

Sinusoidal alternating waveform

introduction

Sinusoidal alternating waveform

- Different Alternative waveforms are shown below

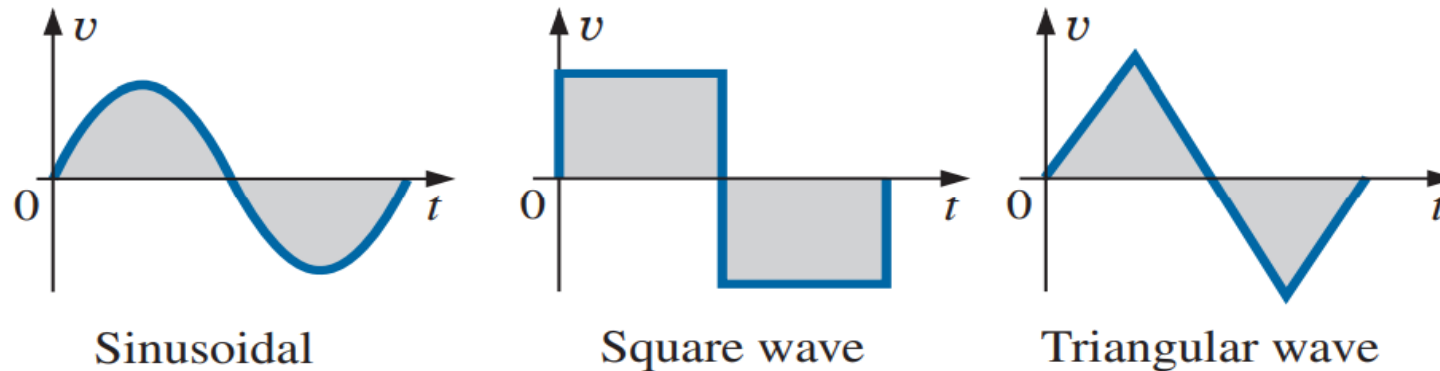


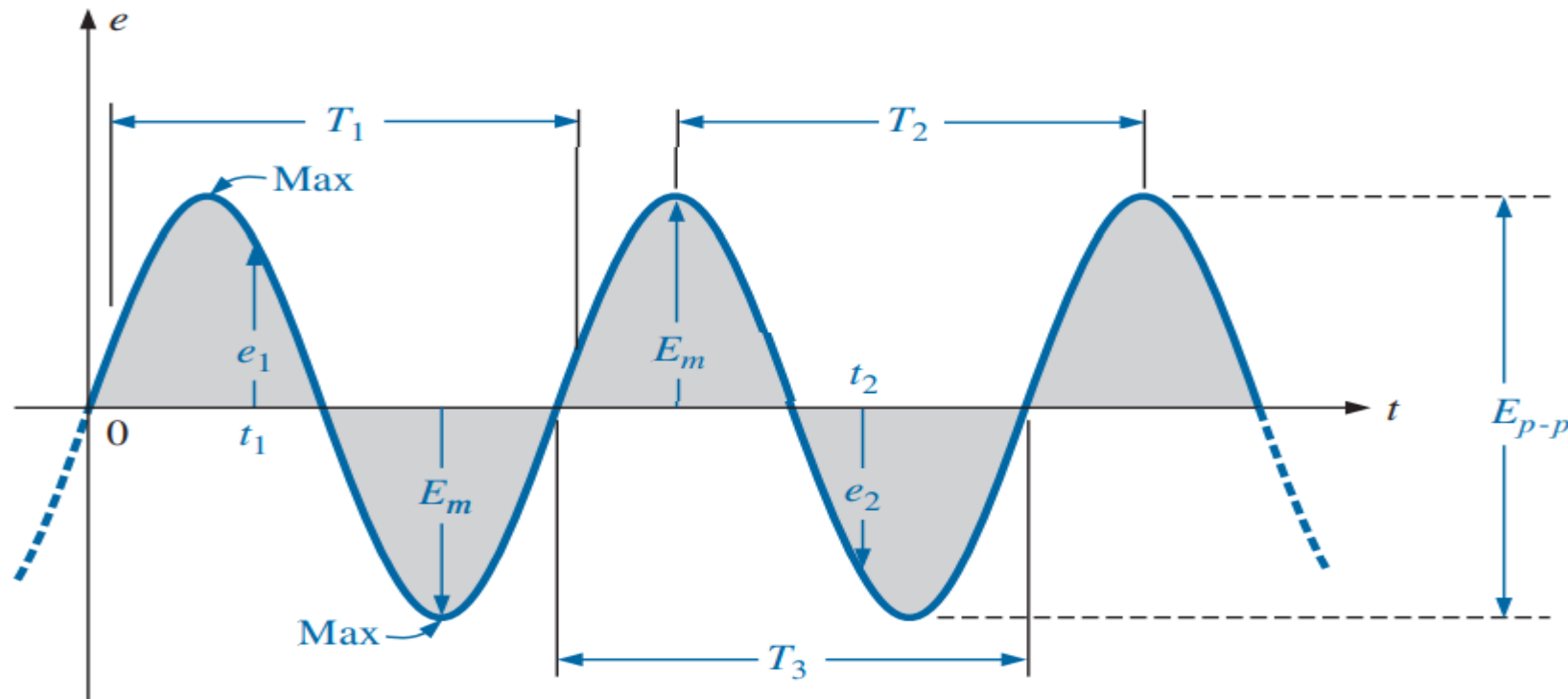
FIG. 1

Alternating waveforms.

SINUSOIDAL ac VOLTAGE CHARACTERISTICS AND DEFINITIONS

- **Definitions**

- The sinusoidal waveform in Fig. with its additional notation will now be used as a model in defining a few basic terms. These terms, however, can be applied to any alternating waveform. It is important to remember, as you proceed through the various definitions, that the vertical scaling is in volts or amperes and the horizontal scaling is in units of time



Definitions

- **Waveform**: The path traced by a quantity, such as the voltage in Fig. 3, plotted as a function of some variable, such as time (as above), position, degrees, radians, temperature, and so on.
- **Instantaneous value**: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1 , e_2 in Fig. 3).
- **Peak amplitude**: The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters [such as E_m (Fig. 3) for sources of voltage and V_m for the voltage drop across a load]. For the waveform in Fig. 3, the average value is zero volts, and E_m is as defined by the figure

continue

- **Peak value:** The maximum instantaneous value of a function as measured from the zero volt level. For the waveform in Fig. 3, the peak amplitude and peak value are the same since the average value of the function is zero volts.
- **Peak-to-peak value:** Denoted by E_{p-p} or V_{p-p} (as shown in Fig. 3), the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks

continue

- **Periodic waveform:** A waveform that continually repeats itself after the same time interval. The waveform in Fig. 3 is a periodic waveform.
- **Period (T):** The time of a periodic waveform.
- **Cycle:** The portion of a waveform contained in one period of time. The cycles within T1, T2, and T3 in Fig. 3 may appear different in Fig. 4, but they are all bounded by one period of time and therefore satisfy the definition of a cycle.

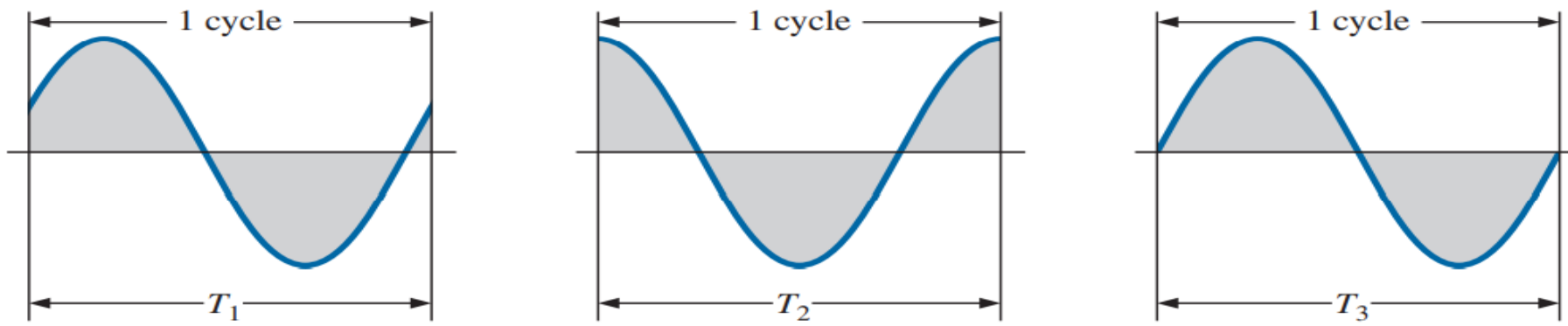


FIG. 4

Defining the cycle and period of a sinusoidal waveform.

Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform in Fig. 5(a) is 1 cycle per second, and for Fig. 5(b), $2\frac{1}{2}$ cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig. 5(c)], the frequency would be 2 cycles per second.

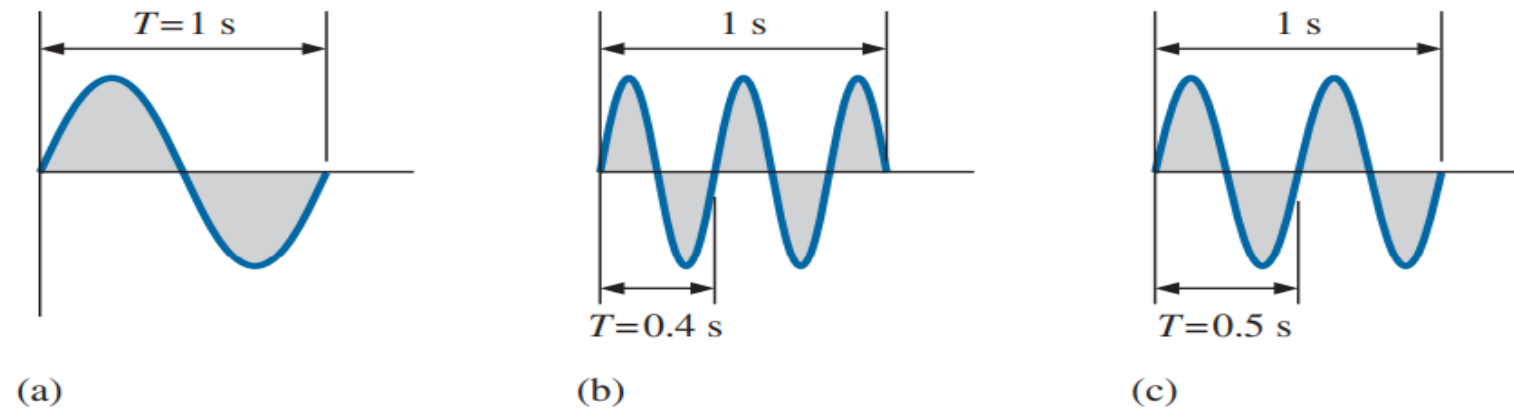


FIG. 5

Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

continue

- The unit of measure for frequency is the hertz (Hz), where
(1) The unit hertz is derived from the surname of Heinrich Rudolph Hertz (Fig. 6), who did original research in the area of alternating currents and voltages and their effect on the basic R, L, and C elements.
- The frequency standard for North America is 60 Hz, whereas for Europe it is predominantly 50 Hz

example

EXAMPLE 1 For the sinusoidal waveform in Fig. 7.

- What is the peak value?
- What is the instantaneous value at 0.3 s and 0.6 s?
- What is the peak-to-peak value of the waveform?
- What is the period of the waveform?
- How many cycles are shown?
- What is the frequency of the waveform?

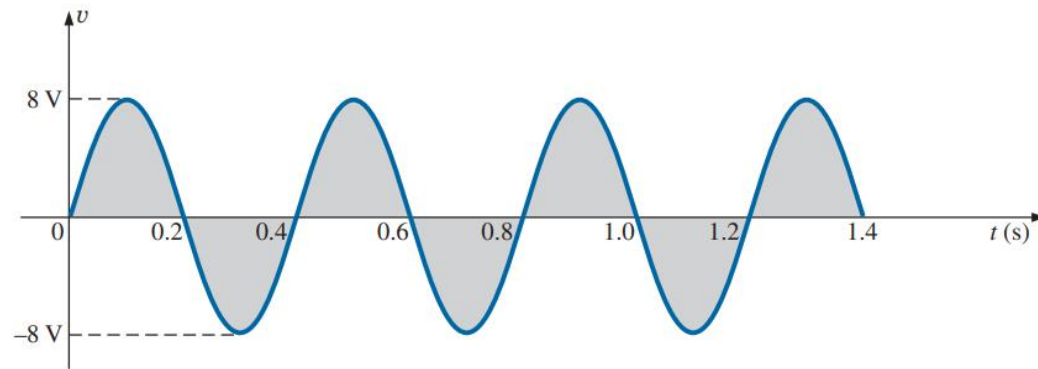


FIG. 7
Example 1.

Solutions:

- 8 V.**
- At 0.3 s, **-8 V**; at 0.6 s, **0 V**.
- 16 V.**
- 0.4 s.**
- 3.5 cycles.**
- 2.5 cps, or 2.5 Hz.**

continue

Since the frequency is inversely related to the period—that is, as one increases, the other decreases by an equal amount—the two can be related by the following equation:

$$\boxed{f = \frac{1}{T}} \quad \begin{array}{l} f = \text{Hz} \\ T = \text{seconds (s)} \end{array} \quad (2)$$

or

$$\boxed{T = \frac{1}{f}} \quad (3)$$

EXAMPLE 2 Find the period of periodic waveform with a frequency of

- a. 60 Hz.
- b. 1000 Hz.

Solutions:

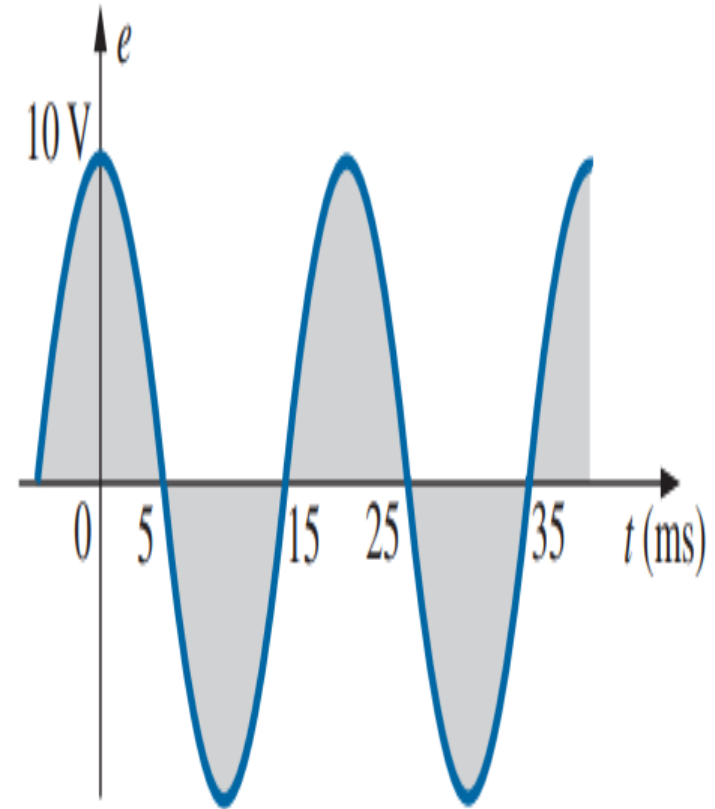
- a. $T = \frac{1}{f} = \frac{1}{60 \text{ Hz}} \cong 0.01667 \text{ s}$ or **16.67 ms**
(a recurring value since 60 Hz is so prevalent)
- b. $T = \frac{1}{f} = \frac{1}{1000 \text{ Hz}} = 10^{-3} \text{ s} = \mathbf{1 \text{ ms}}$

Example

EXAMPLE 3 Determine the frequency of the waveform in Fig. 9.

