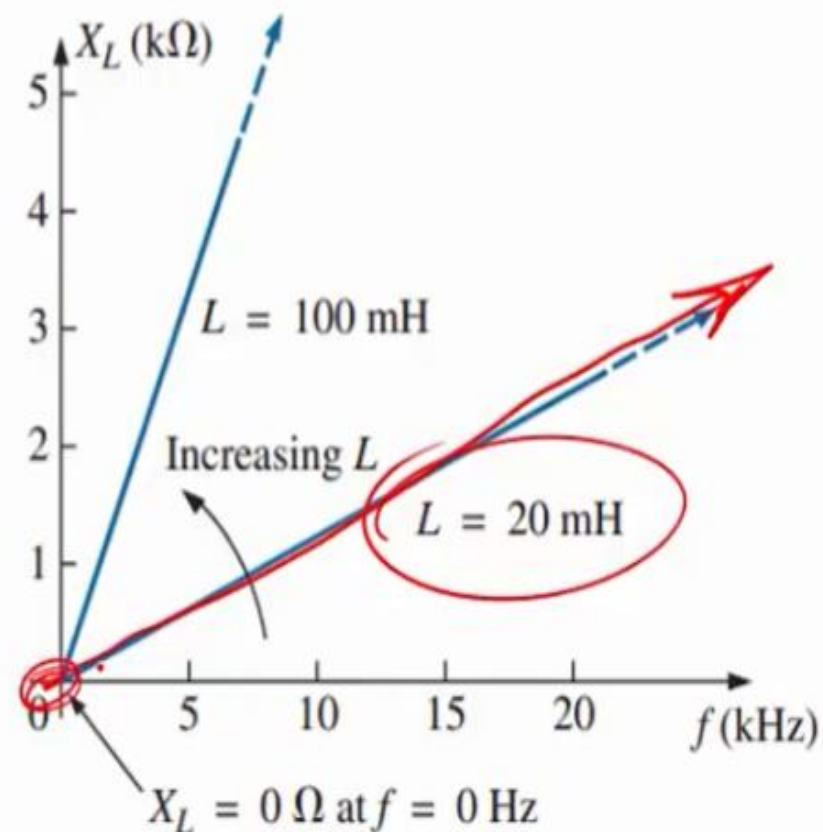


Frequency Response of The Basic Elements

Ideal Response

Inductor L For the ideal inductor, the equation for the reactance can be written as follows to isolate the frequency term in the equation. The result is a constant times the frequency variable that changes as we move down the horizontal axis of a plot.

$$X_L = \omega L = 2\pi f L = (2\pi L)f$$

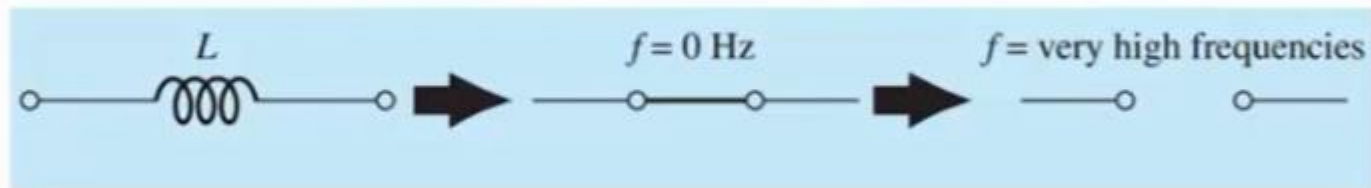


Frequency Response of The Basic Elements

Ideal Response

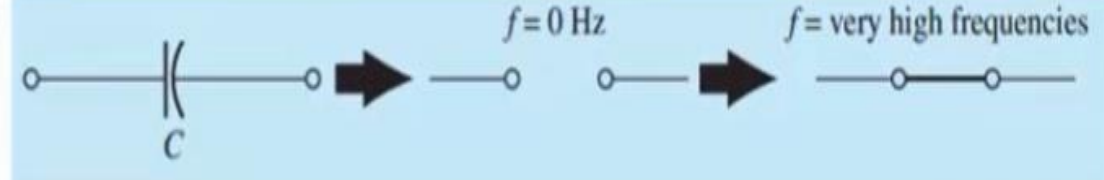
*At a frequency of 0 Hz, an inductor takes on the characteristics of a **short circuit**, as shown in Fig.* $X_L = 2\pi fL = 2\pi(0 \text{ Hz})L = 0 \Omega$

*At very high frequencies, the characteristics of an inductor approach those of an **open circuit**, as shown in Fig.*



