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*Lecture 1

Number System

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* Decimal Numbers

In the decimal number system, each of the ten digits (0 through 9) represents a certain quantity.

The position of each digit in a decimal number indicates the magnitude of the quantity represented and can be assigned a weight. The weights for whole numbers are positive powers of ten that increase from right to left, beginning with $10^0 = 1$.

$$\dots 10^5 10^4 10^3 10^2 10^1 10^0$$

For fractional numbers, the weights are negative powers of ten that decrease from left to right beginning with 10^{-1} .

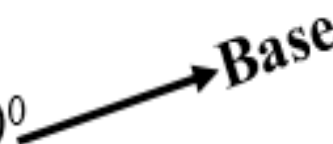
$$10^2 10^1 10^0 . 10^{-1} 10^{-2} 10^{-3} \dots$$

↑
Decimal point

EXAMPLE 1: Express the decimal number 47 as a sum of the values of each digit.

Solution:

Decimal Number:	4	7
Decimal Weight:	10^1	10^0



$$\begin{aligned} 47 &= (4 \times 10^1) + (7 \times 10^0) \\ &= (4 \times 10) + (7 \times 1) = \mathbf{40 + 7} \end{aligned}$$

Note: The digit 4 has a weight of 10, which is 10^1 , as indicated by its position. The digit 7 has a weight of 1, which is 10^0 , as indicated by its position.

EXAMPLE 2: Express the decimal number 568.23 as a sum of the values of each digit.

Solution:

Decimal Number:	5	6	8	.	2	3
Decimal Weight:	10^2	10^1	10^0	.	10^{-1}	10^{-2}

$$\begin{aligned}\mathbf{568.23} &= (5 \times 10^2) + (6 \times 10^1) + (8 \times 10^0) + (2 \times 10^{-1}) + (3 \times 10^{-2}) \\ &= (5 \times 100) + (6 \times 10) + (8 \times 1) + (2 \times 0.1) + (3 \times 0.01) \\ &= \mathbf{500 + 60 + 8 + 0.2 + 0.03}\end{aligned}$$

Note: The whole number digit 5 has a weight of 100, which is 10^2 , the digit 6 has a weight of 10, which is 10^1 , the digit 8 has a weight of 1, which is 10^0 , the fractional digit 2 has a weight of 0.1, which is 10^{-1} , and the fractional digit 3 has a weight of 0.01, which is 10^{-2} .

* Binary Numbers

- * The binary number system is another way to represent quantities. It is less complicated than the decimal system because the binary system has only two digits. The two binary digits (bits) are **1** and **0**.

The Weighting Structure of Binary Numbers:

- * A binary number is a weighted number. The right-most bit is the **LSB** (least significant bit) in a binary whole number. The weights increase from right to left by a power of two for each bit. The left-most bit is the **MSB** (most significant bit); its weight depends on the size of the binary number.

*Decimal-to-Binary Conversion

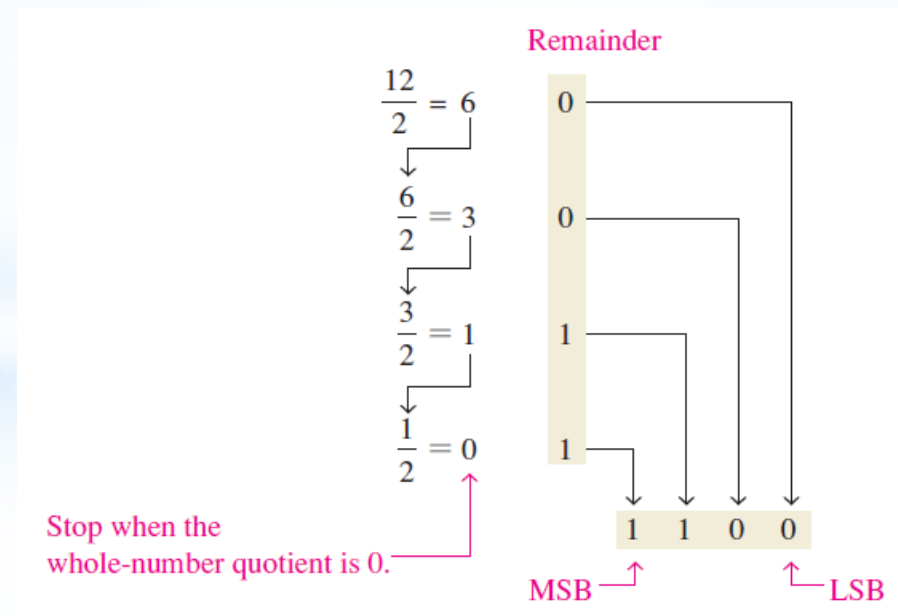
A. Sum-of-Weights Method:

- * One way to find the binary number that is equivalent to a given decimal number is to determine the set of binary weights whose sum is equal to the decimal number.