Solution: From the figure, $T = (25 \text{ ms} - 5 \text{ ms})$ or $(35 \text{ ms} - 15 \text{ ms}) = 20 \text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3} \text{ s}} = \mathbf{50 \text{ Hz}}$$



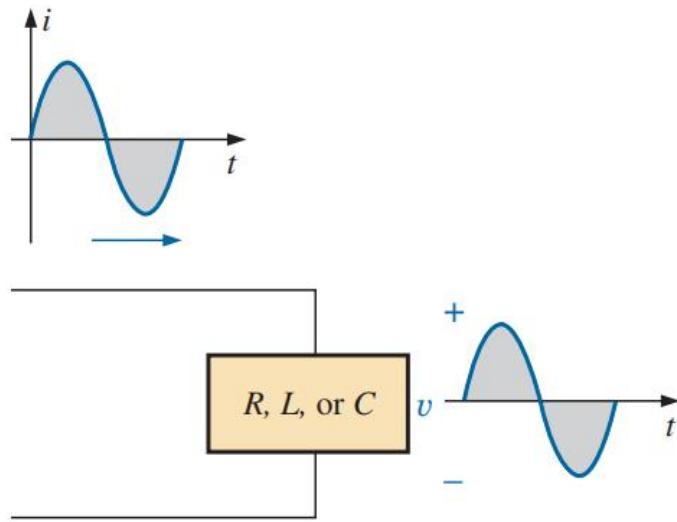


FIG. 12

The sine wave is the only alternating waveform whose shape is not altered by the response characteristics of a pure resistor, inductor, or capacitor.

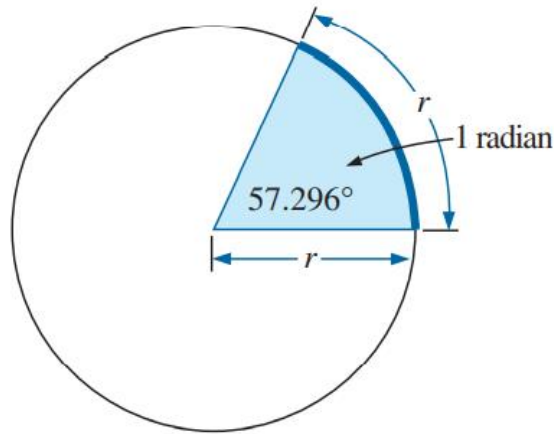


FIG. 13

Defining the radian.

4 THE SINUSOIDAL WAVEFORM

The terms defined in the previous section can be applied to any type of periodic waveform, whether smooth or discontinuous. The sinusoidal waveform is of particular importance, however, since it lends itself readily to the mathematics and the physical phenomena associated with electric circuits. Consider the power of the following statement:

The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R , L , and C elements.

In other words, if the voltage across (or current through) a resistor, inductor, or capacitor is sinusoidal in nature, the resulting current (or voltage, respectively) for each will also have sinusoidal characteristics, as shown in Fig. 12. If any other alternating waveform such as a square wave or a triangular wave were applied, such would not be the case.

The unit of measurement for the horizontal axis can be **time** (as appearing in the figures thus far), **degrees**, or **radians**. The term **radian** can be defined as follows: If we mark off a portion of the circumference of a circle by a length equal to the radius of the circle, as shown in Fig. 13, the angle resulting is called *1 radian*. The result is

$$1 \text{ rad} = 57.296^\circ \approx 57.3^\circ \quad (4)$$

where 57.3° is the usual approximation applied.

One full circle has 2π radians, as shown in Fig. 14. That is,

$$2\pi \text{ rad} = 360^\circ \quad (5)$$

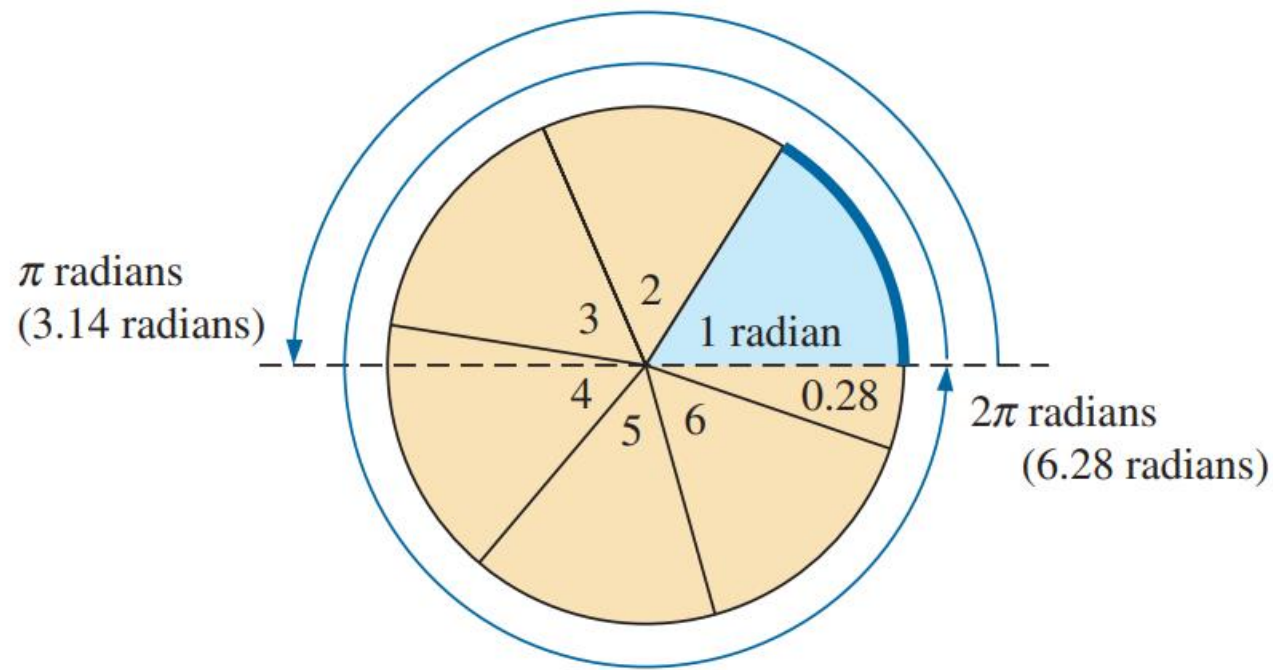


FIG. 14

There are 2π radians in one full circle of 360° .

so that

$$2\pi = 2(3.142) = 6.28$$

and

$$2\pi(57.3^\circ) = 6.28(57.3^\circ) = 359.84^\circ \cong 360^\circ$$

A number of electrical formulas contain a multiplier of π . For this reason, it is sometimes preferable to measure angles in radians rather than in degrees.

The quantity π is the ratio of the circumference of a circle to its diameter.

π has been determined to an extended number of places, primarily in an attempt to see if a repetitive sequence of numbers appears. It does not. A sampling of the effort appears below:

$$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ \dots$$

Although the approximation $\pi \cong 3.14$ is often applied, all the calculations in the text use the π function as provided on all scientific calculators.