Frequency Response of The Basic Elements

Ideal Response

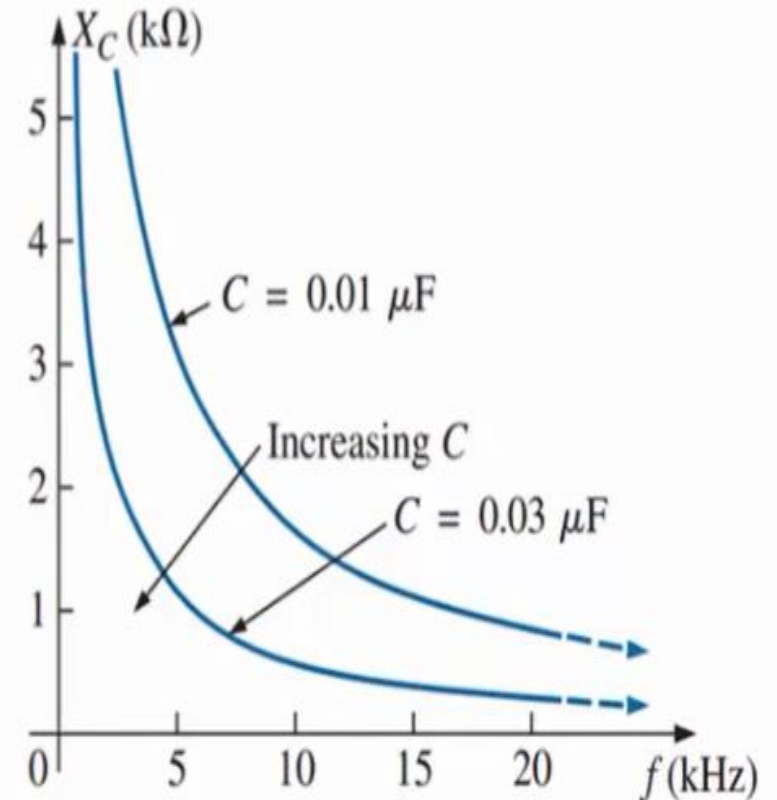


Capacitor C For the capacitor, the equation for the reactance

$$X_C = \frac{1}{2\pi fC}$$

*At or near 0 Hz, the characteristics of a capacitor approach those of an **open circuit**, as shown in Fig.*

*At very high frequencies, a capacitor takes on the characteristics of a **short circuit**, as shown in Fig.*



EXAMPLE At what frequency will the reactance of a 200 mH inductor match the resistance level of a 5 k Ω resistor?

Solution: The resistance remains constant at 5 k Ω for the frequency range of the inductor. Therefore,

$$\begin{aligned} R &= 5000 \, \Omega = X_L = 2\pi fL = 2\pi Lf \\ &= 2\pi(200 \times 10^{-3} \, \text{H})f = 1.257f \end{aligned}$$

and

$$f = \frac{5000 \, \text{Hz}}{1.257} \cong \mathbf{3.98 \, \text{kHz}}$$

EXAMPLE At what frequency will an inductor of 5 mH have the same reactance as a capacitor of $0.1 \mu\text{F}$?

Solution:

$$\begin{aligned}X_L &= X_C \\2\pi fL &= \frac{1}{2\pi fC} \\f^2 &= \frac{1}{4\pi^2 LC}\end{aligned}$$

and

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5 \times 10^{-3} \text{ H})(0.1 \times 10^{-6})}} = \frac{1}{2\pi\sqrt{5 \times 10^{-10}}} = \frac{1}{(2\pi)(2.236 \times 10^{-5})} = \frac{10^5 \text{ Hz}}{14.05} \cong 7.12 \text{ kHz}$$

EXAMPLE The voltage across a $1\ \mu\text{F}$ capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.

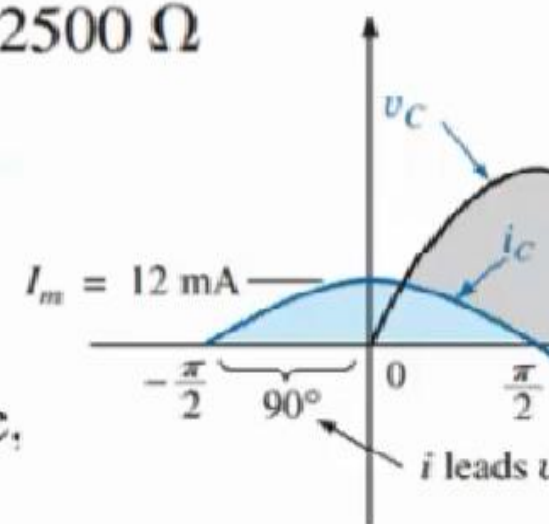
$$v = 30 \sin 400t$$

$$\text{Eq. (6): } X_C = \frac{1}{\omega C} = \frac{1}{(400\ \text{rad/s})(1 \times 10^{-6}\ \text{F})} = \frac{10^6\ \Omega}{400} = 2500\ \Omega$$

$$\text{Eq. (7): } I_m = \frac{V_m}{X_C} = \frac{30\ \text{V}}{2500\ \Omega} = 0.0120\ \text{A} = 12\ \text{mA}$$

and we know that for a capacitor i leads v by 90° . Therefore,

$$i = 12 \times 10^{-3} \sin(400t + 90^\circ)$$



Conclusion

The Basic Elements and Phasors

Phasor notation

Response of The Basic
Elements

Frequency Response of
The Basic Elements