

NETWORK - THEOREMS

SUPERPOSITION THEOREM

INTRODUCTION

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The **superposition theorem** is unquestionably one of the most powerful in this field.

It has such widespread application that people often apply it without recognizing that their maneuvers are valid only because of this theorem.

Superposition Theorem

SUPERPOSITION THEOREM

In general, the theorem can be used to do the following:

- Analyze networks such as introduced in the last chapter that have two or more sources that are not in series or parallel.
- Reveal the effect of each source on a particular quantity of interest.



State of theorem

The superposition theorem states the following:

The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.

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- Analyze networks such as introduced in the last chapter that have two or more sources that are not in series or parallel.
- Reveal the effect of each source on a particular quantity of interest.

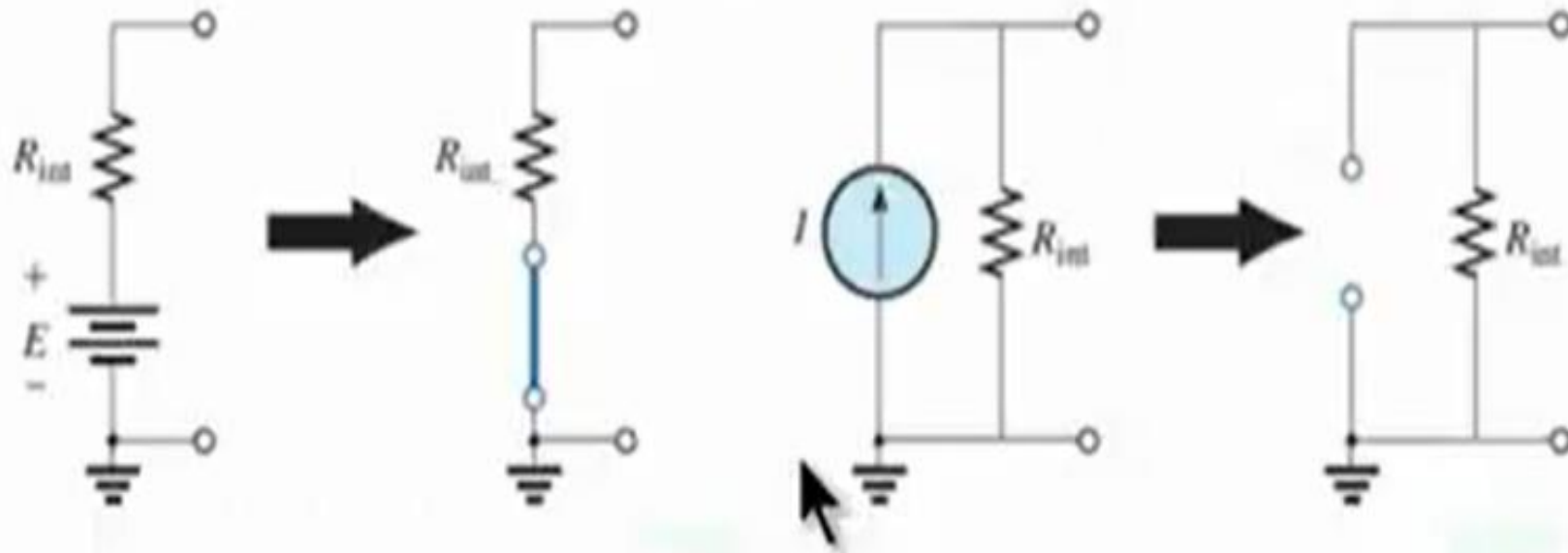
Procedure

SUPERPOSITION THEOREM PROCEDURE

- when removing a voltage source from a network schematic, replace it with a direct connection (**short circuit**) of zero ohms. Any internal resistance associated with the source must remain in the network.
- when removing a current source from a network schematic, replace it by an **open circuit** of infinite ohms. Any internal resistance associated with the source must remain in the network.