For 180° and 360° , the two units of measurement are related as shown in Fig. 14. The conversions equations between the two are the following:

$$\text{Radians} = \left(\frac{\pi}{180^\circ} \right) \times (\text{degrees}) \quad (6)$$

$$\text{Degrees} = \left(\frac{180^\circ}{\pi} \right) \times (\text{radians}) \quad (7)$$

Applying these equations, we find

$$\mathbf{90^\circ: \text{ Radians} = \frac{\pi}{180^\circ}(90^\circ) = \frac{\pi}{2} \text{ rad}}$$

$$\mathbf{30^\circ: \text{ Radians} = \frac{\pi}{180^\circ}(30^\circ) = \frac{\pi}{6} \text{ rad}}$$

$$\mathbf{\frac{\pi}{3} \text{ rad: Degrees} = \frac{180^\circ}{\pi} \left(\frac{\pi}{3} \right) = 60^\circ}$$

$$\mathbf{\frac{3\pi}{2} \text{ rad: Degrees} = \frac{180^\circ}{\pi} \left(\frac{3\pi}{2} \right) = 270^\circ}$$

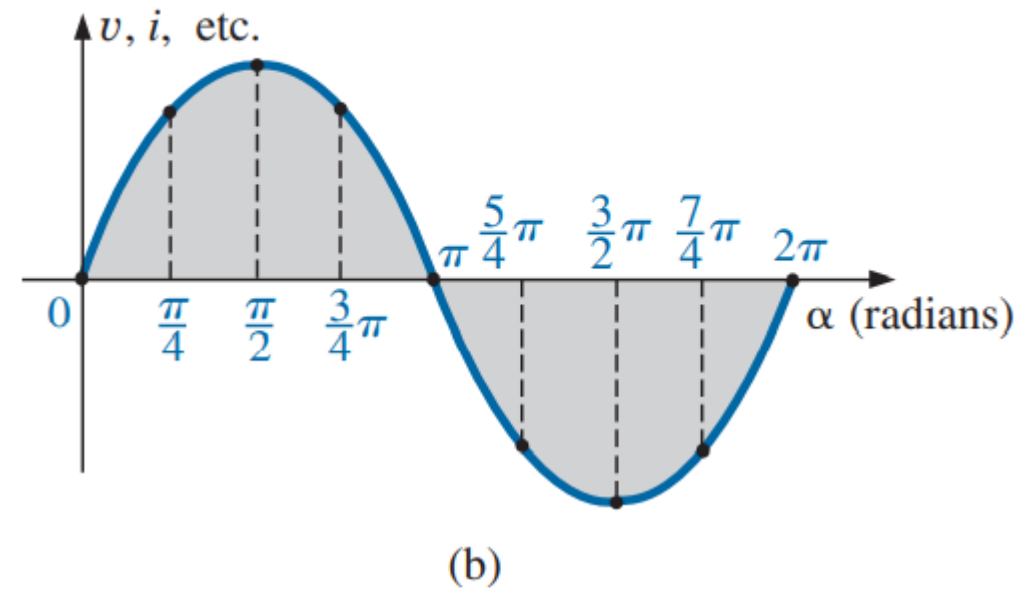
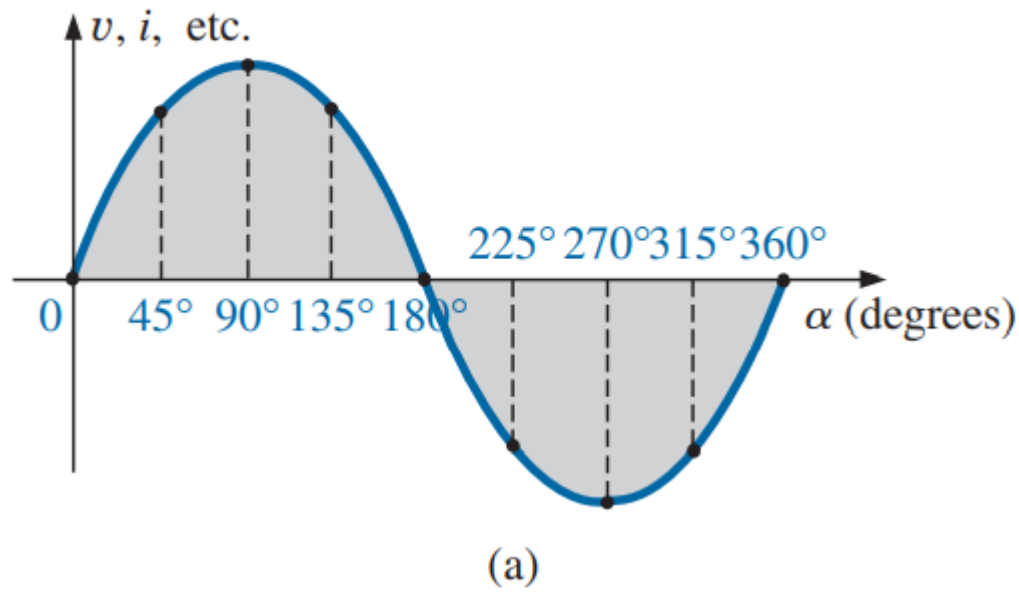


FIG. 15

Plotting a sine wave versus (a) degrees and (b) radians.

For comparison purposes, two sinusoidal voltages are plotted in Fig. 15 using degrees and radians as the units of measurement for the horizontal axis.

The velocity with which the radius vector rotates about the center, called the **angular velocity**, can be determined from the following equation:

$$\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}} \quad (8)$$

Substituting into Eq. (8) and assigning the lowercase Greek letter *omega* (ω) to the angular velocity, we have

$$\omega = \frac{\alpha}{t} \quad (9)$$

and

$$\alpha = \omega t \quad (10)$$

Since ω is typically provided in radians per second, the angle α obtained using Eq. (10) is usually in radians. If α is required in degrees, Eq. (7) must be applied. The importance of remembering the above will become obvious in the examples to follow.

In Fig. 16, the time required to complete one revolution is equal to the period (T) of the sinusoidal waveform in Fig. 16(i). The radians subtended in this time interval are 2π . Substituting, we have

$$\omega = \frac{2\pi}{T} \quad (\text{rad/s}) \quad (11)$$

In words, this equation states that the smaller the period of the sinusoidal waveform of Fig. 16(i), or the smaller the time interval before one complete cycle is generated, the greater must be the angular velocity of the rotating radius vector. Certainly this statement agrees with what we have learned thus far. We can now go one step further and apply the

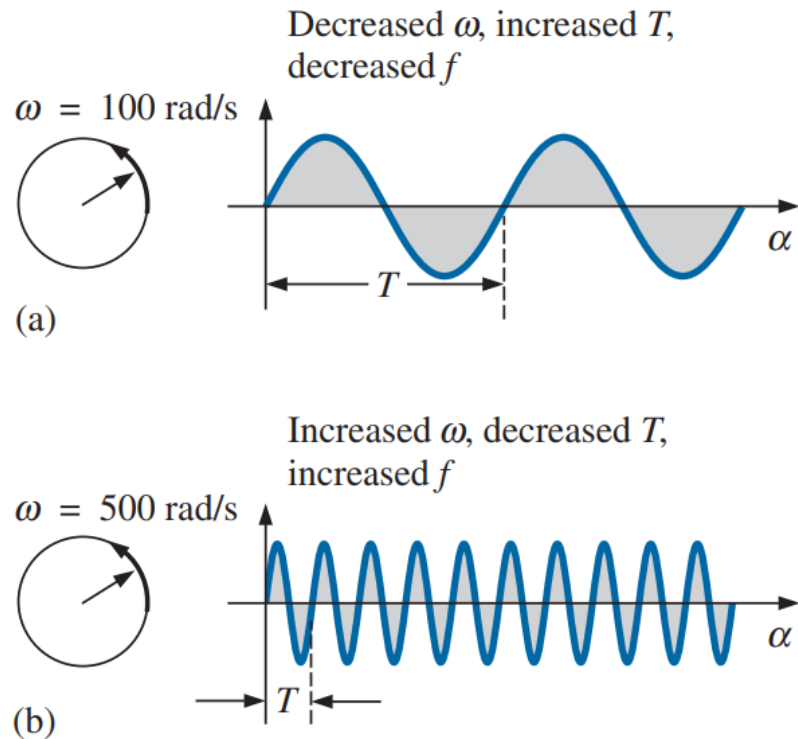


FIG. 17

Demonstrating the effect of ω on the frequency and period.

fact that the frequency of the generated waveform is inversely related to the period of the waveform; that is, $f = 1/T$. Thus,

$$\boxed{\omega = 2\pi f} \quad (\text{rad/s}) \quad (12)$$

This equation states that the higher the frequency of the generated sinusoidal waveform, the higher must be the angular velocity. Eqs. (11) and (12) are verified somewhat by Fig. 17, where for the same radius vector, $\omega = 100$ rad/s and 500 rad/s.