Binary Weight:	2^3	2^2	2^1	2^0
	8	4	2	1
Binary Number:	1	1	0	0
	↓			↓
	MSB			LSB
$\therefore (12)_{10} = (1100)_2$				

B. Repeated Division-by-2 Method:

* A systematic method of converting whole numbers from decimal to binary is the repeated division-by-2 process until there is a 0 whole-number quotient. The remainders generated by each division form the binary number. The first remainder to be produced is the LSB, and the last remainder to be produced is the MSB.



*Decimal Fractions-to-Binary Conversion

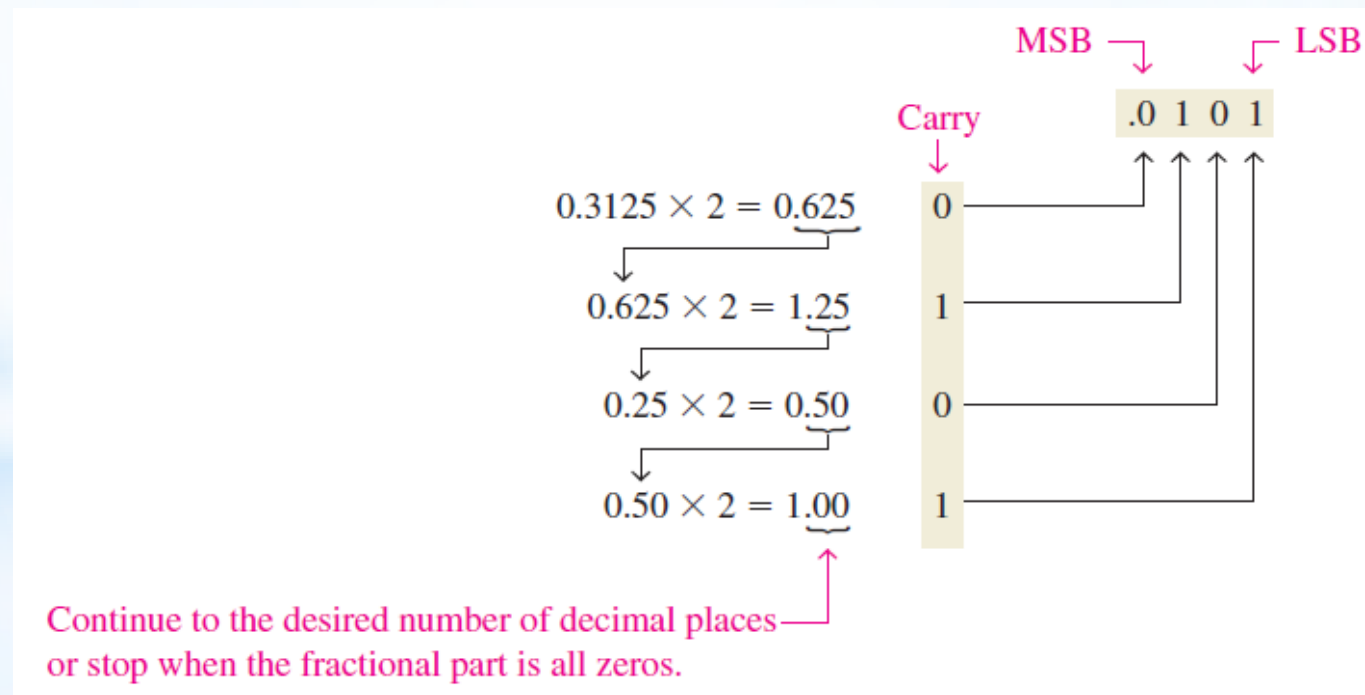
A. Sum-of-Weights Method:

*The sum-of-weights method can be applied to fractional decimal numbers, as shown in the following example:

Binary Weight:	0	.	2^{-1}	2^{-2}	2^{-3}
		.	0.5	0.25	0.125
Binary Number:	0	.	1	0	1
$\therefore (0.625)_{10} = (0.101)_2$					

B. Repeated Multiplication by 2 Method:

- * Decimal fractions can be converted to binary by repeated multiplication by 2. The carry digits, or carries, generated by the multiplications produce the binary number. The first carry produced is the MSB, and the last carry is the LSB.