Illustration

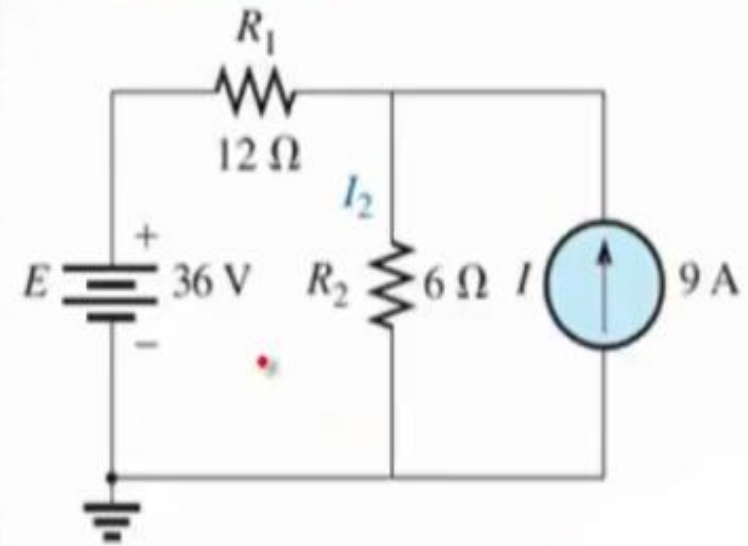
The previous statements are illustrated in Fig shown below



Example

EXAMPLE

Using the superposition theorem, determine the current through resistor R_2 for the network in Fig.



Solutions:

In order to determine the effect of the 36 V voltage source, the current source must be replaced by an open-circuit equivalent.

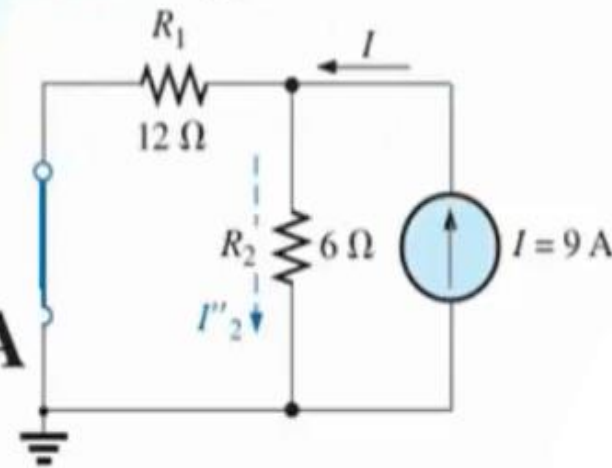
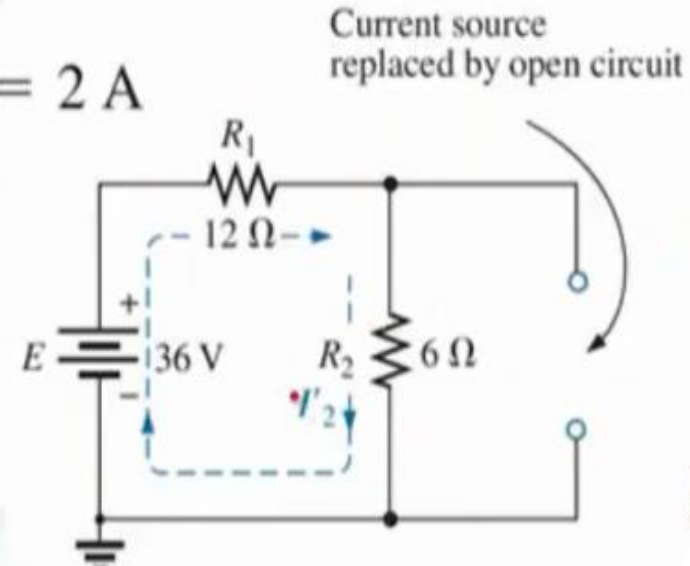
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$$I'_2 = \frac{E}{R_T} = \frac{E}{R_1 + R_2} = \frac{36 \text{ V}}{12 \Omega + 6 \Omega} = \frac{36 \text{ V}}{18 \Omega} = 2 \text{ A}$$

$$I''_2 = \frac{R_1(I)}{R_1 + R_2} = \frac{(12 \Omega)(9 \text{ A})}{12 \Omega + 6 \Omega} = 6 \text{ A}$$