EXAMPLE 4 Determine the angular velocity of a sine wave having a frequency of 60 Hz.

Solution:

$$\omega = 2\pi f = (2\pi)(60 \text{ Hz}) \cong \mathbf{377 \text{ rad/s}}$$

(a recurring value due to 60 Hz predominance).

EXAMPLE 5 Determine the frequency and period of the sine wave in Fig. 17(b).

Solution: Since $\omega = 2\pi/T$,

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{500 \text{ rad/s}} = \frac{2\pi \text{ rad}}{500 \text{ rad/s}} = \mathbf{12.57 \text{ ms}}$$

and
$$f = \frac{1}{T} = \frac{1}{12.57 \times 10^{-3} \text{ s}} = \mathbf{79.58 \text{ Hz}}$$

EXAMPLE 6 Given $\omega = 200 \text{ rad/s}$, determine how long it will take the sinusoidal waveform to pass through an angle of 90° .

Solution: Eq. (10): $\alpha = \omega t$, and

$$t = \frac{\alpha}{\omega}$$

However, α must be substituted as $\pi/2$ ($= 90^\circ$) since ω is in radians per second:

$$t = \frac{\alpha}{\omega} = \frac{\pi/2 \text{ rad}}{200 \text{ rad/s}} = \frac{\pi}{400} \text{ s} = \mathbf{7.85 \text{ ms}}$$

EXAMPLE 7 Find the angle through which a sinusoidal waveform of 60 Hz will pass in a period of 5 ms.

Solution: Eq. (11): $\alpha = \omega t$, or

$$\alpha = 2\pi ft = (2\pi)(60 \text{ Hz})(5 \times 10^{-3} \text{ s}) = \mathbf{1.885 \text{ rad}}$$

If not careful, you might be tempted to interpret the answer as 1.885° . However,

$$\alpha(^{\circ}) = \frac{180^{\circ}}{\pi \text{ rad}}(1.885 \text{ rad}) = \mathbf{108^{\circ}}$$

5 GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is

$$A_m \sin \alpha \quad (13)$$

where A_m is the peak value of the waveform and α is the unit of measure for the horizontal axis, as shown in Fig. 18.

The equation $\alpha = \omega t$ states that the angle α through which the rotating vector in Fig. 16 will pass is determined by the angular velocity of the rotating vector and the length of time the vector rotates. For example, for a particular angular velocity (fixed ω), the longer the radius vector is permitted to rotate (that is, the greater the value of t), the greater is the number of degrees or radians through which the vector will pass. Relating this statement to the sinusoidal waveform, we have that, for a particular angular velocity, the longer the time, the greater is the number of cycles shown. For a fixed time interval, the greater is the angular velocity, the greater is the number of cycles generated.

Due to Eq. (10), the general format of a sine wave can also be written

$$A_m \sin \omega t \quad (14)$$

with ωt as the horizontal unit of measure.

For electrical quantities such as current and voltage, the general format is

$$\begin{aligned} i &= I_m \sin \omega t = I_m \sin \alpha \\ e &= E_m \sin \omega t = E_m \sin \alpha \end{aligned}$$

where the capital letters with the subscript m represent the amplitude, and the lowercase letters i and e represent the instantaneous value of current and voltage, respectively, at any time t . This format is particularly important because it presents the sinusoidal voltage or current as a function of time, which is the horizontal scale for the oscilloscope. Recall that the horizontal sensitivity of a scope is in time per division, not degrees per centimeter.

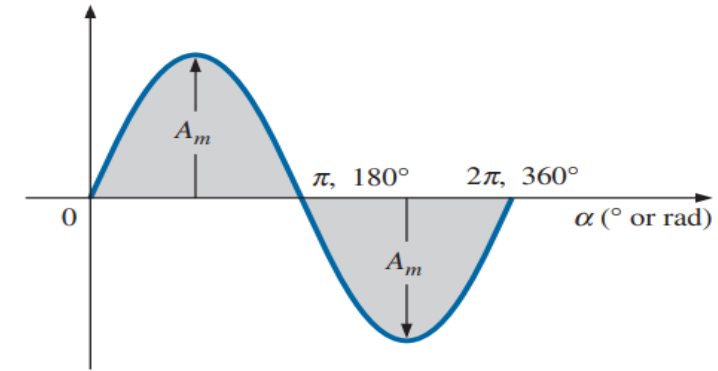


FIG. 18
Basic sinusoidal function.

EXAMPLE 8 Given $e = 5 \sin \alpha$, determine e at $\alpha = 40^\circ$ and $\alpha = 0.8\pi$.

Solution: For $\alpha = 40^\circ$,

$$e = 5 \sin 40^\circ = 5(0.6428) = \mathbf{3.21 \text{ V}}$$

For $\alpha = 0.8\pi$,

$$\alpha(^{\circ}) = \frac{180^{\circ}}{\pi}(0.8 \pi) = 144^{\circ}$$

and

$$e = 5 \sin 144^\circ = 5(0.5878) = \mathbf{2.94 \text{ V}}$$

The angle at which a particular voltage level is attained can be determined by rearranging the equation

$$e = E_m \sin \alpha$$

continue

in the following manner:

$$\sin \alpha = \frac{e}{E_m}$$

which can be written

$$\alpha = \sin^{-1} \frac{e}{E_m} \quad (15)$$

Similarly, for a particular current level,

$$\alpha = \sin^{-1} \frac{i}{I_m} \quad (16)$$

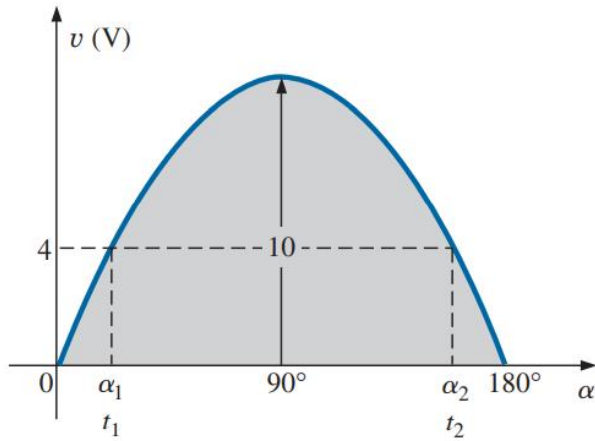


FIG. 19
Example 9.

EXAMPLE 9

- Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.
- Determine the time at which the magnitude is attained.

Solutions:

- Eq. (15):

$$\alpha_1 = \sin^{-1} \frac{v}{E_m} = \sin^{-1} \frac{4 \text{ V}}{10 \text{ V}} = \sin^{-1} 0.4 = \mathbf{23.58^\circ}$$

However, Fig. 19 reveals that the magnitude of 4 V (positive) will be attained at two points between 0° and 180° . The second intersection is determined by

$$\alpha_2 = 180^\circ - 23.578^\circ = \mathbf{156.42^\circ}$$

In general, therefore, keep in mind that Eqs. (15) and (16) will provide an angle with a magnitude between 0° and 90° .

- Eq. (10): $\alpha = \omega t$, and so $t = \alpha/\omega$. However, α must be in radians. Thus,

$$\alpha(\text{rad}) = \frac{\pi}{180^\circ}(23.578^\circ) = 0.412 \text{ rad}$$

and
$$t_1 = \frac{\alpha}{\omega} = \frac{0.412 \text{ rad}}{377 \text{ rad/s}} = \mathbf{1.09 \text{ ms}}$$

For the second intersection,

$$\alpha(\text{rad}) = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73 \text{ rad}$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73 \text{ rad}}{377 \text{ rad/s}} = \mathbf{7.24 \text{ ms}}$$

EXAMPLE 10 Sketch $e = 10 \sin 314t$ with the abscissa a. angle (α) in degrees. b. angle (α) in radians. c. time (t) in seconds

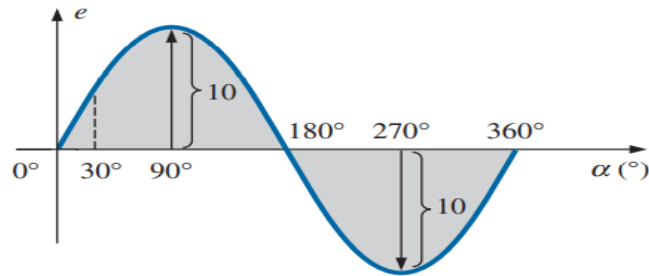


FIG. 24

Example 10, horizontal axis in degrees.