*Binary-to-Decimal Conversion

- * The decimal value of any binary number can be found by adding the weights of all bits that are 1 and discarding the weights of all bits that are 0.**

Binary Weight:	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	64	32	16	8	4	2	1
Binary Number:	1	1	0	1	1	0	1
	↓						↓
	MSB						LSB

$1101101 = 2^6 + 2^5 + 2^3 + 2^2 + 2^0$
 $= 64 + 32 + 8 + 4 + 1 = 109$
 $\therefore (1101101)_2 = (109)_{10}$

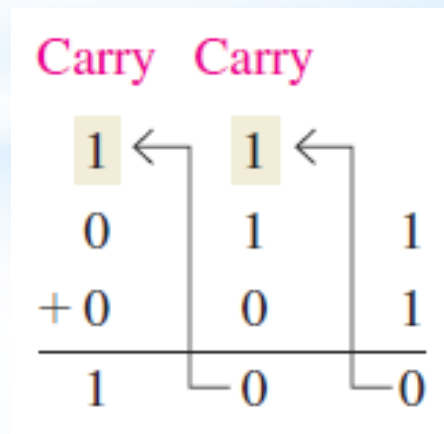
*Binary Arithmetic

- * Binary arithmetic is essential in all digital computers and in many other types of digital systems. To understand digital systems, you must know the basics of binary addition, subtraction, multiplication, and division.

A. Binary Addition:

- * The four basic rules for adding binary digits (bits) are as follows:

$0 + 0 = 0$	Sum of 0 with a carry of 0
$0 + 1 = 1$	Sum of 1 with a carry of 0
$1 + 0 = 1$	Sum of 1 with a carry of 0
$1 + 1 = 10$	Sum of 0 with a carry of 1



* B. Binary Subtraction:

* The four basic rules for subtracting bits are as follows:

$$\begin{array}{l} 0 - 0 = 0 \\ 1 - 1 = 0 \\ 1 - 0 = 1 \\ 10 - 1 = 1 \quad 0 - 1 \text{ with a borrow of } 1 \end{array}$$

Left column:
When a 1 is borrowed, a 0 is left, so $0 - 0 = 0$.

Middle column:
Borrow 1 from next column to the left, making a 10 in this column, then $10 - 1 = 1$.

Right column:
 $1 - 1 = 0$

$$\begin{array}{r} 0101 \\ -011 \\ \hline 010 \end{array}$$

C. Binary Multiplication:

* The four basic rules for multiplying bits are as follows:

$$\begin{array}{l} 0 \times 0 = 0 \\ 0 \times 1 = 0 \\ 1 \times 0 = 0 \\ 1 \times 1 = 1 \end{array}$$

Partial products

$$\begin{array}{r} 11 \\ \times 11 \\ \hline 11 \\ +11 \\ \hline 1001 \end{array} \quad \begin{array}{r} 3 \\ \times 3 \\ \hline 9 \end{array}$$

Partial products

$$\begin{array}{r} 111 \\ \times 101 \\ \hline 111 \\ 000 \\ +111 \\ \hline 100011 \end{array} \quad \begin{array}{r} 7 \\ \times 5 \\ \hline 35 \end{array}$$

* D. Binary Division:

- * Division in binary follows the same procedure as division in decimal.

$$\begin{array}{r} 10 \\ 11 \overline{)110} \\ \underline{11} \\ 000 \end{array}$$

$$\begin{array}{r} 2 \\ 3 \overline{)6} \\ \underline{6} \\ 0 \end{array}$$

$$\begin{array}{r} 11 \\ 10 \overline{)110} \\ \underline{10} \\ 10 \\ \underline{10} \\ 00 \end{array}$$

$$\begin{array}{r} 3 \\ 2 \overline{)6} \\ \underline{6} \\ 0 \end{array}$$