The total solution for current I_2 is the sum of the currents established by the two sources. That is,

$$I_2 = I'_2 + I''_2 = 2 \text{ A} + 6 \text{ A} = \mathbf{8 \text{ A}}$$

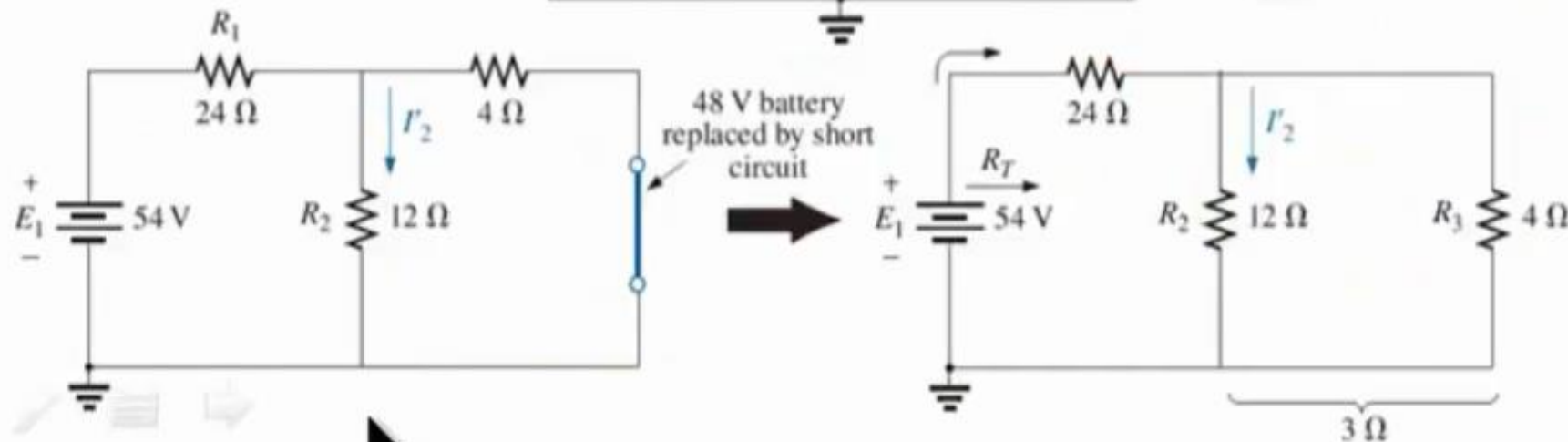
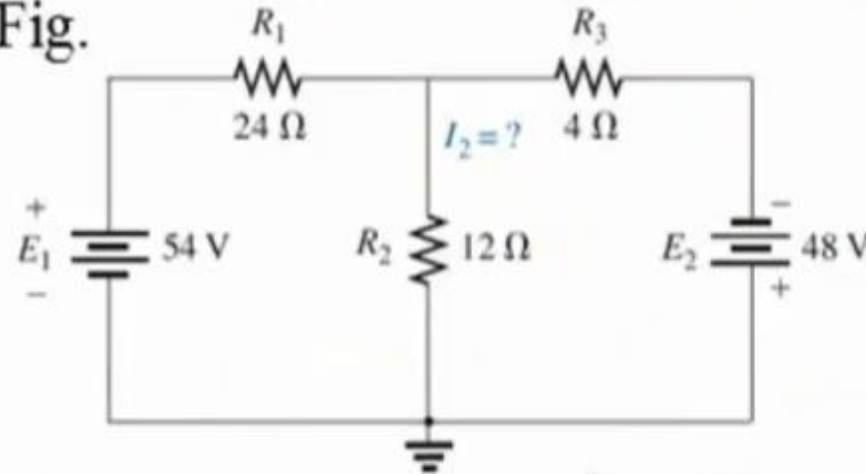


Example

EXAMPLE

Using the superposition theorem, determine the current through the $12\ \Omega$ resistor in Fig.