Solutions:

- a. See Fig. 24. (Note that no calculations are required.)
- b. See Fig. 25. (Once the relationship between degrees and radians is understood, no calculations are required.)
- c. See Fig. 26.

$$360^\circ: \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{314} = 20 \text{ ms}$$

$$180^\circ: \quad \frac{T}{2} = \frac{20 \text{ ms}}{2} = 10 \text{ ms}$$

$$90^\circ: \quad \frac{T}{4} = \frac{20 \text{ ms}}{4} = 5 \text{ ms}$$

$$30^\circ: \quad \frac{T}{12} = \frac{20 \text{ ms}}{12} = 1.67 \text{ ms}$$

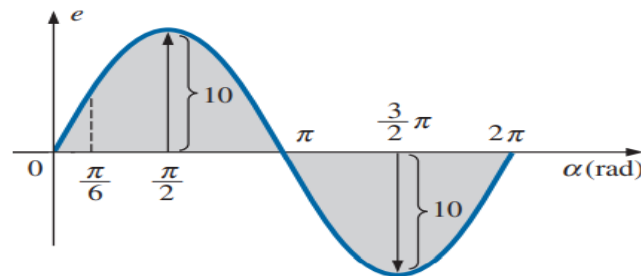


FIG. 25

Example 10, horizontal axis in radians.

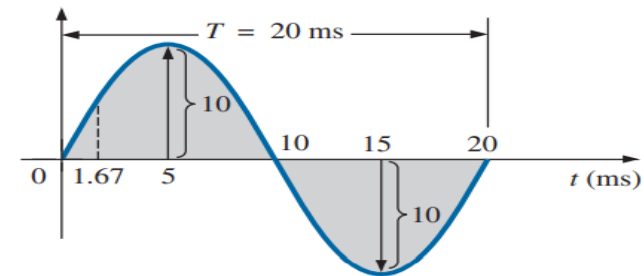


FIG. 26

Example 10, horizontal axis in milliseconds.

Phase shift

6 PHASE RELATIONS

Thus far, we have considered only sine waves that have maxima at $\pi/2$ and $3\pi/2$, with a zero value at 0 , π , and 2π , as shown in Fig. 25. If the waveform is shifted to the right or left of 0° , the expression becomes

$$A_m \sin(\omega t \pm \theta) \quad (17)$$

where θ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a *positive-going* (increasing with time) slope *before* 0° , as shown in Fig. 27, the expression is

$$A_m \sin(\omega t + \theta) \quad (18)$$

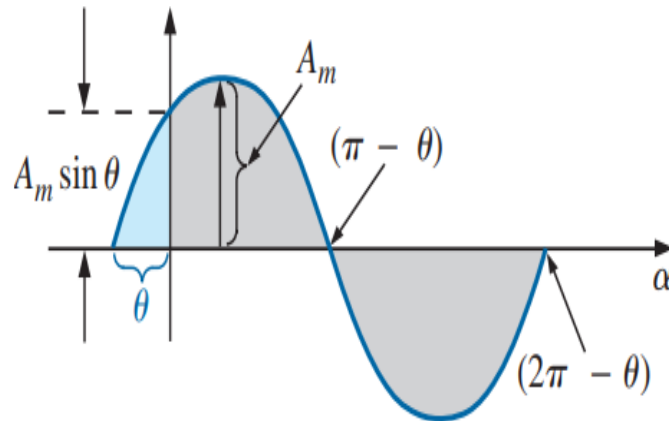


FIG. 27

Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope before 0° .

At $\omega t = \alpha = 0^\circ$, the magnitude is determined by $A_m \sin \theta$. If the waveform passes through the horizontal axis with a positive-going slope *after* 0° , as shown in Fig. 28, the expression is

$$A_m \sin(\omega t - \theta) \quad (19)$$

Finally, at $\omega t = \alpha = 0^\circ$, the magnitude is $A_m \sin(-\theta)$, which, by a trigonometric identity, is $-A_m \sin \theta$.

If the waveform crosses the horizontal axis with a positive-going slope 90° ($\pi/2$) sooner, as shown in Fig. 29, it is called a *cosine wave*; that is,

$$\sin(\omega t + 90^\circ) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t \quad (20)$$

or

$$\sin \omega t = \cos(\omega t - 90^\circ) = \cos\left(\omega t - \frac{\pi}{2}\right) \quad (21)$$

The terms **leading** and **lagging** are used to indicate the relationship between two sinusoidal waveforms of the *same frequency* plotted on the same set of axes. In Fig. 29, the cosine curve is said to *lead* the sine curve by 90° , and the sine curve is said to *lag* the cosine curve by 90° . The 90° is referred to as the phase angle between the two waveforms. In language commonly applied, the waveforms are *out of phase* by 90° . Note that the phase angle between the two waveforms is measured between those two points on the horizontal axis through which each passes with the *same slope*. If both waveforms cross the axis at the same point with the same slope, they are *in phase*.

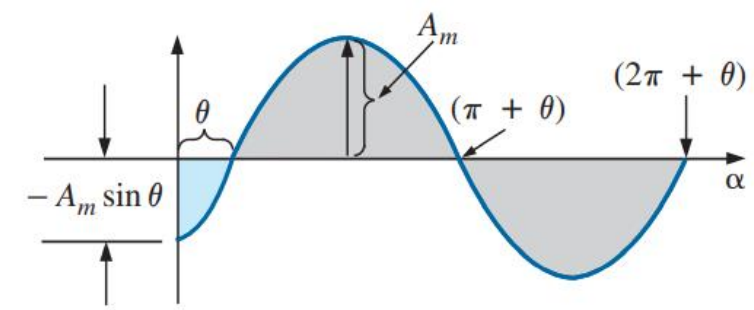


FIG. 28

Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0° .

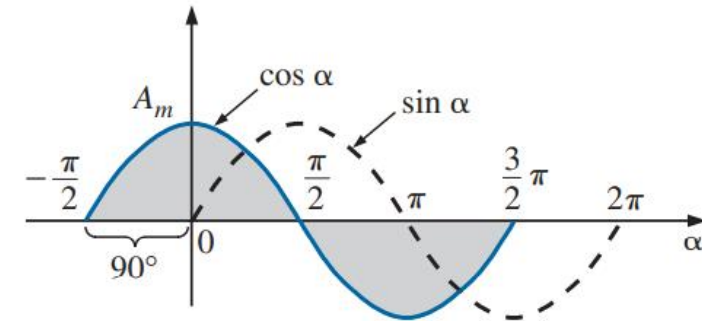


FIG. 29

Phase relationship between a sine wave and a cosine wave.