Solutions:



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Therefore the total resistance seen by 54 V source is

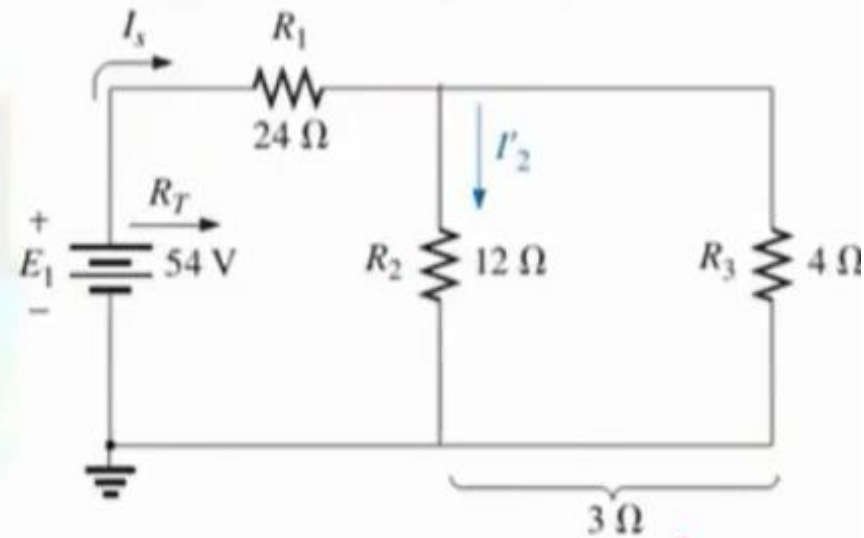
$$R_T = R_1 + R_2 \parallel R_3 = 24 \, \Omega + 12 \, \Omega \parallel 4 \, \Omega = 24 \, \Omega + 3 \, \Omega = 27 \, \Omega$$