The geometric relationship between various forms of the sine and cosine functions can be derived from Fig. 30. For instance, starting at the $+\sin \alpha$ position, we find that $+\cos \alpha$ is an additional 90° in the counterclockwise direction. Therefore, $\cos \alpha = \sin(\alpha + 90^\circ)$. For $-\sin \alpha$ we must travel 180° in the counterclockwise (or clockwise) direction so that $-\sin \alpha = \sin(\alpha \pm 180^\circ)$, and so on, as listed below:

Graphic
sl

$$\begin{aligned}\cos \alpha &= \sin(\alpha + 90^\circ) \\ \sin \alpha &= \cos(\alpha - 90^\circ) \\ -\sin \alpha &= \sin(\alpha \pm 180^\circ) \\ -\cos \alpha &= \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ) \\ &\text{etc.}\end{aligned}\tag{22}$$

In addition, note that

$$\begin{aligned}\sin(-\alpha) &= -\sin \alpha \\ \cos(-\alpha) &= \cos \alpha\end{aligned}\tag{23}$$

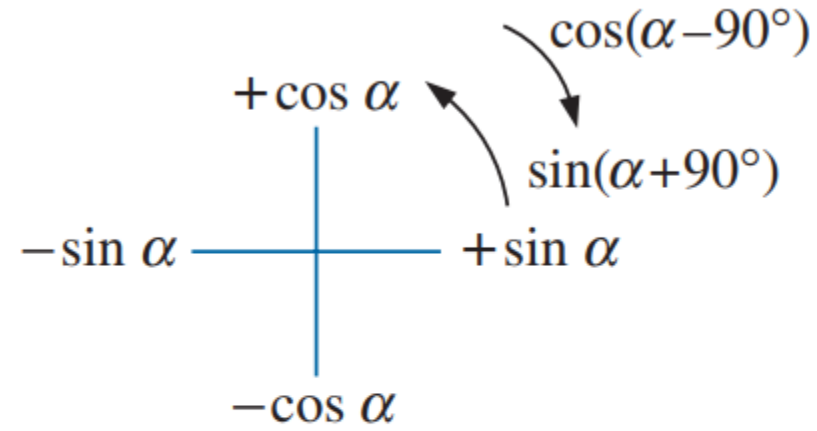


FIG. 30

Graphic tool for finding the relationship between specific sine and cosine functions.

If a sinusoidal expression appears as

$$e = -E_m \sin \omega t$$

the negative sign is associated with the sine portion of the expression, not the peak value E_m . In other words, the expression, if not for convenience, would be written

$$e = E_m(-\sin \omega t)$$

Since $-\sin \omega t = \sin(\omega t \pm 180^\circ)$

the expression can also be written

$$e = E_m \sin(\omega t \pm 180^\circ)$$

revealing that a negative sign can be replaced by a 180° change in phase angle (+ or -); that is,

$$e = -E_m \sin \omega t = E_m \sin(\omega t + 180^\circ) = E_m \sin(\omega t - 180^\circ)$$

A plot of each will clearly show their equivalence. There are, therefore, two correct mathematical representations for the functions.

The **phase relationship** between two waveforms indicates which one leads or lags the other and by how many degrees or radians.

Example 12

EXAMPLE 12 What is the phase relationship between the sinusoidal waveforms of each of the following sets?

- a. $v = 10 \sin(\omega t + 30^\circ)$
 $i = 5 \sin(\omega t + 70^\circ)$
- b. $i = 15 \sin(\omega t + 60^\circ)$
 $v = 10 \sin(\omega t - 20^\circ)$
- c. $i = 2 \cos(\omega t + 10^\circ)$
 $v = 3 \sin(\omega t - 10^\circ)$
- d. $i = -\sin(\omega t + 30^\circ)$
 $v = 2 \sin(\omega t + 10^\circ)$
- e. $i = -2 \cos(\omega t - 60^\circ)$
 $v = 3 \sin(\omega t - 150^\circ)$

solution

Solutions:

a. See Fig. 31.

***i* leads *v* by 40° , or *v* lags *i* by 40° .**

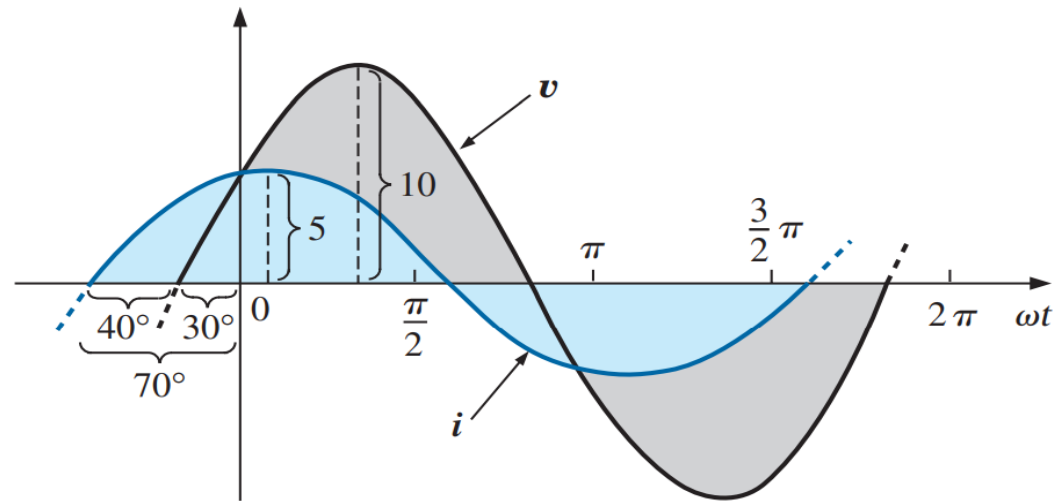


FIG. 31

Example 12(a): i leads v by 40° .

b. See Fig. 32. i leads Y by 80° , or Y lags i by 80°

continue

b. See Fig. 32.

***i* leads *v* by 80°, or *v* lags *i* by 80°.**

c. See Fig. 33.

$$\begin{aligned}i &= 2 \cos(\omega t + 10^\circ) = 2 \sin(\omega t + 10^\circ + 90^\circ) \\&= 2 \sin(\omega t + 100^\circ)\end{aligned}$$

***i* leads *v* by 110°, or *v* lags *i* by 110°.**

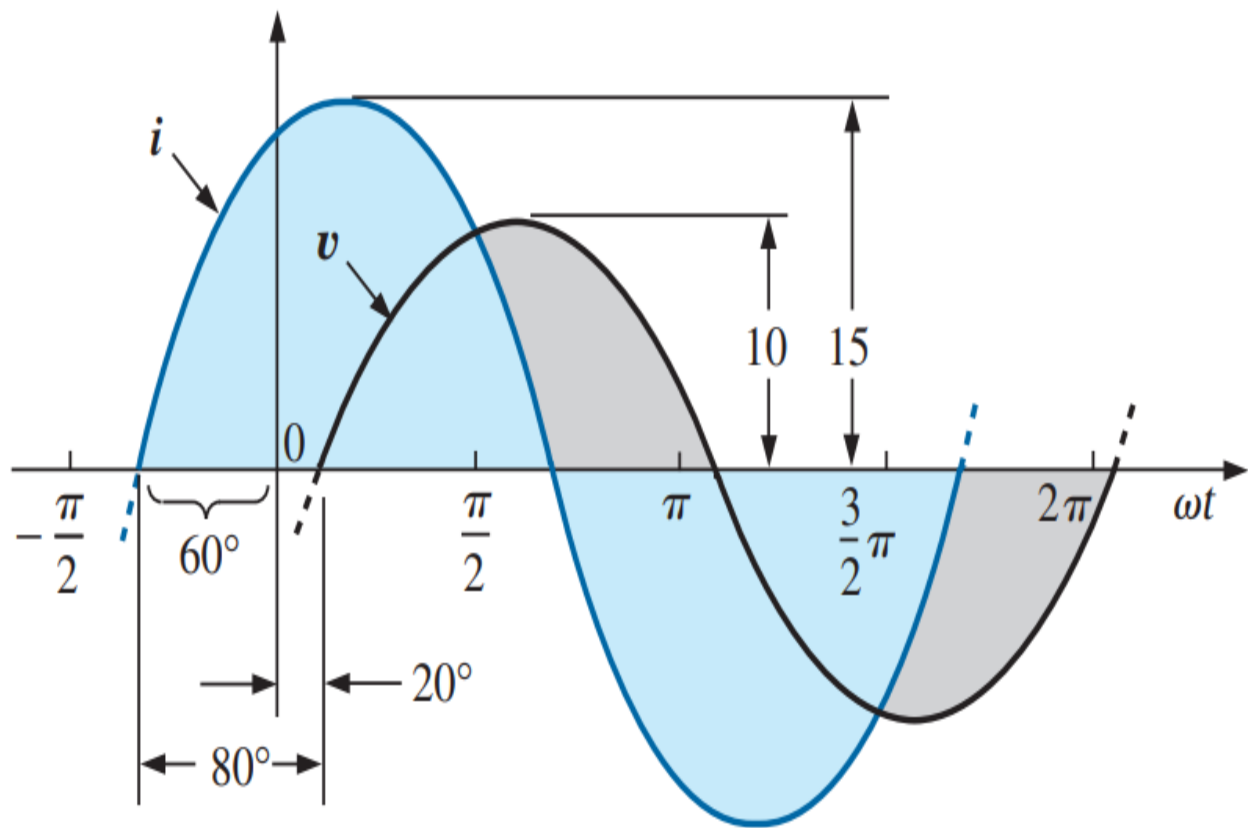


FIG. 32

Example 12(b): i leads v by 80° .