and the source current is

$$I_s = \frac{E_1}{R_T} = \frac{54 \, \text{V}}{27 \, \Omega} = 2 \, \text{A}$$

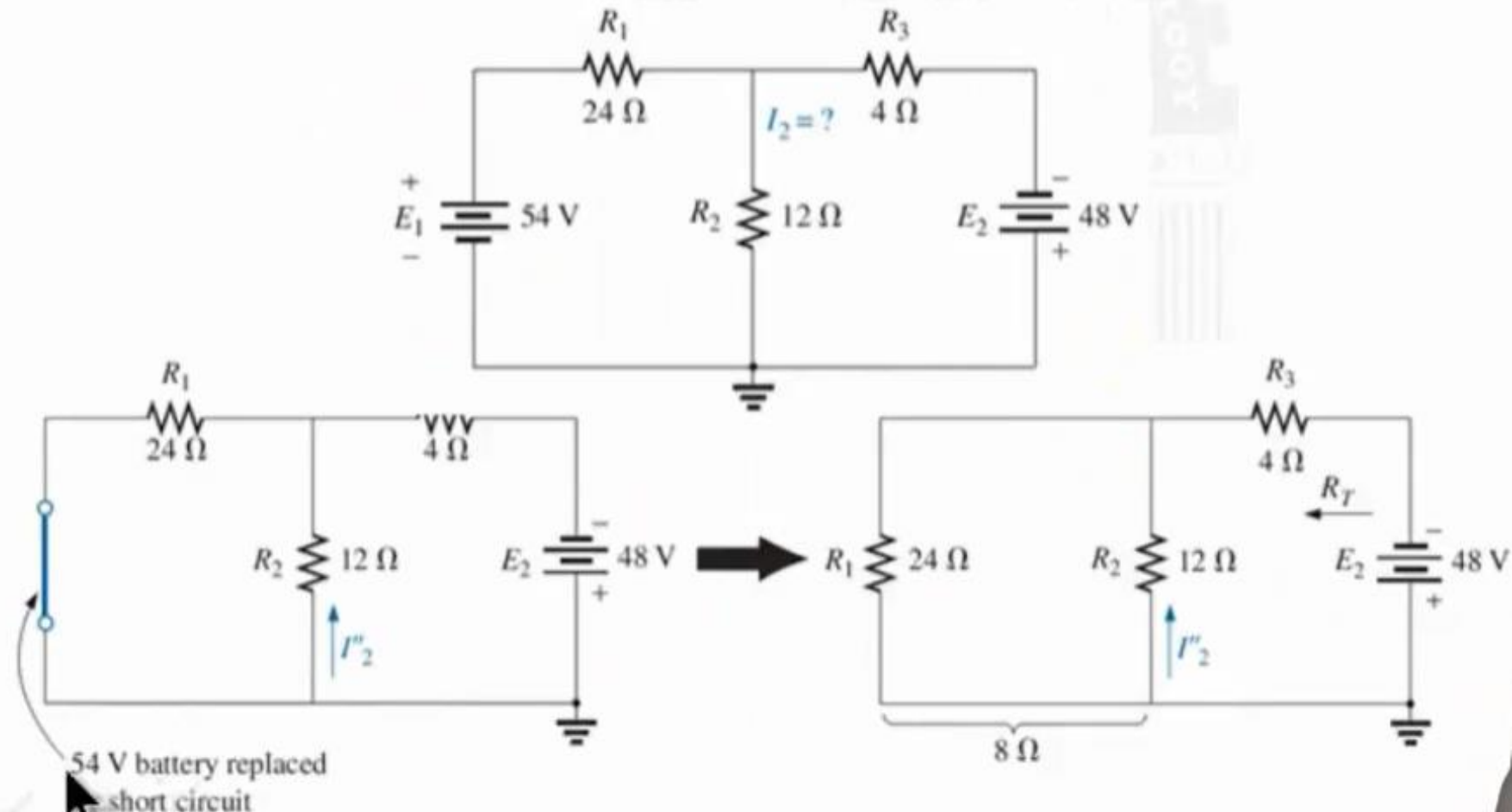
Using the current divider rule

$$I'_2 = \frac{R_3 I_s}{R_3 + R_2} = \frac{(4 \, \Omega)(2 \, \text{A})}{4 \, \Omega + 12 \, \Omega} = 0.5 \, \text{A}$$



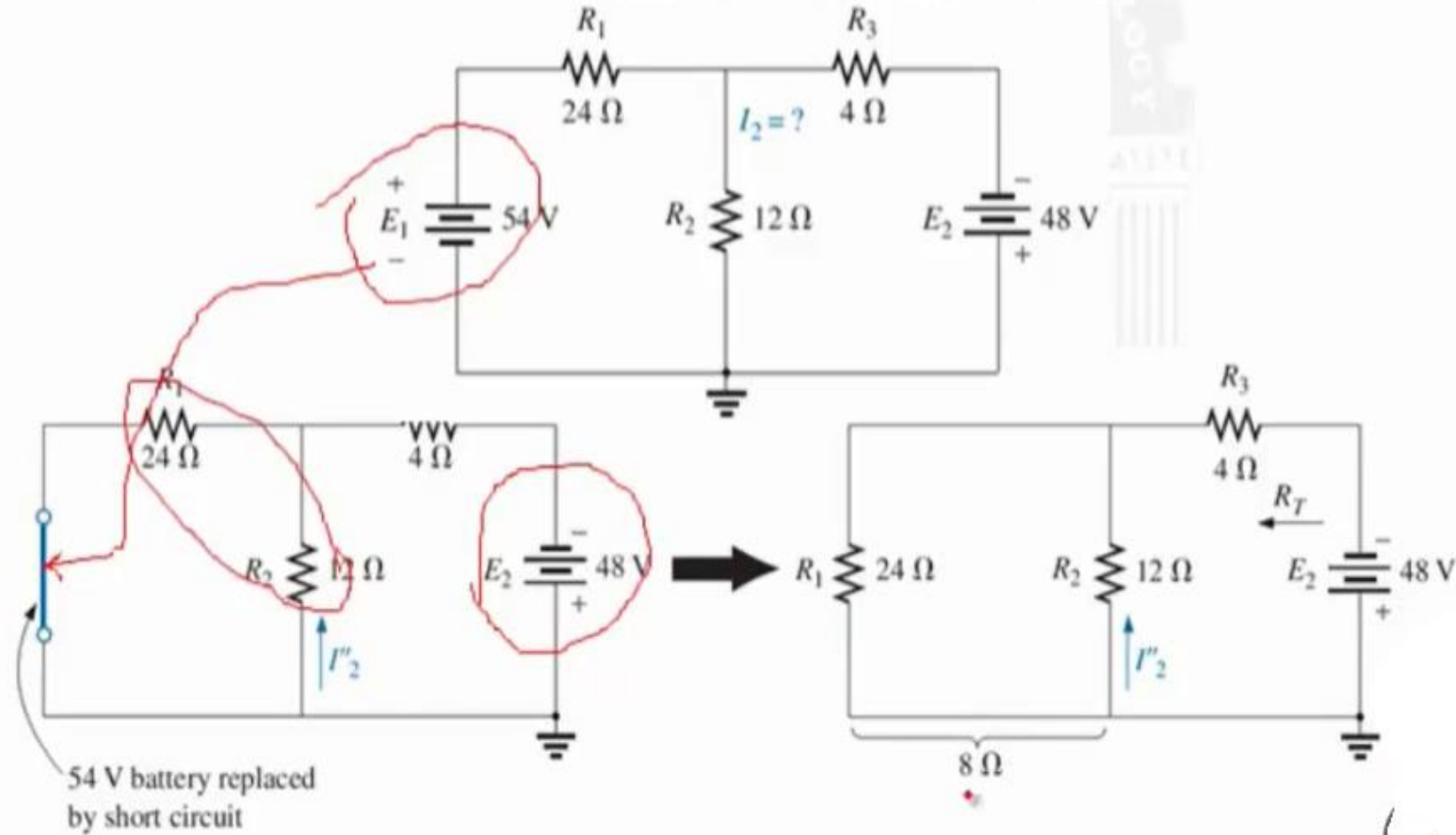
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we now replace the 54 V source by a short-circuit equivalent.



continue

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continue

Therefore the total resistance seen by 48 V source is

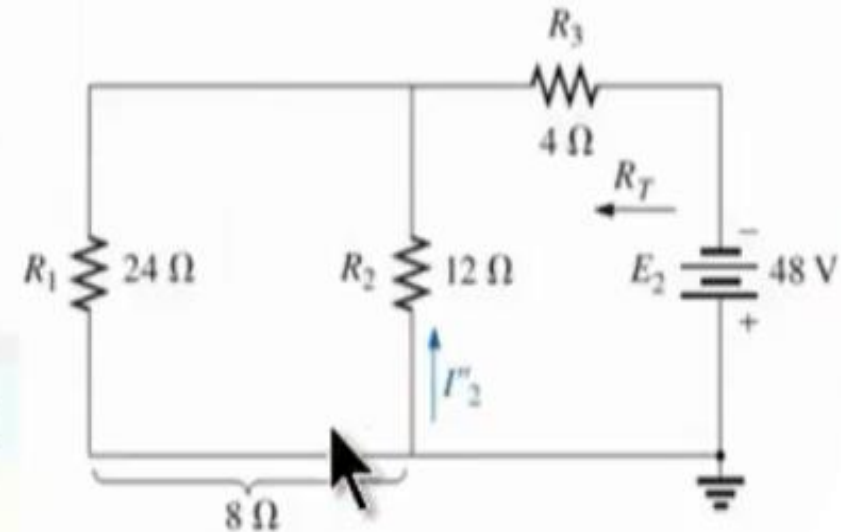
$$R_T = R_3 + R_2 \parallel R_1 = 4 \, \Omega + 12 \, \Omega \parallel 24 \, \Omega = 4 \, \Omega + 8 \, \Omega = 12 \, \Omega$$

and the source current is

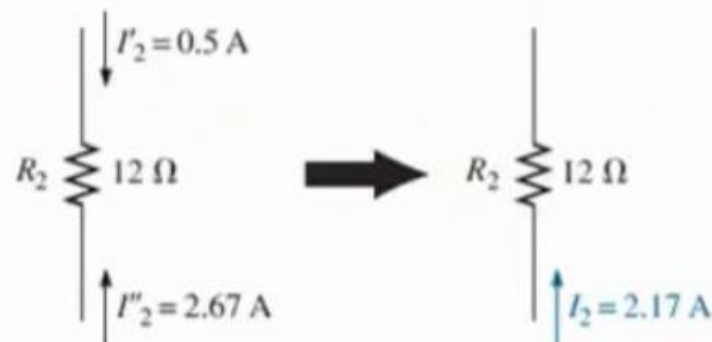
$$I_s = \frac{E_2}{R_T} = \frac{48 \, \text{V}}{12 \, \Omega} = 4 \, \text{A}$$