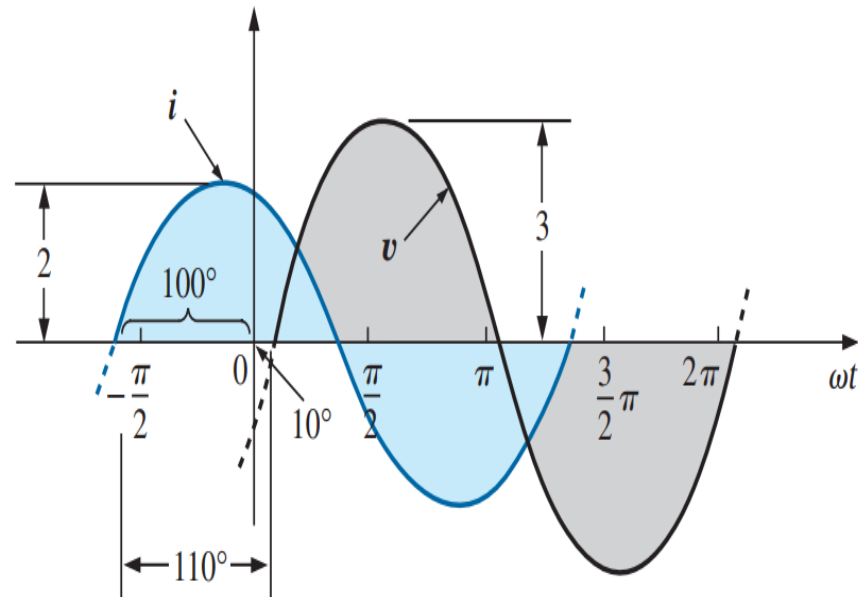


FIG. 33

Example 12(c): i leads v by 110° .

continue

d. See Fig. 34.

Note

$$\begin{aligned}-\sin(\omega t + 30^\circ) &= \sin(\omega t + 30^\circ - 180^\circ) \\ &= \sin(\omega t - 150^\circ)\end{aligned}$$

***v* leads *i* by 160°, or *i* lags *v* by 160°.**

Or using

Note

$$\begin{aligned}-\sin(\omega t + 30^\circ) &= \sin(\omega t + 30^\circ + 180^\circ) \\ &= \sin(\omega t + 210^\circ)\end{aligned}$$

***i* leads *v* by 200°, or *v* lags *i* by 200°.**

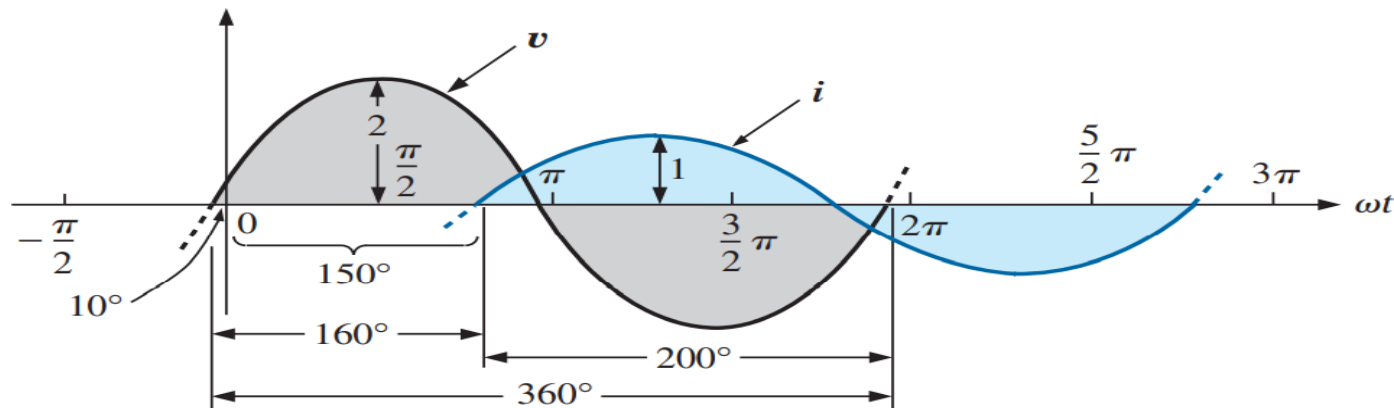


FIG. 34

Example 12(d): v leads i by 160°.

e. See Fig. 35.

By choice

$$\begin{aligned} i &= -2 \cos(\omega t - 60^\circ) = 2 \cos(\omega t - 60^\circ - 180^\circ) \\ &= 2 \cos(\omega t - 240^\circ) \end{aligned}$$

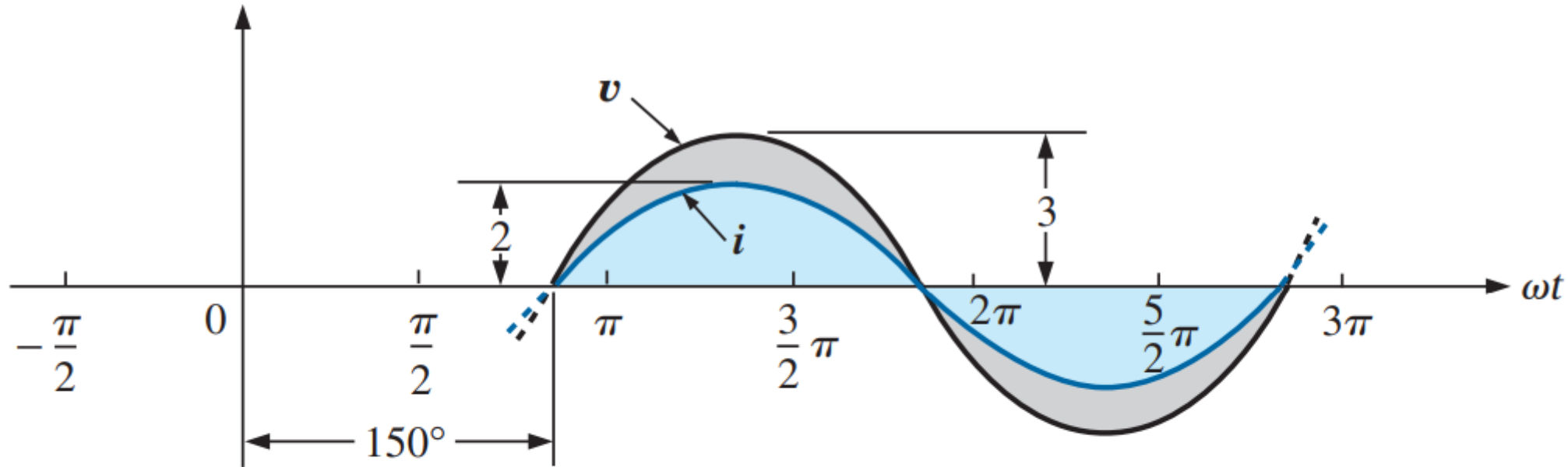


FIG. 35

Example 12(e): v and i are in phase.

continue

However, $\cos \alpha = \sin(\alpha + 90^\circ)$

so that
$$\begin{aligned} 2 \cos(\omega t - 240^\circ) &= 2 \sin(\omega t - 240^\circ + 90^\circ) \\ &= 2 \sin(\omega t - 150^\circ) \end{aligned}$$

***v* and *i* are in phase.**