Applying the current divider rule results in

$$I''_2 = \frac{R_1(I_s)}{R_1 + R_2} = \frac{(24 \, \Omega)(4 \, \text{A})}{24 \, \Omega + 12 \, \Omega} = 2.67 \, \text{A}$$



It is now important to realize that current I_2 due to each source has a different direction, as shown in Fig. The net current therefore is the difference of the two and in the direction of the larger as follows:



$$I_2 = I''_2 - I'_2 = 2.67\text{ A} - 0.5\text{ A} = \mathbf{2.17\text{ A}}$$

Example

EXAMPLE

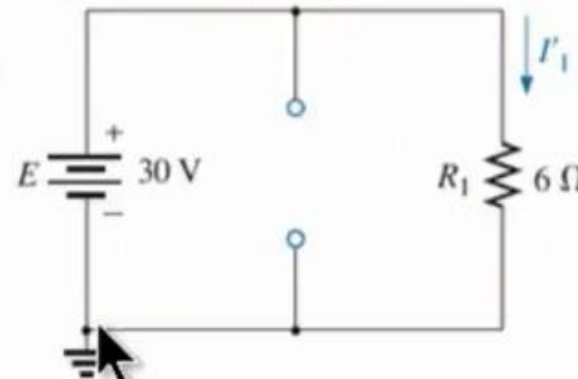
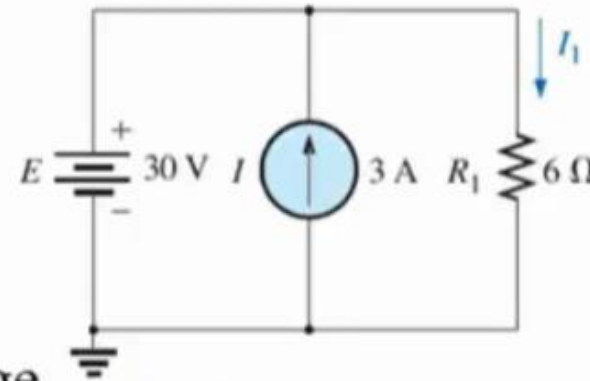
Using the superposition theorem, determine current I_1 for the network in Fig.

Solutions:

First let us determine the effects of the voltage source by setting the current source to zero amperes (open circuit) as shown in Fig below,

Therefore :

$$I'_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{30 \text{ V}}{6 \Omega} = 5 \text{ A}$$



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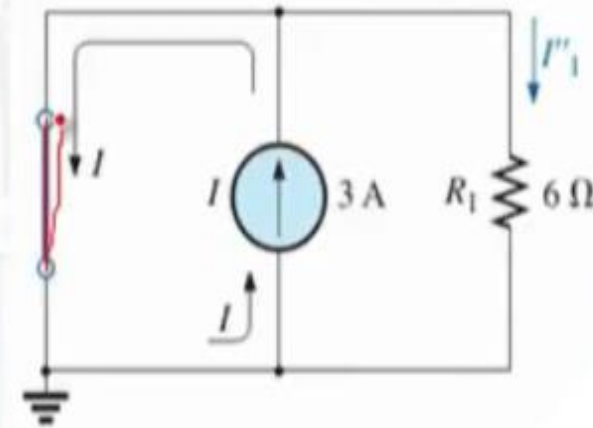
Now, setting the voltage source to zero volts results (short circuit) in the network in Fig.

Since the source current takes the path of least resistance, it chooses the zero ohm path of the inserted short-circuit equivalent, and the current through R_1 is zero amperes.

$$I''_1 = \frac{R_{sc}I}{R_{sc} + R_1} = \frac{(0\ \Omega)I}{0\ \Omega + 6\ \Omega} = 0\ \text{A}$$

The total current is defined by

$$I_1 = I'_1 + I''_1 = 5\ \text{A} + 0\ \text{A} = \mathbf{5\ \text{A}}$$



CONCLUSION

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This lecture introduced the superposition theorem that has application throughout the field of electricity and electronics. Not only can it be used to solve networks such as those previously encountered, but it also provide an opportunity to determine the impact of a particular source or element on the response of the entire system.